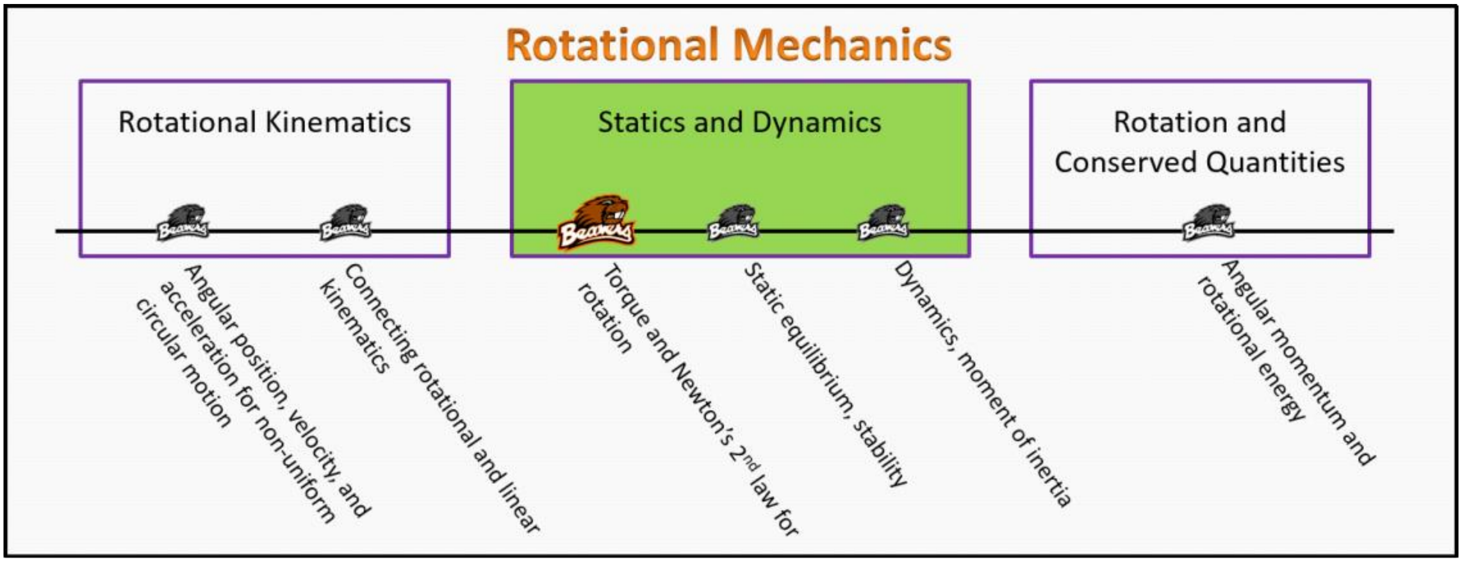
Solutions

Friday, January 11, 2019 10:02 AM

Statics and Dynamics
Foundation Stage (SD.2.L1)

lecture 1
Torque and Newton's 2nd law for rotation



Textbook Chapters (* Calculus version)

- **BoxSand** :: KC videos (statics & dynamics)
- **Knight** (College Physics : A strategic approach 3rd) :: 7.3
- ***Knight** (Physics for Scientists and Engineers 4th) :: 12.5
- **Giancoli** (Physics Principles with Applications 7th) :: 8-4

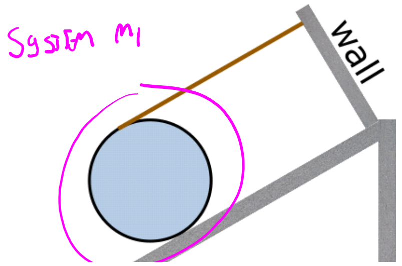
Warm up

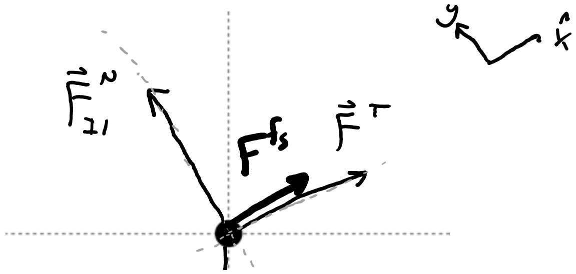
SD.2.L1-1:

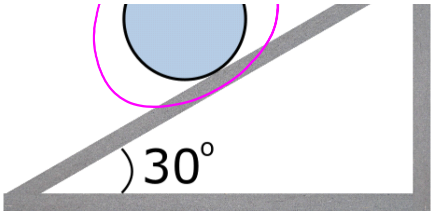
Description: Draw a FBD.

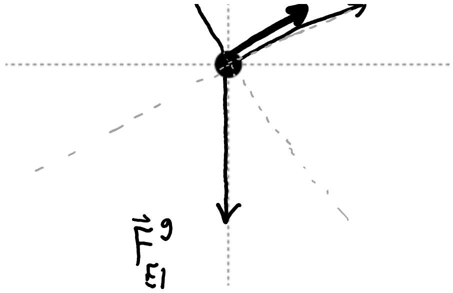
Learning Objectives: [?] - Can you identify the objectives from the previous lecture, and this lecture, that this question is relevant to?

Problem Statement: A disk with uniform mass distribution sits in static equilibrium on an incline with the aid of a rope as shown in the figure below. Draw a FBD for the disk.









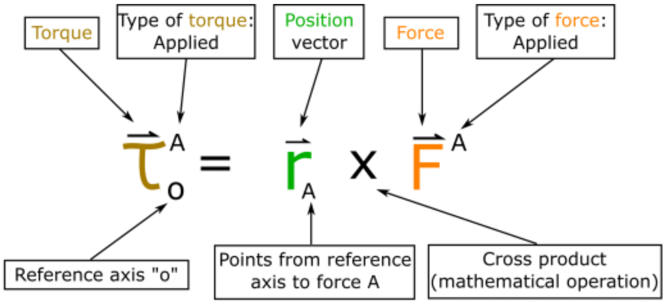
Selected Learning Objectives

1. Coming soon to a lecture template near you.

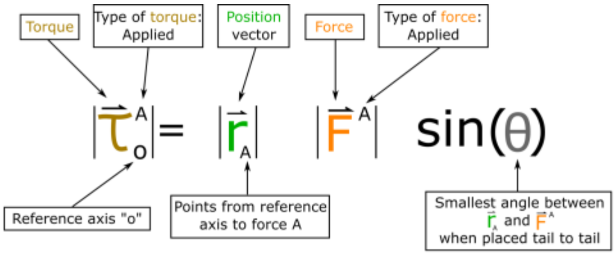
Key Terms

- Reference axis
- Cross product
- Torque
- Extended Free Body Diagram (e-FBD)

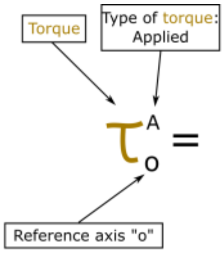
Key Equations



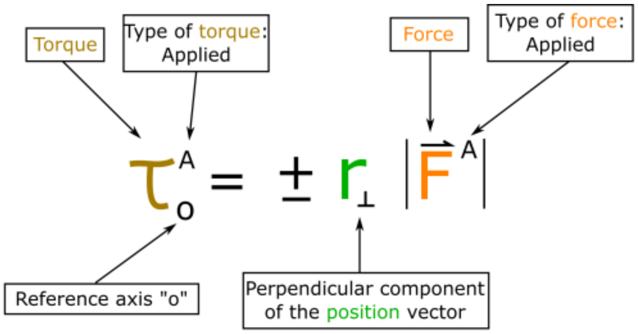
In words: The torque from the applied force about reference axis "o" is equal to the cross product between the position vector that points from "o" to the applied force with the applied force.



In words: The magnitude of torque from the applied force about reference axis "o" is equal to the magnitude of the position vector that points from "o" to the applied force times the magnitude of the applied force times the sin of the smallest angle between the two vectors when placed tail to tail.

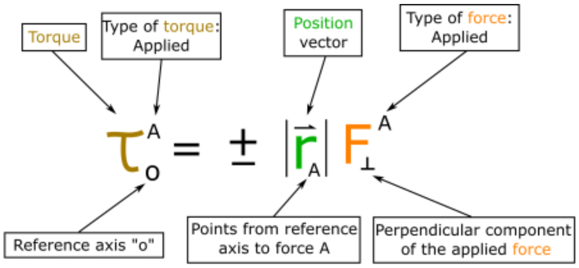


In words: The component of torque from the applied force about reference axis "o" is equal to plus or minus the magnitude of the torque. The plus or minus is determined by the desired rotation about "o": ccw(+), cw(-).



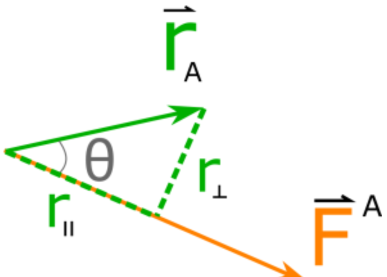
$$\tau_o^A = \pm\, r_\perp \left|\vec{F}^A\right|$$

In words: The component of torque from the applied force about reference axis "o" is equal to plus or minus the perpendicular component of the position vector with respect the the applied force times the magnitude of the applied force.

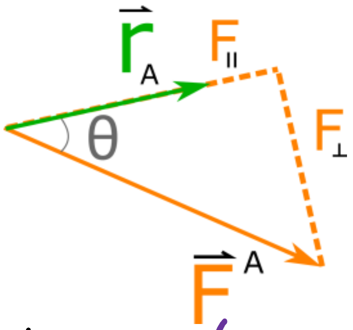


$$\tau_o^A = \pm \left|\vec{r}_A\right| F_\perp^A$$

In words: The component of torque from the applied force about reference axis "o" is equal to plus or minus the magnitude of the position vector that points from "o" to the applied force times the perpendicular component of the applied force with respect to the position vector.



$$|\tau| = (|r|\sin\theta)\,|F|$$



$$|\tau| = |r|\,(|F|\sin\theta)$$

Key Concepts

- A reference axis is analogous to an origin of a coordinate system. You can place the reference axis anywhere you like. A reference axis differs from an origin because it is an axis, while the origin is just a point. Imagine a thin cylindrical rod as the reference axis, thus this rod (reference axis) can be oriented in different directions. Remember that the reference axis does not need to be at the location where an object is rotating about, you can choose your reference axis location and orientation anywhere you'd like.
- The cross product is a mathematical operation that takes in two vectors, and returns a vector. Conceptually, you can think of the cross product as asking, "how perpendicular two vectors are to each other". The more perpendicular the two vectors are, the larger the magnitude of the cross product, reaching its maximum magnitude when the two vectors are perpendicular, and is zero when the two vectors are parallel.
- Torque is a vector quantity found by taking the cross product between the position vector from a reference axis and the force applied to an object. The magnitude of torque depends on the magnitude of the applied force, the magnitude of the position vector, and the angle between the two.
- The vector nature of torque is not covered in this template. We will only be working with a component of torque about a reference axis. Recall that component of vectors can be positive or negative. To determine if the component of torque is positive or negative use the following convention: if the force is trying to rotate the object ccw about the reference axis, then the torque from this force is positive ; if the force is trying to rotate the object cw about the reference axis, then the torque from this force is negative.
- An extended free body diagram is a tool to help organize information when a torque analysis is required. The e-FBD should include an outline of the system, the reference axis, all of the forces acting on the system with their tails at the locations they act, and the position vectors from the reference axis to all of the forces.

Questions

Act I: Torque

SD.2.L1-2:

Description: Determine dimensions of torque. Determine which physical quantity has same dimensions as torque. Compare quantities with same dimensions. (2 minute + 2 minutes + 1 minute)

Learning Objectives: [1, 12, 13]

Problem Statement: Recall that the magnitude of torque is: $|\vec{\tau}_o| = |\vec{r}_o|\,|\vec{F}|\,\sin(\theta)$.

(a) Which of the following are the correct dimensions for torque?

(1) N m

(2) $\frac{kg\,m}{s}$

(3) $\frac{kg\,m^2}{s^2}$

(4) $\frac{[M]^2\,[L]}{[T]^2}$

(5) $\frac{[M]^2\,[L]^2}{[T]}$

(6) $\frac{[M]\,[L]}{[T]^2}$

(7) $\frac{[M]\,[L]}{[T]}$

[✓ (8)] $\frac{[M]\,[L]^2}{[T]^2}$

$[\vec{r}] = [L]$

$[\vec{F}] = \frac{[M][L]}{[T]^2}$

$[\sin\theta]$ = DIMENSIONLESS

$\therefore$

$[\vec{\tau}] = \frac{[M][L]^2}{[T]^2}$

SI UNITS

$\frac{kg\,m^2}{s^2} = Nm$

(b) Which of the following physical quantities have the same dimensions as torque?

F (1) Momentum

T [✓ (2)] Energy SCALAR

T [✓ (3)] Work SCALAR

F (4) Impulse

F (5) Force

F (6) Displacement

F (7) Velocity

F (8) Acceleration

T [✓ (9)] Heat SCALAR

(c) What is the difference between torque and your answer(s) to part b?

F (1) Torque is related to rotational motion and part b answer(s) are only related to translational motion.

F (2) Torque is a scalar and part b answer(s) are vectors.

T [✓ (3)] Torque is a vector and part b answer(s) are scalars.

SD.2.L1-3:

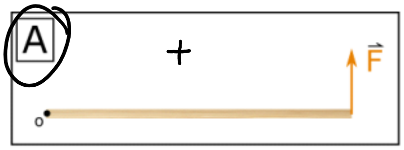
Description: Ranking torque. (3 minutes + 3 minutes)

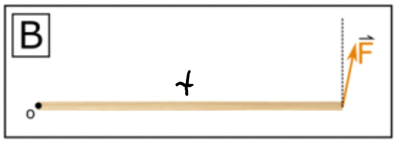
Learning Objectives: [1, 12, 13]

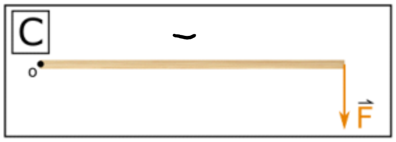
Problem Statement: Consider a block of wood being pushed/pulled on by a force as shown in the images below. All forces have the same magnitude and the images are drawn to scale.

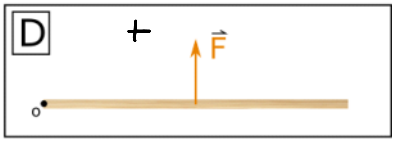
(a) In which situation is the torque produced by the force about axis **o** the most positive?

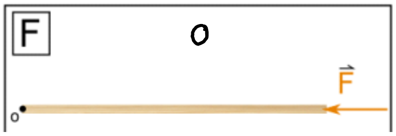
$|\vec{\tau}_o| = |\vec{r}|\,|\vec{F}|\sin\theta$





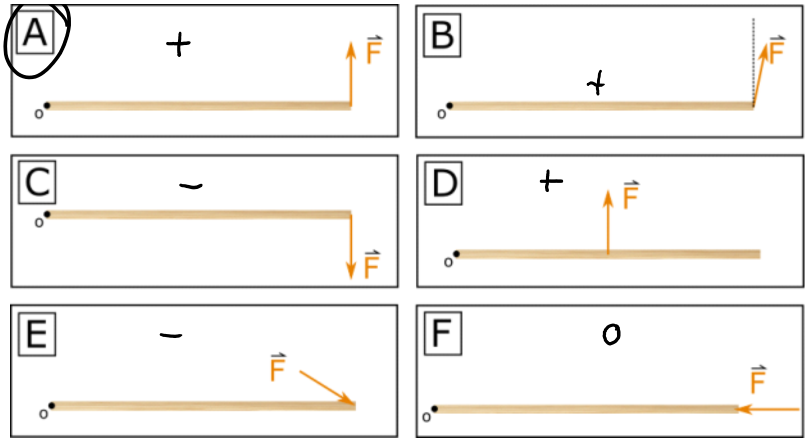






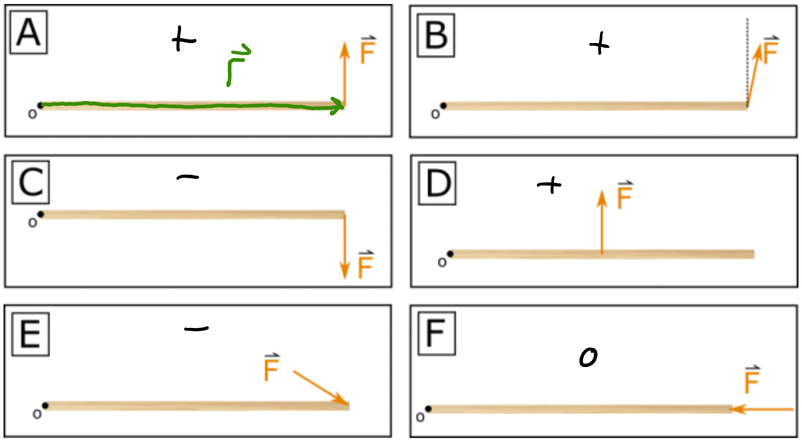
(a) In which situation is the torque produced by the force about axis **o** the most positive?

$|\vec{\tau}_o| = |\vec{r}||\vec{F}|\sin\theta$



(b) Rank the torques in each situation from most negative to most positive.

$$\boxed{\tau^C < \tau^E < \tau^F < \tau^D < \tau^B < \tau^A}$$



$\tau_o = \pm|\vec{r}||\vec{F}|\sin\theta$ OR $\tau_o = \pm|\vec{r}|F_\perp$

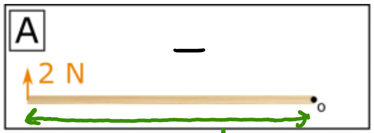
SD.2.L1-4:

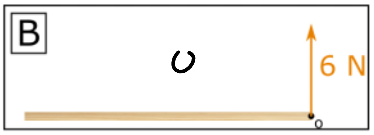
Description: Ranking torque. (5 minutes)

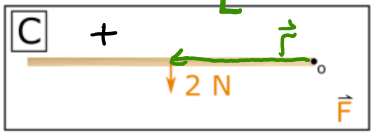
Learning Objectives: [1, 12, 13]

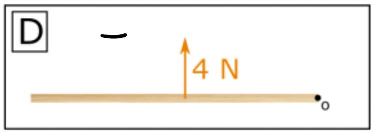
$\tau_o = \pm|\vec{r}||\vec{F}|\sin\theta$

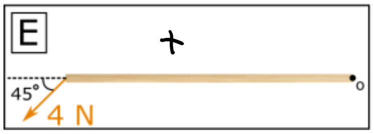
Problem Statement: Consider a block of wood being pushed/pulled on by a force as shown in the images below. The images are drawn to scale. Rank the torques in each situation from most negative to most positive.

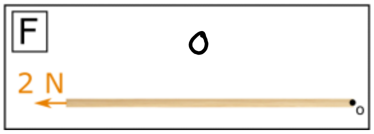












A

$L(2N)\sin(90)$

$-2L$

B

$0(6N)$

0

C

$\frac{L}{2}(2N)\sin(90)$

L

D

$\frac{L}{2}(4N)\sin(90)$

$-2L$

E

$L(4N)\sin(45)$

$4L\frac{\sqrt{2}}{2}$

$+2\sqrt{2}L$

F

$L(2N)\sin(0)$

0

$$\boxed{\tau^A = \tau^D < \tau^F = \tau^B < \tau^C < \tau^E}$$

SD.2.L1-5:

Description: Find torque about given reference axis. (1 minute + 4 minutes)

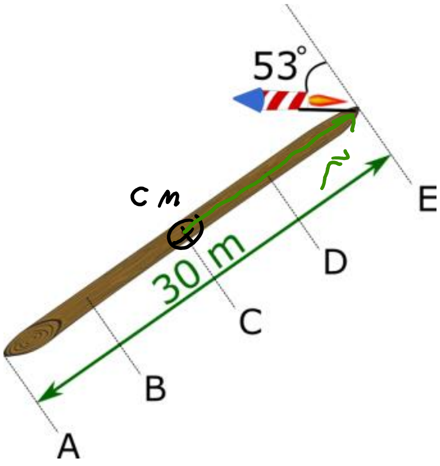
Learning Objectives: [1, 12, 13]

Problem Statement: Benny and Bernice the beavers are building a space dam in outer space far away from other massive objects. While moving a 24000 kg log into the correct place, Benny accidently nudges the constant 1990 N thruster off the log's axis as shown in the image below.

Part I: Spinning out of control.

(a) At what location is the center of mass of the log assuming uniform mass distribution?

(1) A
(2) B
[✓ (3) C]
(4) D
(5) E



(b) What torque does the constant 1990 N rocket thruster provide to the log about the center of mass of the log?

[✓ (1) 18000 N·m]
(2) 23800 N·m
(3) 29900 N·m
(4) 36000 N·m
(5) 47700 N·m

$$\tau_{cm} = +|\vec{r}||\vec{F}|\sin\theta$$

$$= +\left(\frac{30}{2}\right)m\,(1990\,N)\,\sin(53+90)$$

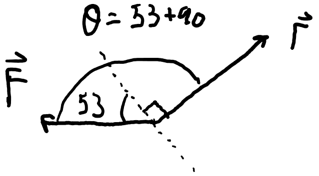
(3) 29900 N·m
(4) 36000 N·m
(5) 47700 N·m
(6) 59700 N·m

$$= +\left(\frac{30}{2}\right)m\,(1990\,N)\,\sin(53+90)$$

$$\approx 18000\ Nm$$

A

VEC. SP.



Part II: Bernice to the rescue.

(c) , **(d)** , **(e)** , **(f)** : See problem SD.2.L1-8

SD.2.L1-6:

Description: Find torque about a given reference axis. (4 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: A 3,000-kg crane is supporting a 20,000-kg shipping container as seen in the image below. Find the torque due to the 20,000 kg container on the crane about the reference axis **o**. Recall **g** = 9.8 m/s^2.

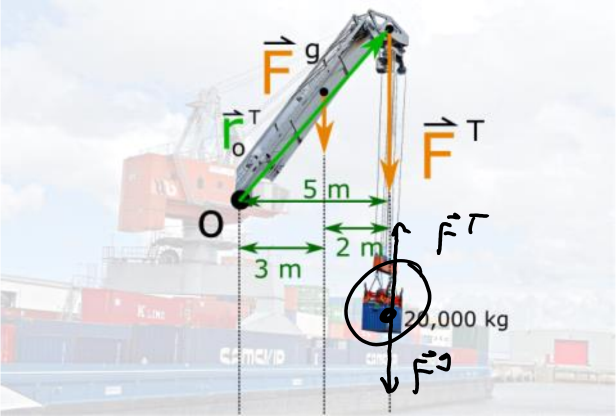
(1) -100,000 N·m
(2) 392,000 N·m
(3) -588,000 N·m
(4) -980,000 N·m
(5) 1,068,000 N·m

$$\tau_o^T = -|\vec{r}_T|\,|\vec{F}^T|\,\sin\overset{?}{\dot{\theta}}$$

$$\tau_o^T = -r_\perp\,|\vec{F}^T|$$

$$= -5m\,(20000\,kg\cdot g)$$

$$= -980000\ Nm$$



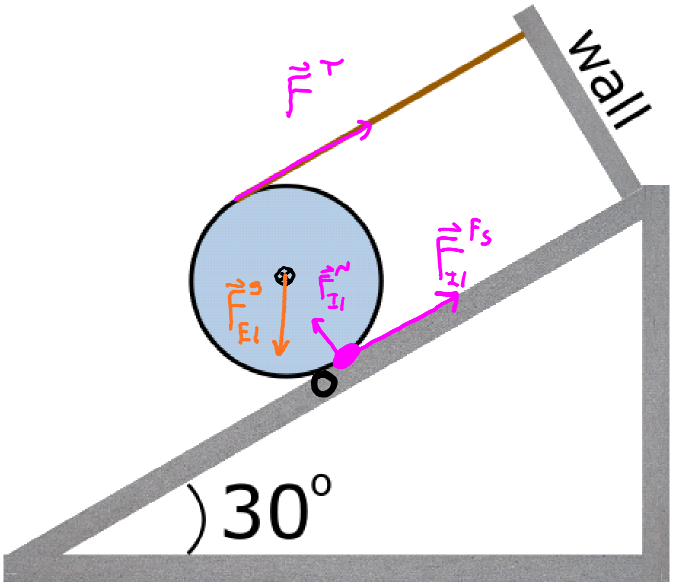
Act II: Newton's 2nd law for rigid rotators

SD.2.L1-7:

Description: Label forces and their locations of action on an object. (3 minutes + 1 minute + 3 minutes + 2 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: A disk with uniform mass distribution sits in static equilibrium on an incline with the aid of a rope as shown in the figure below.



(a) There are 4 forces acting on the disk: tension, normal, static friction, and gravity. On top of the image, draw tension, normal, and static friction by placing the tail of each force vector at the location which they act on the disk.

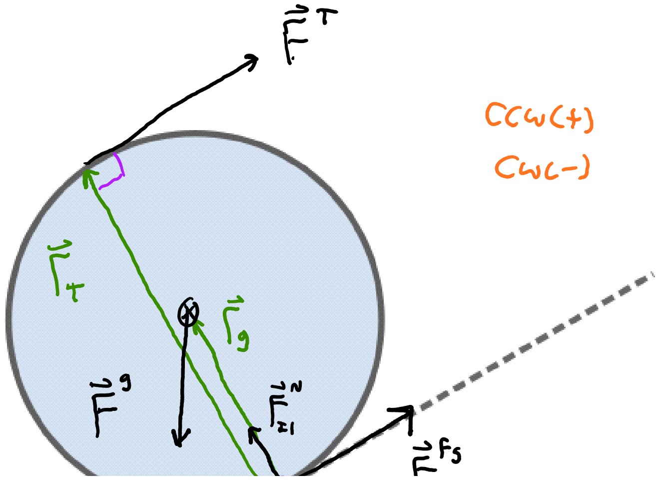
(b) With your neighbors, take 1 minute to discuss where you think the force of gravity acts on this disk. Then draw the force of gravity vector on the image by placing the tail of the vector at the location that it is acting on.

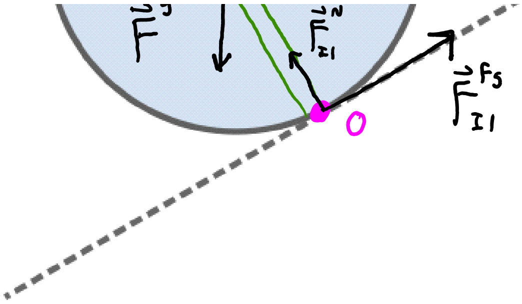
(c) If we wished to solve for the tension, what location would you choose to place the reference axis?

[✓ (1)] At location of contact between disk and incline.
(2) At location of contact of rope and disk.
[✓ (3)] At center of mass of the disk. ← ok, but awful math!
(4) At contact point of rope and wall.

$\tau_o = \pm |\vec{r}||\vec{F}| \sin\theta$ $\tau_o = 0$ IF $|\vec{r}| = 0$

(d) Create an e-FBD to organize all of the information that is needed to solve for the tension in the rope.





SD.2.L1-8:

Description: Calculate net torque. (3 minutes + 1 minute + 3 minutes + 2 minutes)

Learning Objectives: [1, 12, 13]

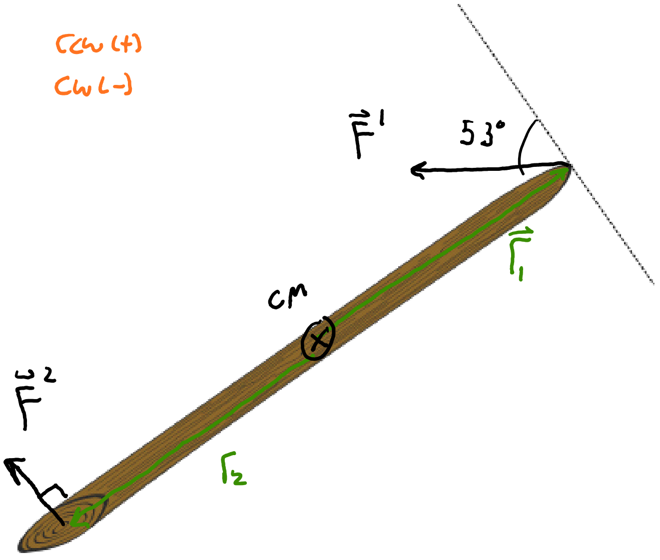
Problem Statement: Benny and Bernice the beavers are building a space dam in outer space far away from other massive objects. While moving a 24000 kg log into the correct place, Benny accidently nudges the 1990 N thruster off the log's axis as shown in the image below.

Part I: Spinning out of control.

(a) and **(b)**: See problem SD.2.L1-5

Part II: Bernice to the rescue.

(c) With the 24000 kg log now spinning out of control, Bernice quickly springs to action by placing a 2nd rocket of unknown force on the log such that the rockets uses the minimum amount of constant thrust to cancel out the torque. Draw an e-FBD for the log with both Benny's and Bernice's rockets acting on the log.



(d) If we wish to determine the minimum force the rocket needs to apply, where should we place the reference axis?

$$\text{MIN } \vec{F}\ r_2 \longrightarrow \text{MAX } r$$

(e) Calculate the minimum amount of force of the rocket the Bernice attached to cancel out the torque on the 30 m long log.

$$\Sigma \tau_{cm} = 0$$

$$\tau_1 + \tau_2 = 0$$

$$\underbrace{18000\ \text{NM}}_{\text{FROM PART D}} - |\vec{r}_2||\vec{F}^2| \overset{1}{\cancel{\sin\theta_2}} = 0$$

$$18000\ \text{NM} - 15\text{m}\,|\vec{F}^2| = 0$$

$$\boxed{|\vec{F}^2| = 1200\ \text{N}}$$

$$\boxed{|\vec{F}^2| = 1200\ N}$$

(f) What happens to the log after Bernice places the rocket on the log?

F (1) The log stops spinning
F (2) The log continues spinning but slows down and eventually reverses direction.
T [✓ (3)] The log continues to spin at the same rate as it was the moment the 2nd rocket was added.

NET FORCE CAUSES OBJECTS TO $\underline{\underline{\text{CHANGE}}}$ VELOCITY

NET TORQUE CAUSES OBJECTS TO $\underline{\underline{\text{CHANGE}}}$ ANGULAR VELOCITY.

SD.2.L1-9:

Description: Conceptual question regarding Newton's laws of motion in rotational form. (4 minutes)

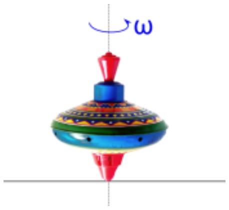
Learning Objectives: [1, 12, 13]

Problem Statement: A few students are discussing a top that is spinning vertically in one place, but the rotational speed is slowing down. Which student(s) do you agree with?

T [✓ (1)] If you look at a FBD for the top, $\overset{T}{\underline{\text{there is no net external force}}}$, $\overset{T}{\underline{\text{thus the velocity of the center of mass is constant}}}$.

F (2) The $\underset{T}{\underline{\text{angular velocity of the top is changing}}}$, $\underset{F}{\underline{\text{so there must be a non-zero net external force acting on the top}}}$.

T [✓ (3)] I agree that $\underset{T}{\underline{\text{there is no net external force}}}$, but $\underset{T}{\underline{\text{since the angular velocity is changing there must be a net external torque on the top}}}$.



T [✓ (4)] The top spins for a long time because the angular impulse is small, thus the top loses angular momentum slowly.

T [✓ (5)] The top spins for a long time because the friction between the top and table is small, thus the rotational kinetic energy of the top slowly converts to thermal energy of the top+table system.

$$\Sigma \vec{F}_{EXT} \longrightarrow \Delta \vec{V}_{cm} \qquad \Sigma \vec{\tau}_{EXT} \longrightarrow \Delta \omega$$

SD.2.L1-10:

Description: Conceptual question regarding equilibrium. (4 minutes)

Learning Objectives: [1, 12, 13]

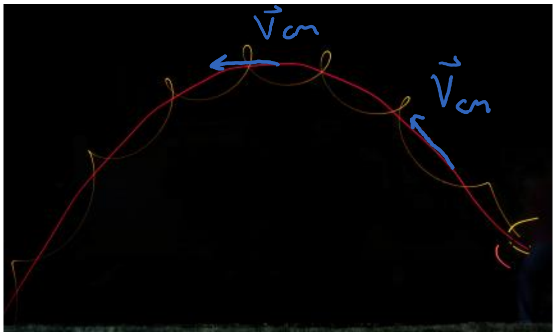
Problem Statement: An axe is thrown into the air near the surface of the earth. A red LED is attached to the center of mass of the axe and a yellow LED is attached to one end of the axe so that a picture taken at night can be used to create a trajectory of each part of the axe as seen in the image below. Which statement(s) is(are) true? Ignore air resistance and consider only the time that the axe has left your hand and before it hits the ground.

F (1) The axe has no net external force acting on it.

T [✓ (2)] The axe has no net external torque acting on it.

F (3) The energy of the axe is constant.

F (4) The axes' momentum is constant.



Conceptual questions for discussion

1. Is a torque a force?
2. A friend of yours claims that it is easiest to open a door by pushing at a location furthest away from the hinges. Do you agree? Would you add anything to this statement if you don't necessarily agree?
3. Do you agree with the following statement: A small force can never apply a torque larger in magnitude than a larger force on the same object.
4. How might you experimentally find the center of mass of an object near the surface of the earth?
5. Is a torque a force?
6. If you wish to stop a door from closing using a door stop that wedges between the floor and the bottom of the door, where should you place the door stop and why?
7. Do you agree with the following statement: If an object is not rotating, then it has no torques acting on it.
8. Do you agree with the following statement: If an object is rotating, then there must be a net external torque on it.

Hints

SD.2.L1-1: No hints.

SD.2.L1-2: No hints.

SD.2.L1-3: Determine sign of torques first, then focus on magnitudes.

SD.2.L1-4: Determine sign of torques first, then focus on magnitudes.

SD.2.L1-5: Draw a vector operation to determine the angle that goes into the sin function in the definition of torque.

SD.2.L1-6: Recall that a cross product asks, "how perpendicular two vectors are".

SD.2.L1-7: No hints.

SD.2.L1-8: Talk to your neighbors about what angle relative to the log would give the largest torque due to a force.

SD.2.L1-9: No hints.

SD.2.L1-10: No hints.