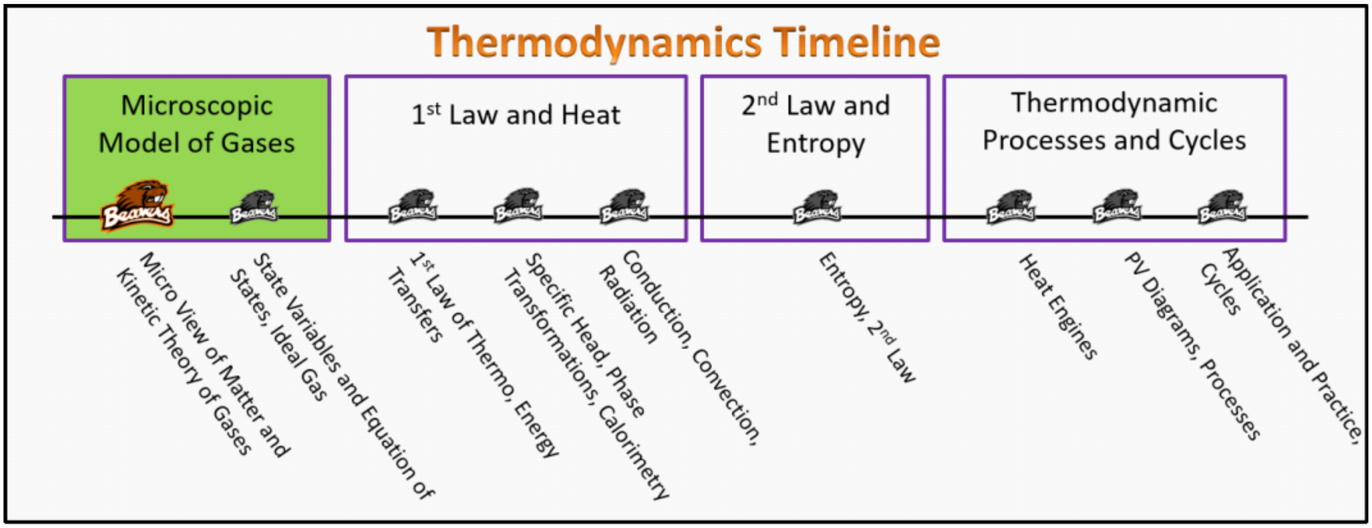
MG.2.L1 - Solutions

Wednesday, January 23, 2019 6:48 AM

Thermodynamics
Foundation Stage (MG.2.L1)

Lecture 1
Microscopic Model of Gases



Textbook Chapters (* Calculus version)

- **BoxSand** :: KC videos (kinetic theory of gases)
- **Knight** (College Physics : A strategic approach 3rd) :: 11.3 ; 12.1 ; 12.2
- ***Knight** (Physics for Scientists and Engineers 4th) :: 18.1 ; 18.2 ; 18.3
- **Giancoli** (Physics Principles with Applications 7th) :: 13-1 ; 13-2 ; 13-3 ; 13-9 ; 13-10

Warm up

MG.2.L1-1:

Description: Apply energy analysis to determine change in thermal energy.

Learning Objectives: [?] - Can you identify the objectives from the previous lecture, and this lecture, that this question is relevant to?

Problem Statement: A 0.145 kg baseball is traveling 35 m/s horizontally before it collides with a glove. The baseball is not rotating. Assuming all of the translational kinetic energy of the baseball goes into changing the thermal energy of the baseball and glove, how much increase in thermal energy is this in joules?

$$KE \rightarrow E_{th}$$

$$\frac{1}{2}mv^2 = \Delta E_{th}$$

2

$$\frac{1}{2}(0.145)(35)^2 = 88.8\ \mathrm{J}$$

Selected Learning Objectives

1. Coming soon to a lecture template near you.

Key Terms

- Average translational kinetic energy per particle
- Average mass
- Root mean square speed
- Thermal energy
- Boltzmann's postulate
- Boltzmann's constant
- Temperature
- Thermodynamic equilibrium

Key Equations

Kinetic theory of gasses

Average translational kinetic energy per particle | Average mass of gas particle | Root mean squared speed of particle

$$\overline{ke}_T = \frac{1}{2}\,\overline{m}\,v_{rms}^2$$

In words: The average translational kinetic energy per particle of a system is equal to one half of the average mass of particles in the system multiplied by the root mean squared speed squred of the particles.

Thermal energy (a.k.a. internal energy) | Average translational kinetic energy per particle

$$E^{Th} = N\,\overline{ke}_T$$

Number of particles in system

*This definition is only valid for monatomic gases.

In words: The thermal energy of a system is equal to the number of particles in the system times the average translational kinetic energy per particle in the system.

Boltzmann's postulate

Thermal energy (a.k.a. internal energy) | Boltzmann's constant | Temperature

$$E^{Th} = N\,\frac{3}{2}\,k_B\,T$$

Number of particles in system

*This definition is only valid for monatomic gases.

In words: The thermal energy of a system is equal to the number of particles in the system times three halves of the Boltzmann's constant multiplied by temperature.

Change in thermal energy (a.k.a. internal energy) | Boltzmann's constant | Change in temperature

$$\Delta E^{Th} = N\,\frac{3}{2}\,k_B\,\Delta T$$

Number of particles in system

*This definition is only valid for monatomic gases.

In words: The change in thermal energy of a system is equal to the number of particles in the system times three halves of the Boltzmann's constant multiplied by the change in temperature.

Combining kinetic theory and Boltzmann's postulate

Average translational kinetic energy per particle | Boltzmann's constant | Temperature

$$\overline{ke}_T = \frac{3}{2} k_B T$$

In words: The average translational kinetic energy per particle in a system is equal to three halves Boltzmann's constant multiplied by the temperature of the system.

Key Concepts

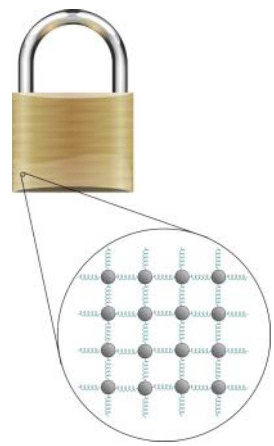
- Stay tuned...

Questions

Act I: Thermodynamics Background Information

Common States of Matter

Solid

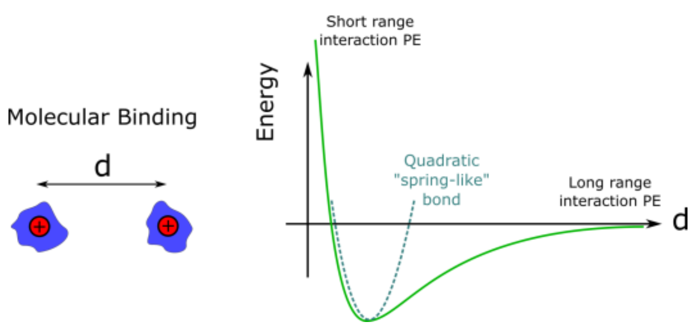


Macroscopic View

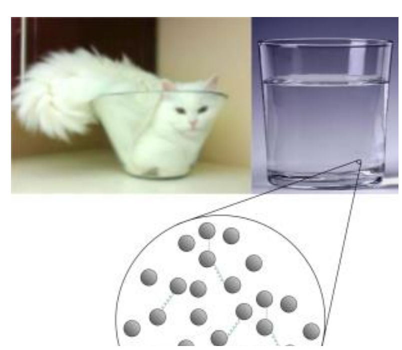
- Objects that maintain their size and shape and usually require a large force to deform it.

Microscopic View

- Crystalline lattice; regularly repeated sequence of atomic structure (range of symmetry varies).
- Motion of atoms is vibrations about a fixed point.
- Strong spring-like bonds which represent chemical bonds between atoms.



Liquid

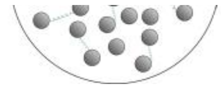


Macroscopic View

- Something that doesn't maintain a fixed shape, rather it conforms to an open container holding it. Not easily compressed.

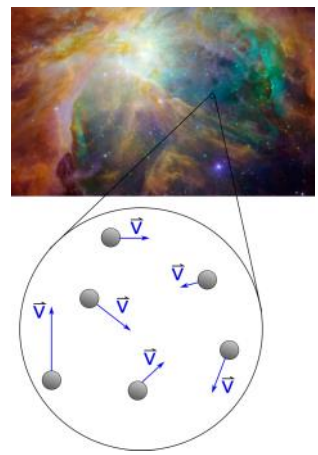
Microscopic View

- Molecules vibrate and "roll" around each other.
- Weak bonding.



- Motion of molecules is more random than solid atoms but less random that the motion of gas molecules.
- Little free space between molecules.

Gas



Macroscopic View

- Something that has neither fixed size or shape and expands to fill a closed container. Easily compressible.

Microscopic View

- Random direction and random speeds of molecules (random motion).
- Lots of free space between each particle/molecule.
- Very weak or no long range interactions between particles.

State Variables

Now that we have a more detailed picture of the macroscopic and microscopic descriptions of states of matter, let's look at the common ways to quantify state properties. Thermodynamics uses **state variables**, which are properties used to describe the equilibrium state of matter. Examples of state variables include: number of particles (**N**), pressure (**P**), volume (**V**), temperature (**T**), thermal energy ($\mathbf{E^{th}}$), entropy (**S**), chemical potential (**μ**), magnetization (**M**), and many more. When studying matter, the trick is to choose the state variables that are relevant to the physics of the observation/experiment. For the remainder of our thermodyanmic studies, we will not study magnetic materials and only consider systems with no particles entering or leaving our system; thus we will not consider magentization and chemical potential. The state variables that we will work with quantitatively are: number of particles, pressure, volume, and temperature.

Number of Particles (N) and Moles (n)

There are two common ways to represent the amount of particles/molecules in a system: number of particles (**N**), and moles (**n**). One mole is defined as the ratio of number of particles to 6.022 x 10^{23} particles: **n** = **N**/6.022x10^{23} . Both number of particles and moles can represent individual atoms or molecules depending on the system of interest.

Often times in thermodynamics the periodic table is needed to go back and forth with a laboratory measurement and number of particles or moles. This requires us to expand our vocabulary into the realm of chemistry. Below are the basic definitions that may be useful when studying thermodynamics.

Atomic mass number (**A**) - The number of protons and neutrons in an element. Found on the superscript of any atomic symbol.

Example: The atomic number for ^{12}C is **A** = 12.

Atomic mass unit (**u**) - Measure of mass defined to be exactly equal to one twelfth the mass of a 12C atom.

Example: The atomic mass unit of ^{12}C is equal to 12 **u**.

1 **u** = 1.66×10^{-27} kg

Atomic mass (**m**$_{\text{atom}}$) - The mass of an atom. For our purposes, the atomic mass is basically equal to the atomic mass number **A**.

Example: The atomic mass of carbon 12 is $\mathbf{m}_{^{12}\mathrm{C}} = 12\ \mathbf{u}$

Molecular mass (**m**$_{\text{molecule}}$) - The mass of a molecule which is the sum of the atomic masses of the atoms that form the molecule.

Example: The molecular mass of an oxygen gas molecule O_2 is $\mathbf{m}_{\mathrm{O}_2} = \mathbf{m}_{^{16}\mathrm{O}} + \mathbf{m}_{^{16}\mathrm{O}} = 16\ \mathbf{u} + 16\ \mathbf{u} = 32\ \mathbf{u}$

Molar mass (**M**$_{\text{mol,atom or molecule}}$) - The mass in grams for 1 mole of substance. For this class we can get away with using the atomic or molecular mass as the molar mass.

Example: The molar mass of oxygen gas O_2 is $\mathbf{M}_{\mathrm{mol,O}_2} = 32\ \mathrm{g/mol}$

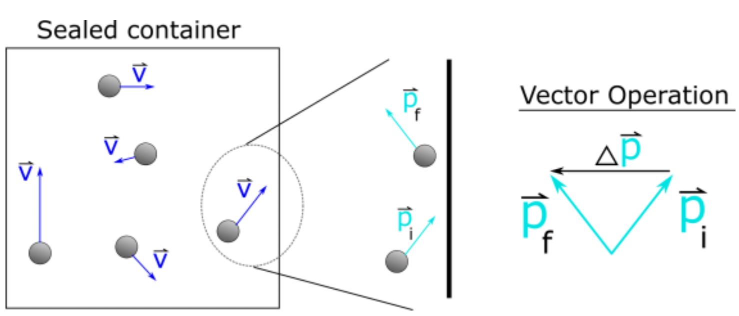
With the above basic definitions, we can also define moles as the ratio of the atomic or molecular mass to the molar mass as shown below.

$$\mathbf{n} = \frac{\mathbf{m}_{\text{atom or molecule}}}{\mathbf{M}_{\text{mol,atom or molecule}}}$$

Pressure (P)

Microscopic view

Consider a gas held in a sealed container. Pressure is a collective force per area from impulses due to the collisions of th e particles with the container walls. As seen in the image below, when a particle hits the wall, the change in momentum of the particle is roughly horizontally to the left, thus the particle imparted a force on the wall to the right. The collectiv e forces from all of the collisions on the area of the wall create the pressure.



Macroscopic view

As seen from the microscopic view of matter, the collisions from the particles on the wall exert a force on the wall and thus pressure. Unfortunately there are a lot of particles hitting the wall, so much that it's impossible to track the motion of all the particles. Thus the macroscopic view of pressure is the net effect of these microscopic collisions. The macroscopic definition for pressure is force/area. The SI units of pressure are Pascals (Pa) which are newtons/meters ^{2}.

Volume (V)

Volume is a measure of a physical length cubed. There are various types of systems that one might encounter when studying thermodynamics based off of their energy and mass transfer which best fit under this volume section.

Open system - System can exchange mass and energy with surroundings.

Example: A plastic container with a hole in is such that gas can escape.

Closed system - System cannot exchange mass with surroundings, but can exchange energy with surroundings.

Example: A plastic container with no holes such that the gas cannot escape and other gases outside can't enter.

Isolated system - System cannot exchange mass and energy with surroundings.

Example: A well-constructed thermos insulated from its surroundings.

Temperature (T)

Temperature is a macroscopic quantity that is a measure of the average microscopic translational kinetic energy of an object. The SI unit of temperature is Kelvin (**K**). Two other common units are Celsius (°**C**) and Fahrenheit (°**F**). Conversions between each unit are readily found online.

Act II: Kinetic Theory of Gases

MG.2.L1-2:

Description: Proportional reasoning with temperature and kinetic energy. (3 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: If the temperature of a gas triples, by what factor does the average translational kinetic energy of the gas change by?

$\overline{KE_t}$

(1) 1/9
(2) 1/6
(3) 1/3
(4) 1
(5) 3
(6) 6
(7) 9

$$\overline{KE_t} = \frac{3}{2} k_B T$$

$$\text{new } \overline{KE_t} = \frac{3}{2} k_B (3T_{old}) = 3 \times \overline{KE_t}_{old}$$

MG.2.L1-3:

Description: Given initial moles, pressure, and temperature, find largest average translational kinetic energy per molecule. (4 minutes)

Learning Objectives: [1, 12, 13]

$\overline{KE_t}$

Problem Statement: Consider the table of gases below. Which system has the largest average translational kinetic energy per molecule? Assume the temperatures are sufficiently low enough that the diatomic molecules acts like monatomic molecules (i.e. rotational and vibrational effects are negligible).

(1) A
(2) B
(3) C
(4) D
(5) E

MC Letter	Quantity (mol)	Gas	Pressure (atm)	Temperature (K)
A	2	He	2	300
B	2	N_2	0.5	450
C	1	He	1	300
D	1	N_2	0.5	600
E	1	Ar	0.5	450

$$\overline{KE_t} = \frac{3}{2} k_B \underbrace{T}_{\text{only quantity that } \overline{KE_t} \text{ depends on!}}$$

MG.2.L1-4:

Description: Apply an energy analysis involving thermal energy and temperature. (4 minutes)

Learning Objectives: [1, 12, 13]

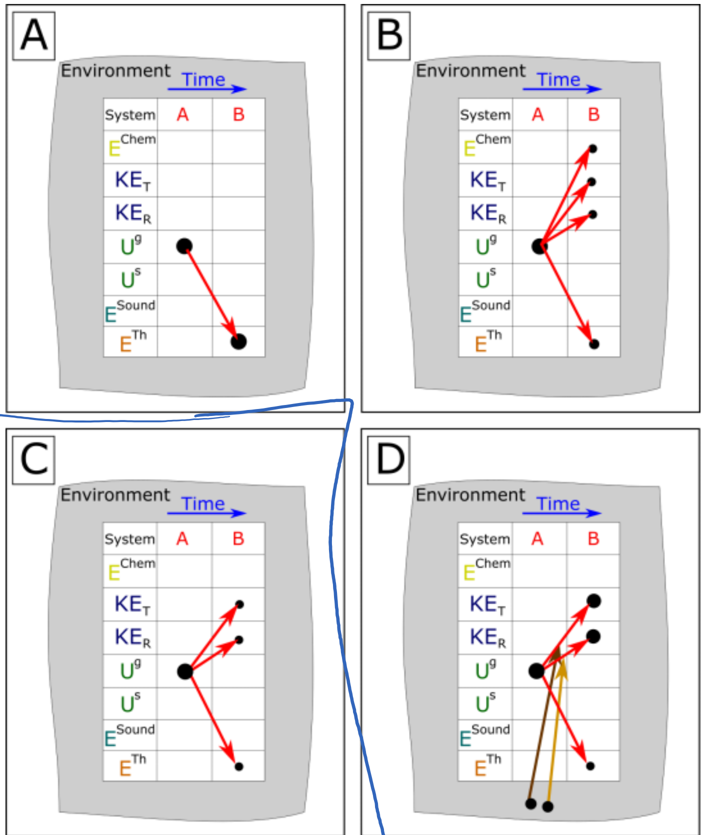
Problem Statement: A paddle wheel frictionally adds thermal energy to 5.0 moles of an ideal monatomic gas in a sealed isolated container. The paddle wheel is driven by a cord connected to a falling 2.0 kg mass falling shown in the figure. We wish to determine how far the 2.0 kg mass has fallen when the temperature of the gas increases by 10 K.

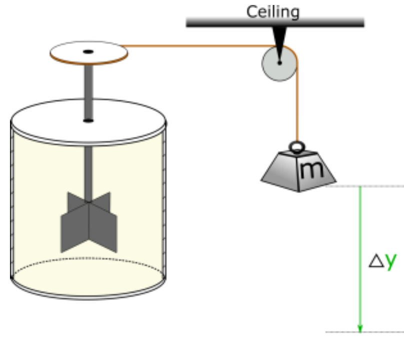
(a) Which of the energy flow diagrams best represents this scenario when snapshots are taken at the following times:

A: The 2 kg mass is released from rest at some height above the ground.

B: The 2 kg mass is falling downwards, the pulleys are spinning, the paddle wheel is spinning, and the gas has increased in temperature.

System: 2 kg mass + pulleys + rope + paddle wheel + Earth + gas





(b) If the if all of the kinetic energies (rotational and translational) are negligible, which of the following mathematical representations best describes an energy analysis for this situation as the mass falls at a constant speed.

(1) $\Delta E^{th} + \Delta KE_T + \Delta KE_R + \Delta U^g + \Delta E^{Chem} = 0$

(2) $\Delta E^{th} + \Delta KE_T + \Delta U^g = 0$

(3) $\Delta E^{th} + \Delta U^g = 0$

(4) $\Delta KE_T + \Delta U^g + \Delta E^{Chem} = 0$

or equiv. $\Delta E_{th} = -\Delta U^g$

thermal E gain

amount of grav. pot. E lost

(c) Which of the following is a simplification for the correct answer in part (b) that allows us to solve for the change in height required?

(1) $(3/2)\, N k_B T + (1/2)\, m v^2 + m g \Delta y = 0$

(2) $(3/2)\, N k_B \Delta T + m g \Delta y = 0$

(3) $-\mu_k F^N d + m g \Delta y = 0$

(4) $(1/2)\, m v^2 + m g \Delta y = 0$

$$\Delta E_{th} = N\,\overline{\Delta KE_t} = N \tfrac{3}{2} k_B \Delta T$$

$$\Delta U^g = mg\Delta y$$

$$\Rightarrow 0 = \Delta E_{th} + \Delta U^g = \boxed{\tfrac{3}{2} N k_B \Delta T + mg\Delta y = 0}$$

(d) A paddle wheel frictionally adds thermal energy to 5.0 moles of an ideal monatomic gas in a sealed isolated container. The paddle wheel is driven by a cord connected to a falling 2.0 kg mass. Assume all kinetic energies are negligible compared to the change in thermal and gravitational potential energy. Calculate how far the 2.0 kg mass has fallen when the temperature of the gas increases by 10 K assuming negligible rotational kinetic energy from the pulleys and paddle wheel.

$$N k_B = nR \qquad N = n(6.02\times10^{23}) \qquad k_B = \frac{R}{6.02\times10^{23}}$$

$$mg\Delta y = -\tfrac{3}{2} N k_B \Delta T$$

$$\Delta y = -\frac{3}{2}\frac{N k_B \Delta T}{mg} = -\frac{3}{2}\frac{nR\Delta T}{mg}$$

$$\Delta y = -\frac{3}{2}\,\frac{(5)(8.32)(10)}{(2)(9.8)} = -31.8\ m$$

Act III: Thermodynamic Equilibrium

MG.2.L1-5:

Description: Compare temperatures of gases when in equilibrium (2 minutes).

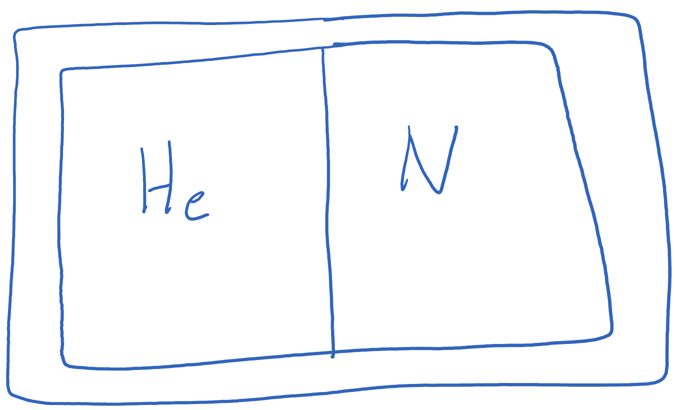
Learning Objectives: [1, 12, 13]

Problem Statement: You have two isolated containers in contact with each other with a shared closed wall. One container is full of helium gas. The other holds a larger number of nitrogen gas molecules. Both gases are in equilibrium with each other. How does the temperature of the helium compare to the temperature of the nitrogen gas?

(1) $T_{He} > T_N$

(2) $T_{He} < T_N$

(3) $T_{He} = T_N$



MG.2.L1-6:

Description: Compare thermal energies of gases when in equilibrium. (4 minutes)

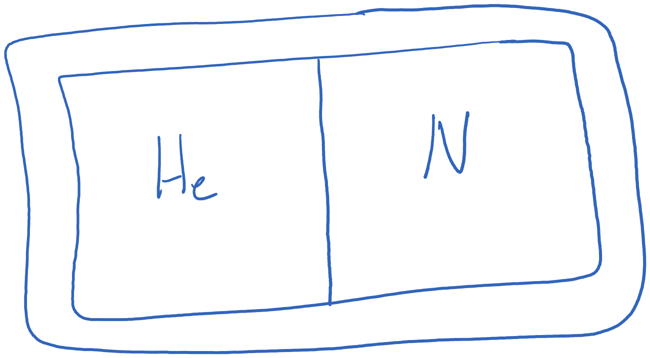
Learning Objectives: [1, 12, 13]

Problem Statement: You have two isolated containers in contact with each other with a shared closed wall. One container is full of helium gas. The other holds a larger number of nitrogen gas molecules. Both gases are in equilibrium with each other. How does the total internal energy of the helium compare to the total internal energy of the nitrogen?

(1) $E_{He} > E_N$

$\boxed{(2)\ E_{He} < E_N}$

(3) $E_{He} = E_N$



$$E_{th} = N\,\overline{KE_t} = N\left(\tfrac{3}{2}k_B T\right)$$

$$\underbrace{N_{He}}\left(\tfrac{3}{2}k_B \underbrace{T_{He}}\right) \overset{?}{\lesseqgtr} \underbrace{N_N}\left(\tfrac{3}{2}k_B \underbrace{T_N}\right)$$

$T_{He} = T_N$

$N_{He} < N_N$

MG.2.L1-7:

Description: Compare temperature, average speed, kinetic energy, and thermal energy of gases in equilibrium. (8 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: A closed system with 2 regions (**A** and **B**) are isolated from the surroundings. Regions **A** and **B** both contain an equal number of different monatomic gas particles. The gases are in thermal equilibrium with each other. The mass of the gas in **A** is four times that of the mass of the gas in **B**. Which of the following statements are ***true*** regarding this situation?

~~(1)~~ The temperature of **B** is four times that of **A.**

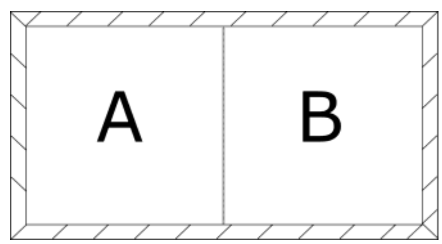
T (2) The temperature of **B** is equal to the temperature of **A**.

~~(3)~~ The average molecular speed of **B** is equal to that of **A**.

T (4) The average molecular speed of **B** is twice that of **A**.

~~(5)~~ The average kinetic energy of **A** is a fourth that of **B**. $\tfrac{3}{2}k_B T_A = \tfrac{3}{2}k_B T_B$

T (6) The total internal energy of **A** is equal to **B**. $T_A = T_B,\ N_A = N_B \Rightarrow E_{th,A} = E_{th,B}$



equilib. $\Rightarrow \overline{KE_t} = \overline{KE_t}$ or $\tfrac{3}{2}k_B T_A = \tfrac{3}{2}k_B T_B$

$\Rightarrow T_A = T_B$

$\Rightarrow \tfrac{1}{2}\overline{m}_A v_{rms,A}^2 = \tfrac{1}{2}\overline{m}_B v_{rms,B}^2$

$v_{rms} \Rightarrow$ root-mean-squared

$$v_{rms} = \sqrt{\sum_{i=1}^{N}(v_i)^2}$$

$m_A = 4\,m_B$

$\Rightarrow v_{rms,B}^2 = 4\,v_{rms,A}^2$

$\Rightarrow v_{rms,B} = 2\,v_{rms,A}$

Conceptual questions for discussion

1. Stay tuned

Hints

MG.2.L1-1: Recall that average acceleration is the change in velocity divided by change in time. Also, a change in a vector quantity can be found by placing the initial and final vectors tail to tail, with the change pointing from the initial to the final.

MG.2.L1-2: Recall that the net external force causes an acceleration of the center of mass of the object, thus first determine which direction the acceleration of the puck is.

MG.2.L1-3: Use a vector operation diagram to help determine the direction of the average acceleration.

MG.2.L1-4: If an object is speeding up and moving in a circle, there must be a component of acceleration in the tangential direction. Discuss with your neighbors what "speeding up at a decreasing rate" means with regards to tangential acceleration.

MG.2.L1-5: No hints.

MG.2.L1-6: No hints.

MG.2.L1-7: No hints.