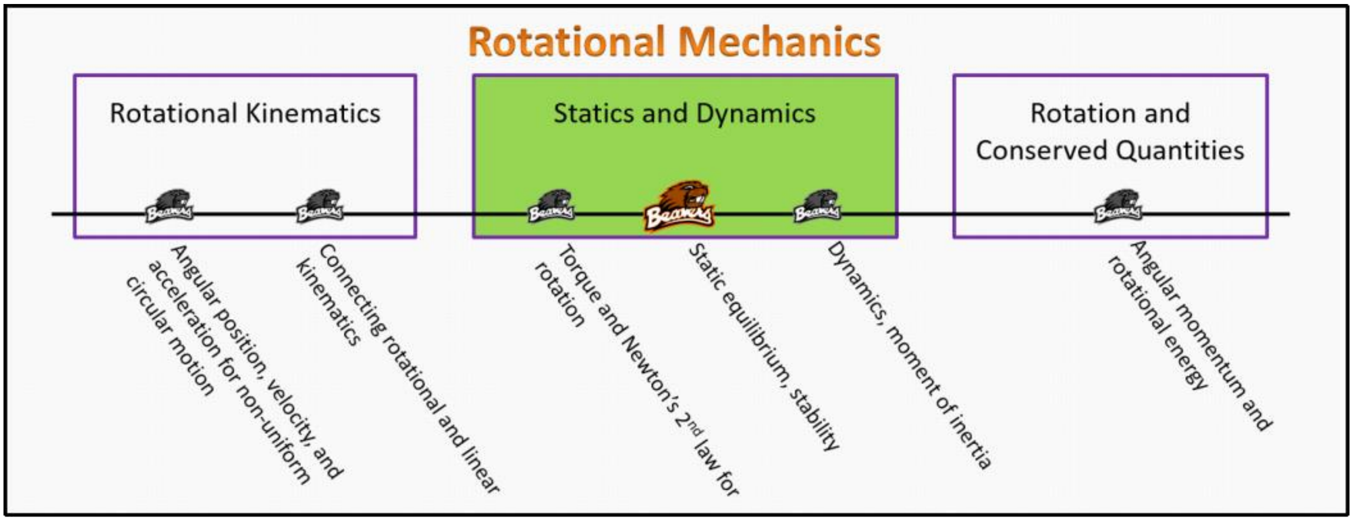
SD.2.L2 - Solutions

Wednesday, January 16, 2019 7:44 PM

Statics and Dynamics
Foundation Stage (SD.2.L2)

lecture 2
Static equilibrium, stability



Textbook Chapters (* Calculus version)

- **BoxSand** :: KC videos (statics & dynamics)
- **Knight** (College Physics : A strategic approach 3rd) :: 7.3 ; 7.4
- ***Knight** (Physics for Scientists and Engineers 4th) :: 12.2 ; 12.5 ; 12.7 ; 12.8
- **Giancoli** (Physics Principles with Applications 7th) :: 9-1 ; 9-2 ; 9-3 ; 9-4

Warm up

SD.2.L2-1:

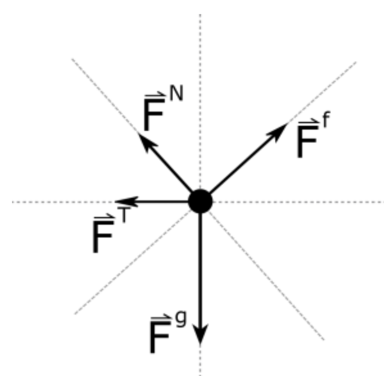
Description: Identify external forces that should be on an eFBD.

Learning Objectives: [?] - Can you identify the objectives from the previous lecture, and this lecture, that this question is relevant to?

Problem Statement: Below shows a FBD for a system in equilibrium. All forces are drawn to scale.

(a) If a torque analysis is to be carried out for this same system, which forces would also show up on an eFBD?

(1) The same forces as on the FBD.
(2) Only the forces on the FBD that produce a torque.
(3) The same forces as on the FBD, plus the other forces that cause torque.
(4) Unable to determine without knowing the actual situation.



FORCE ANALYSIS

$$\sum \vec{F}_{EXT} = m\vec{a}$$

EXTERNAL FORCES

TORQUE ANALYSIS

$$\sum \tau_{EXT,o} = I_o \alpha$$

EXTERNAL TORQUES

TORQUE = $\vec{r} \times \vec{F}$

EXTERNAL FORCES

EXTERNAL TORQUES

TORQUE $= \vec{r} \times \vec{F}$

Selected Learning Objectives

1. Coming soon to a lecture template near you.

Key Terms

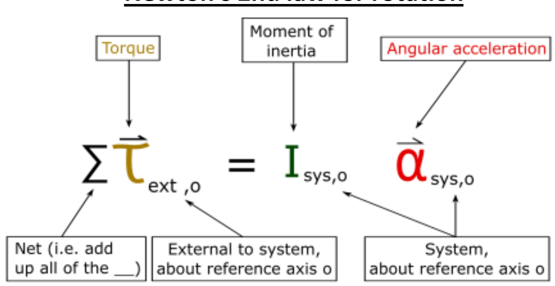
- Torque analysis
- Rotational static equilibrium
- Rotational dynamic equilibrium
- Rotational dynamics
- Translational static equilibrium
- Translational dynamic equilibrium
- Translational dynamics
- Center of mass
- Stability condition and critical point

Key Equations

Equilibrium definitions

	Translational	Rotational
Static Equilibrium	$\vec{v}_{cm} = \vec{0}$	$\vec{\omega}_o = \vec{0}$
Dynamic Equilibrium	$\vec{v}_{cm} = \overline{\text{constant}} \neq \vec{0}$ $\vec{a}_{cm} = \vec{0}$ $\Sigma\vec{F}_{ext} = \vec{0}$	$\vec{\omega}_o = \overline{\text{constant}} \neq \vec{0}$ $\vec{\alpha}_o = \vec{0}$ $\Sigma\vec{\tau}_{ext,o} = \vec{0}$
Dynamics	$\Sigma\vec{F}_{ext} = m_{sys}\vec{a}_{cm}$	$\Sigma\vec{\tau}_{ext,o} = I_{sys,o}\vec{\alpha}_o$

Newton's 2nd law for rotation



In words: The net torque external to the system about reference axis o is equal to the momentum of inertia of the sysem about reference axis o multiplied by the angular acceleration of the system about reference axis o.

Center of mass

Position of center of mass for system

Position of center of mass of object 1

Mass of object 1

$$\vec{r}^{\,sys}_{cm} = \frac{\vec{r}^{\,1}_{cm} m_1 + \vec{r}^{\,2}_{cm} m_2 + \ldots + \vec{r}^{\,n}_{cm} m_n}{m_1 + m_2 + \ldots + m_n} = \frac{\sum \vec{r}^{\,i}_{cm} m_i}{\sum m_i}$$

Net (i.e. add up all of the __)

In words: The position of the center of mass for a system of "n" number of rigid objects is equal to the sum of the products between the position of each object's center of mass and its mass all divided by the total mass of the system.

Key Concepts

- A torque analysis utilizes Newton's 2nd law of motion for rotation. The major components of a torque analysis are: identifying a system, drawing an eFBD, choosing a reference axis, summing up all of the external torques on the system and setting the sum equal to the moment of inertia times the angular acceleration. The moment of inertia plays a similar role as mass does in Newton's 2nd law for translation of the center of mass.
- In general there are two ways to classify the status of a system: equilibrium, and dynamics. Equilibrium can be split into two sub-categories: static equilibrium, and dynamic equilibrium. We can break all of the categories and sub-categories into rotational and translational versions if the system is a mix of different rotational and translational equilibrium/dynamics. See the table in the key equations for definitions of each.
- The center of mass of an object/system is the point that the object/system will rotate around when there is no net force acting on the object/system. Mathematically, the center of mass is a mass-weighted average of the geometric center of an object/system. Mass-weighted average of the geometric center what??...For example, if two equal masses are located a distance d apart, the center of mass is at the geometric center, d/2. Now if the object on the left is more massive, the center of mass is closer to the left object.
- Often times we can use the stability condition, rather than a torque analysis, when a system is in static equilibrium but just about ready to transition into dynamics. An example of such a scenario is a book balanced on the edge of a table, about to tip over if moved any further out. The stability condition for objects that are not glued or fastened to the ground is the following: when a system is at the critical point, the center of mass of the system is above or below the furthest most normal force point. If the center of mass of the system goes beyond the furthest most normal force point then the system will transition in dynamics (e.g. tip over).

Questions

Act I: Equilibrium

SD.2.L2-2:

Description: Determine sign of net torque and angular acceleration. Determine what additional torque would place system in equilibrium. (2 minutes + 1 minute + 2 minutes + 1 minute)

Learning Objectives: [1, 12, 13]

Problem Statement: Benny and Bernice are traveling on their matching Scooty Puff Jrs., far away from massive objects, when they decide to play a fun physics game. Bernice attaches 3 rocket thrusters to a 20,000 kg log of uniform mass distribution as shown below. The magnitude of torque applied by each thruster on the log is also shown. It's now Benny's task to put a 4th thruster on the log such that the log remains in equilibrium, he only gets one shot, if he fails Bernice gets one Beaver point.

(a) The initial net torque on the log is $\Sigma \tau_{CM} = \tau^1 + \tau^2 + \tau^3$

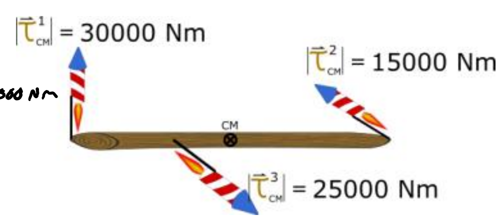
CCW(+)

CW(-)

(1) Positive
(2) Negative
(3) Zero

$= -30000\ Nm + 15000\ Nm + 25000\ Nm$

$= +10000\ Nm$



(b) What is the sign of the angular acceleration of the log?

(1) Positive

$\Sigma \tau_{c} = I_{cm} \alpha$

(b) What is the sign of the angular acceleration of the log?

(1) Positive
(2) Negative
(3) Zero

$$\Sigma \tau_{cm} = \underset{\substack{\uparrow \\ \text{ALWAYS + LIKE MASS}}}{I_{cm}} \alpha$$

(c) At some time later, Benny places the 4th thruster on the log so that the log is in rotational equilibrium. What torque, including sign, will put the log in equilibrium?

$$\Sigma \tau_{cm} = I_{cm} \cancel{\alpha}^{\,0}$$

$$\underbrace{\tau^1 + \tau^2 + \tau^3}_{10000\ NM} + \tau^4 = 0$$

$$10000\ NM + \tau^4 = 0$$

$$\boxed{\tau^4 = -10000\ NM}$$

(d) After Benny has placed the rocket thruster, thus placing the log in equilibrium is the log in rotational static equilibrium or rotational dynamic equilibrium?

$$\alpha = 0 \rightarrow \frac{\Delta \omega}{\Delta t} = 0$$

SD.2.L2-3:

Description: Identify equilibrium status of given systems. (1 minute + 1 minute + 1 minute)

Learning Objectives: [1, 12, 13]

Problem Statement: Consider the three scenarios shown below. Match each scenario to their equilibrium status.

(a) Rotational static equilibrium

(1) Earth
(2) Seesaw with heavy dog and cat
(3) Roulette wheel

(b) Rotational dynamic equilibrium

(1) Earth
(2) Seesaw with heavy dog and cat
(3) Roulette wheel

(c) Rotational Dynamics (not in equilibrium)

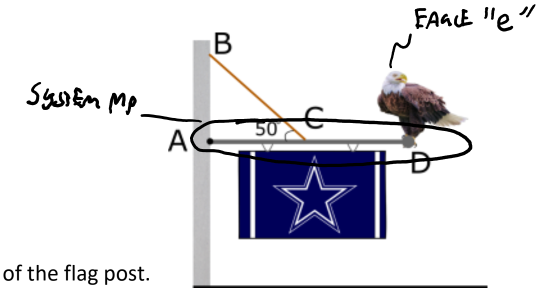
(1) Earth
(2) Seesaw with heavy dog and cat
(3) Roulette wheel

SD.2.L2-4:

Description: Determine the tension in rope using torque analysis. (1 minute + 3 minutes + 1 minute + 4 minutes + 5 minutes + 2 minutes + 3 minutes + 5 minutes)

Learning Objectives: [1, 12, 13]

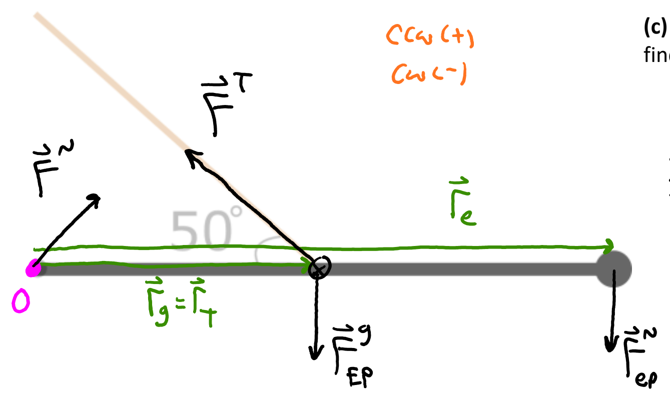
Problem Statement: Elgae the Eagle is 5 kg and perched at the end of a 3-m long flag post of mass 10 kg being held up by a rope as seen in the figure below. The flag post connects to the wall via a pivot. The rope connect to the flag post at the center of the flag post. The flag has negligible mass. We wish to eventually determine the tension in the rope.



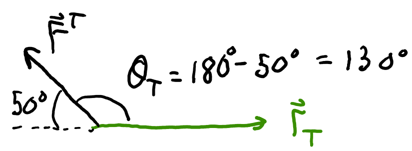
(a) Where would you choose your reference axis from this list of likely candidates?

(1) A
(2) B
(3) C
(4) D

(b) With the system defined as the flag post, draw the e-FBD on the provided outline of the flag post.



(c) Draw the vector operation diagram that would be useful when finding the torque from the tension on the flag post.



(d) Which of the following mathematical representations correctly represents a 2nd law analysis for this scenario?

(1) $|\vec{r}_N||\vec{F}^N|\sin(\theta_N) + |\vec{r}_T||\vec{F}^T|\sin(\theta_T) + |\vec{r}_g||\vec{F}^g_{EP}|\sin(\theta_g) + |\vec{r}_e||\vec{F}^N_{ep}|\sin(\theta_e) = I_o\alpha$

(2) $|\vec{r}_T||\vec{F}^T|\sin(\theta_T) + |\vec{r}_g||\vec{F}^g_{EP}|\sin(\theta_g) + |\vec{r}_e||\vec{F}^N_{ep}|\sin(\theta_e) = I_o\alpha$

$$\Sigma \tau_o = I_o \alpha$$

(3) $-|\vec{r}_T||\vec{F}^T|\sin(\theta_T) + |\vec{r}_g||\vec{F}^g_{EP}|\sin(\theta_g) + |\vec{r}_e||\vec{F}^N_{ep}|\sin(\theta_e) = I_o\alpha$

(4) $|\vec{r}_T||\vec{F}^T|\sin(\theta_T) - |\vec{r}_g||\vec{F}^g_{EP}|\cancelto{1}{\sin(\theta_g)} - |\vec{r}_e||\vec{F}^N_{ep}|\cancelto{1}{\sin(\theta_e)} = I_o\cancelto{0}{\alpha}$

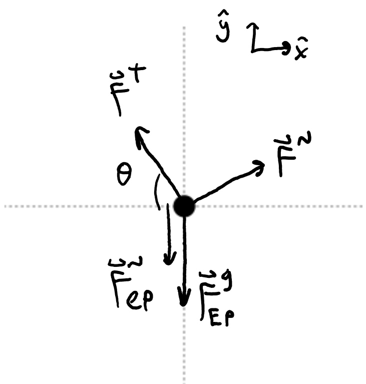
$|\vec{F}^N_{ep}| = |\vec{F}^g_{ep}|$ ... From FBD of e

(e) Elgae the Eagle is 5 kg and perched at the end of a 3-m long flag post of mass 10 kg being held up by a rope as seen in the figure below. The flag post connects to the wall via a pivot. The rope connect to the flag post at the center of the flag post. The flag has negligible mass. **What is the magnitude of the tension in the rope?**

$$(1.5\,m)|\vec{F}^T|\sin(130°) - (1.5\,m)(10\,kg)(9.8\,m/s^2) - (3\,m)(5\,kg)(9.8\,m/s^2) = 0$$

$$\boxed{|\vec{F}^T| \approx 256\ N}$$

(f) Draw a FBD for the flag post.



(g) What are the x and y components of the reaction force from the wall on the flag post?

$$\Sigma F_x = m_p \cancel{a_x}^{0}$$

$$F^N_x - |\vec{F}^T|\cos\theta = 0$$

$$F^N_x - (255.86\,N)\cos(50^\circ) = 0$$

$$\boxed{F^N_x \approx 164\ N}$$

$$\Sigma F_y = m_p \cancel{a_y}^{0}$$

$$F^N_y + |\vec{F}^T|\sin\theta - |\vec{F}^N_{ep}| - |\vec{F}^g_{EP}| = 0$$

$$F^N_y + (255.86\,N)\sin(50) - (5\,kg)(9.8\,m/s^2) - (10\,kg)(9.8\,m/s^2) = 0$$

$$\boxed{F^N_y \approx 49.0\ N}$$

$$\boxed{\vec{F}^N = \langle 164, 49.0 \rangle\ N}$$

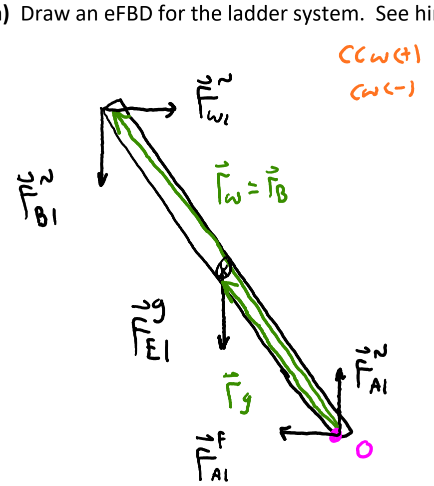
SD.2.L2-5:

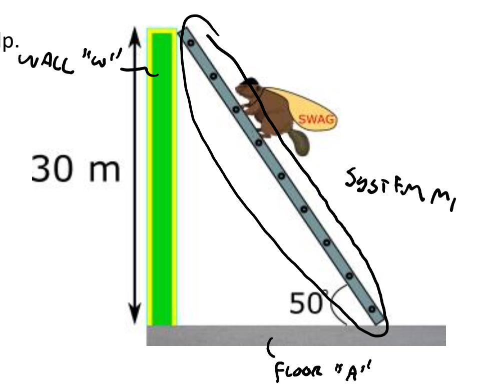
Description: Statics problem solving for a magnitude force. (3 minutes + 3 minute + 6 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: Benny, 20 kg, is looting Autzen stadium of all of Phil Knight's moneys. He places a 130 kg ladder against an icy wall with the top of the ladder at the top of the 30 meter icy wall as seen below. Icy wall here implies negligible friction. We eventually wish to determine the maximum normal force from the icy wall on the ladder.

(a) Draw an eFBD for the ladder system. See hints for help.

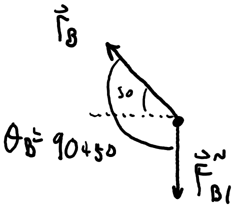




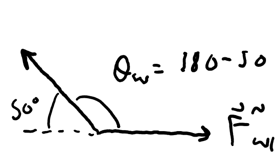
GEOMETRY

$$\frac{30}{\sin 50} \approx 39.16\ m$$

(b) Draw any vector operations to help find angles that are related to cross products.



$\theta_B = \theta_g = 140°$



$\theta_w = 180 - 50 = 130°$

(c) Benny, 20 kg, is looting Autzen stadium of all of Phil Knight's moneys. He places a 130 kg ladder against an icy wall with the top of the ladder at the top of the 30 meter icy wall as seen below. Icy wall here implies negligible friction. **What is the maximum normal force from the icy wall on the ladder?**

$$\Sigma \tau_o = I_o \cancel{\alpha}^{\,0}$$

$$\tau^N_{w1} + \tau^N_{B1} + \tau^g_{E1} + \cancel{\tau^F_{A1}}^{\,0} + \cancel{\tau^N_{A1}}^{\,0} = 0$$

$$-|\vec{r}_w||\vec{F}^N_{w1}|_{max}\sin\theta_w + |\vec{r}_B||\vec{F}^N_{B1}|\sin\theta_B + |\vec{r}_g||\vec{F}^g_{E1}|\sin\theta_g = 0$$

$$-(39.16\,m)|\vec{F}^N_{w1}|_{max}\sin(130°) + (39.16\,m)(20\,kg)(9.8\,m/s^2)\sin(140°) + \left(\frac{39.16\,m}{2}\right)(130\,kg)(9.8\,m/s^2)\sin(140°) = 0$$

$$\boxed{|\vec{F}^N_{w1}| \approx 699\,N}$$

Act II: Stability

SD.2.L2-6:

Description: Apply torque analysis and center of mass stability analysis to determine critical point location. (4 minutes + 4 minutes + 5 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: After being caught stealing moneys, the Ducks take 20 kg Benny to the high seas to walk the plank. The 15 kg plank is made from a 5-m-long piece of wood resting (not fastened) on two triangular pivots. We eventually which to determine the location Benny would be when the plank begins to tip over (the critical point).

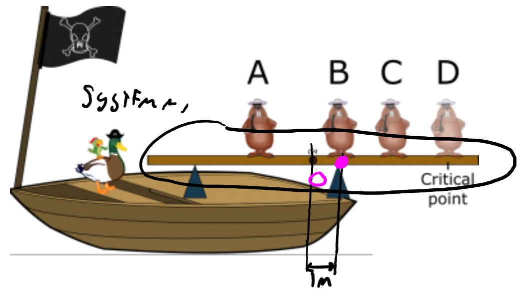
(a) The image below shows snapshots of Benny at 4 different locations as he slowly walks out to the edge. Four students calculate the torque from the left-most triangle on the plank using their reference axis at the right-most triangle. Their answers are written in the following form: $[\tau^{N_L}_A, \tau^{N_L}_B, \tau^{N_L}_C, \tau^{N_L}_D]$, where N_L is the normal force from the left-most pivot and A-D represent the snapshots. Which of the following set of torques could be correct?

(1) [-10 , -7 , -4 , 0]

(2) [4 , 0 , -2 , 4] ~ COULD BE τ^{N_L} FROM BENNY

(3) [-4 , 0 , 4, 10]

(4) [-10 , 0 , -7 , -10]



(b) Which of the following mathematical representations could represent a correct 2nd law application about the right most pivot when Benny is at the critical point? The 15 kg plank is made from a 15-m-long piece of wood resting (not fastened) on two triangular pivots.

(1) $|\vec{\mathbf{r}}_{N_L}||\vec{\mathbf{F}}^{N_L}|\sin(\boldsymbol{\theta}_{N_L}) + |\vec{\mathbf{r}}_{g}||\vec{\mathbf{F}}^{g}|\sin(\boldsymbol{\theta}_{g}) + |\vec{\mathbf{r}}_{N_R}||\vec{\mathbf{F}}^{N_R}|\sin(\boldsymbol{\theta}_{N_R}) + |\vec{\mathbf{r}}_{N_B}||\vec{\mathbf{F}}^{N_B}|\sin(\boldsymbol{\theta}_{N_B}) = \mathbf{I}_o\boldsymbol{\alpha}$

(2) $|\vec{\mathbf{r}}_{N_L}||\vec{\mathbf{F}}^{N_L}|\sin(\boldsymbol{\theta}_{N_L}) - |\vec{\mathbf{r}}_{g}||\vec{\mathbf{F}}^{g}|\sin(\boldsymbol{\theta}_{g}) + |\vec{\mathbf{r}}_{N_R}||\vec{\mathbf{F}}^{N_R}|\sin(\boldsymbol{\theta}_{N_R}) - |\vec{\mathbf{r}}_{N_B}||\vec{\mathbf{F}}^{N_B}|\sin(\boldsymbol{\theta}_{N_B}) = \mathbf{I}_o\boldsymbol{\alpha}$ $\quad \Sigma \tau_o = I_o \alpha$

(3) $|\vec{\mathbf{r}}_{g}||\vec{\mathbf{F}}^{g}|\sin(\boldsymbol{\theta}_{g}) + |\vec{\mathbf{r}}_{N_R}||\vec{\mathbf{F}}^{N_R}|\sin(\boldsymbol{\theta}_{N_R}) - |\vec{\mathbf{r}}_{N_B}||\vec{\mathbf{F}}^{N_B}|\sin(\boldsymbol{\theta}_{N_B}) = \mathbf{I}_o\boldsymbol{\alpha}$ $\quad \cancel{\tau^{N_L}}^{0} + \tau^g + \cancel{\tau^{N_R}}^{0} + \tau^{NB} = 0$

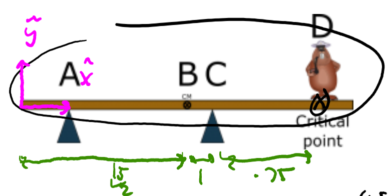
(4) $|\vec{\mathbf{r}}_{g}||\vec{\mathbf{F}}^{g}|\cancel{\sin(\boldsymbol{\theta}_{g})}^{1} - |\vec{\mathbf{r}}_{N_B}||\vec{\mathbf{F}}^{N_B}|\cancel{\sin(\boldsymbol{\theta}_{N_B})}^{1} = \cancel{\mathbf{I}_o\boldsymbol{\alpha}}^{0}$ $\quad |\vec F^N_{BI}| = |\vec F^g_{EB}| \quad \tau^g + \tau^{NB} = 0$

$$(1m)(15kg)(9.8m/s^2) - |\vec r_{NB}|(20kg)(9.8m/s^2) = 0$$

$$\boxed{|\vec r_{NB}| = 0.75\,m}$$

(c) With Benny (20 kg) at the critical point, what is the center of mass of the Benny + plank system using a coordinate system on the left hand side of the 15-m 15 kg plank?

(1) A
(2) B
(3) C
(4) D



$$X^{sys}_{cm} = \frac{x^1_{cm} m_1 + x^B_{cm} m_B}{m_1 + m_B} = \frac{\left(\frac{15m}{2}\right)(15kg) + \left(\frac{15}{2} + 1 + 0.75\right)m\,(20kg)}{(15+20)\,kg}$$

$$\boxed{X^{sys}_{cm} = 8.5\,m}$$

* NOTE RIGHT POST @ $\left(\frac{15}{2} + 1\right)m = 8.5\,m$ //

STABILITY CONDITION.

WHERE IS BENNY ALT. SOLUTION w/ STABILITY CONDITION

$$X_{cm}^{sys} = \frac{X_{cm}^{1} m_1 + X_{cm}^{B} m_B}{m_1 + m_B}$$

$$8.5\ m = \frac{\left(\frac{15}{2} m\right)(15\ kg) + X_{cm}^{B}\ (20\ kg)}{(15 + 20)\ kg}$$

$$\boxed{X_{cm}^{B} \approx 9.25\ m}$$

w/ RIGHT POST AT 8.5 m

$9.25 - 8.5 = \boxed{0.75\ m \text{ FROM RIGHT POST}}$

SD.2.L2-7:

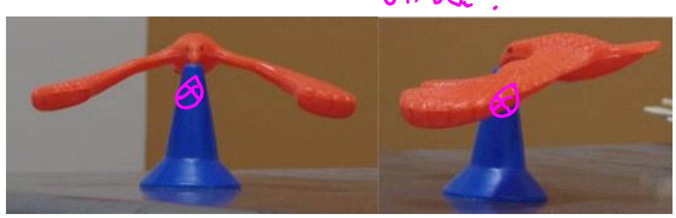
Description: Estimate center of mass given images of systems in static equilibrium. (6 minutes)

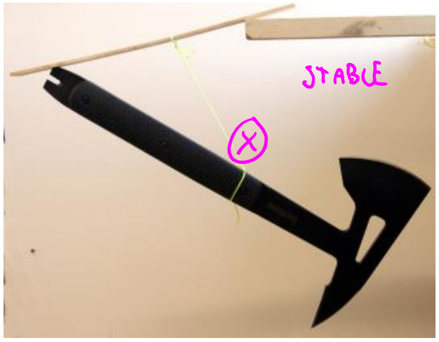
Learning Objectives: [1, 12, 13]

Problem Statement: Below are four images of objects in equilibrium. Estimate where the center of mass is located for each.

UNSTABLE!

STABLE!





Conceptual questions for discussion

1. If the net torque on a system is zero, is the system static?
2. A ladder is leaning against a wall on one end, and on a horizontal floor on the other end. As a person climbs the ladder, at which point along the length of the ladder is it most likely to slide on the horizontal floor?
3. Do you agree with the following statement: If the net torque on a system is zero, then the net force must also be zero. Provide examples if you don't agree.
4. Do you agree with the following statement: If the net force on a system is zero, then the net torque must also be zero. Provide example if you don't agree.
5. Do you agree with the following statement: All systems will tip over if the collective center of mass is beyond the furthest most normal point. Provide examples if you don't agree.

Hints

SD.2.L2-1: No hints.

SD.2.L2-2: Recall the standard convention for torque is ccw(+) and cw(-).

SD.2.L2-3: No hints.

SD.2.L2-4: When choosing reference axis, determine which forces you know the least about.

SD.2.L2-5: When picking the location of the reference axis, think about what forces you know and don't know. When drawing the normal force from Benny on the ladder, where would Benny be located on the ladder when the wall provides the largest normal force on the ladder?.

SD.2.L2-6: No hints.

SD.2.L2-7: No hints.