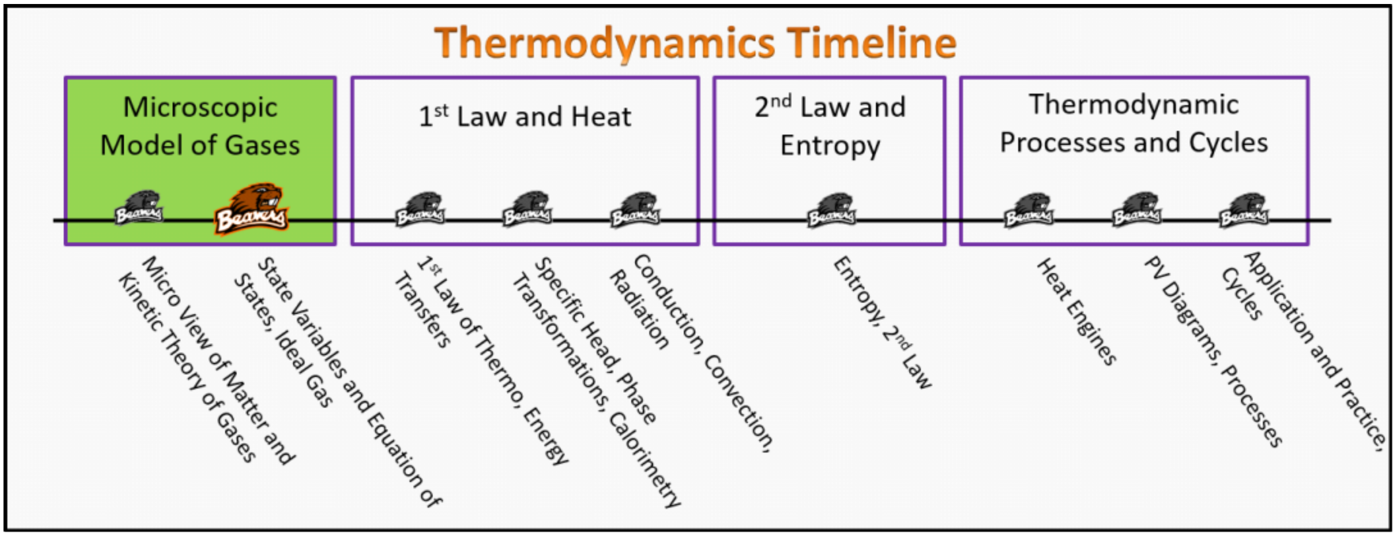
MG.2.L2 - Solutions

Friday, January 25, 2019 7:31 AM

Thermodynamics
Foundation Stage (MG.2.L2)

Lecture 2
State Variables and Equation of States, Ideal Gas



Textbook Chapters (* Calculus version)

- **BoxSand** :: KC videos (ideal gas law)
- **Knight** (College Physics : A strategic approach 3rd) :: 12.2
- ***Knight** (Physics for Scientists and Engineers 4th) :: 18.6
- **Giancoli** (Physics Principles with Applications 7th) :: 13-5 ; 13-6 ; 13-7

Warm up

MG.2.L1-1:

Description: Apply energy analysis to determine change in thermal energy.

Learning Objectives: [?] - Can you identify the objectives from the previous lecture, and this lecture, that this question is relevant to?

Problem Statement: A 0.145 kg baseball is traveling 35 m/s horizontally before it collides with a glove. The baseball is not rotating. Assuming all of the translational kinetic energy of the baseball goes into changing the thermal energy of the baseball and glove, how much increase in thermal energy is this in joules?

Selected Learning Objectives

1. Coming soon to a lecture template near you.

Key Terms

- Average translational kinetic energy per particle
- Average mass
- Root mean square speed
- Thermal energy
- Boltzmann's postulate
- Boltzmann's constant
- Temperature
- Thermodynamic equilibrium

Key Equations

Key Concepts

- Stay tuned...

Questions

Act I: Connecting micro to macro

MG.2.L1-2:

Description: Proportional reasoning with temperature and kinetic energy. (3 minutes)

Learning Objectives: [1, 12, 13]

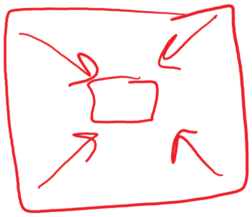
Problem Statement: Determine if the pressure increases, decreases, or stays the same in the following scenarios. Try to use the microscopic arguments of the gas particles to support your decision to your neighbors.

(a) The volume of a sealed container decreases while the number of particles and temperature stays the same. (1) Pressure increases (2) Pressure decreases (3) Pressure remains the same.	**(b)** The temperate of a gas increases while the volume and number of particles stays the same. (1) Pressure increases (2) Pressure decreases (3) Pressure remains the same.	**(c)** The number of particles increase while the temperature and volume remain constant. (1) Pressure increases (2) Pressure decreases (3) Pressure remains the same.

Same temp ⇒ same

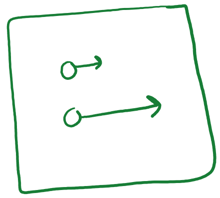
T ↑ ⇒ $\overline{KE}$ ↑ ⇒ more

N ↑ ⇒ more particles

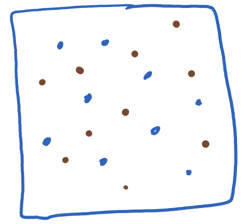


Same temp ⇒ same avg KE ⇒ same avg speed ⇒ less time between collisions with walls ⇒ P↑

T↑ ⇒ $\overline{KE_t}$↑ ⇒ more speed ⇒ less time between collisions with walls ⇒ P↑



N↑ ⇒ more particles @ same speed ⇒ more collisions ⇒ P↑



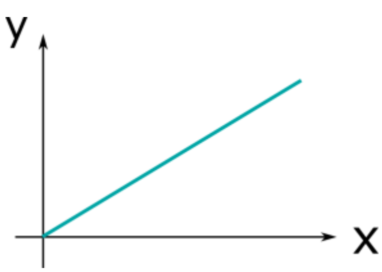
MG.2.L1-3:

Description: Given initial moles, pressure, and temperature, find largest average translational kinetic energy per molecule. (4 minutes)

Learning Objectives: [1, 12, 13]

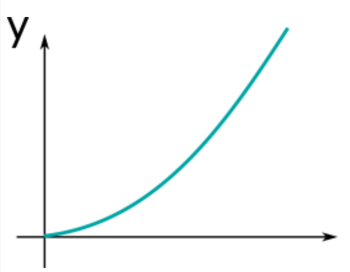
Problem Statement: Consider the three graphs below. Identify the proportionality of each plot (e.g. If you think the plot represents **y** = 5 $\mathbf{x}^3$, then the proportionality would be $\mathbf{x}^3$).

(a)

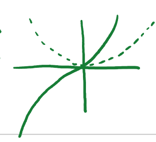


(1) **x**
(2) $\mathbf{x}^2$
(3) $\mathbf{x}^3$
(4) 1/**x**

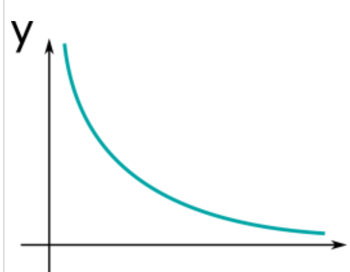
(b)



(1) **x**
(2) $\mathbf{x}^2$
(3) $\mathbf{x}^3$
(4) 1/**x**



(c)



(1) **x**
(2) $\mathbf{x}^2$
(3) $\mathbf{x}^3$
(4) 1/**x**

Act II: Ideal gas equation of state

MG.2.L1-4:

Description: Apply an energy analysis involving thermal energy and temperature. (4 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: One mole of an ideal gas has a set of state variables: $\mathbf{T}_1$, $\mathbf{P}_1$, $\mathbf{V}_1$. If the pressure is tripled while the temperature is held constant, what is the final volume in terms of the initial?

(1) $9\mathbf{V}_1$
(2) $3\mathbf{V}_1$
(3) $\mathbf{V}_1$
(4) $\mathbf{V}_1/3$
(5) $\mathbf{V}_1/9$

$$P_1 V_1 = nRT_1$$

$$V_1 = nR\frac{T_1}{P_1}$$

$V(P) \propto \frac{1}{P} \Rightarrow$ previous quest.

$$V_2 = nR\frac{T_2}{P_2} = nR\frac{T_1}{3P_1} = \frac{1}{3}\left(nR\frac{T_1}{P_1}\right) = \frac{1}{3}V_1$$

MG.2.L1-5:

Description: Compare temperatures of gases when in equilibrium (2 minutes).

Learning Objectives: [1, 12, 13]

Problem Statement: One mole of an ideal gas has a set of state variables: $\mathbf{T}_1$, $\mathbf{P}_1$, $\mathbf{V}_1$. If the volume is doubled while the temperature is tripled, what is the final pressure in terms of the initial?

(1) $\mathbf{P}_1/3$
(2) $\mathbf{P}_1/2$
(3) $2\mathbf{P}_1/3$
(4) $\mathbf{P}_1$
(5) $3\mathbf{P}_1/2$
(6) $3\mathbf{P}_1/4$
(7) $2\mathbf{P}_1$
(8) $3\mathbf{P}_1$

$$P_1 V_1 = nRT_1$$

$$P_1 = nR\frac{T_1}{V_1}$$

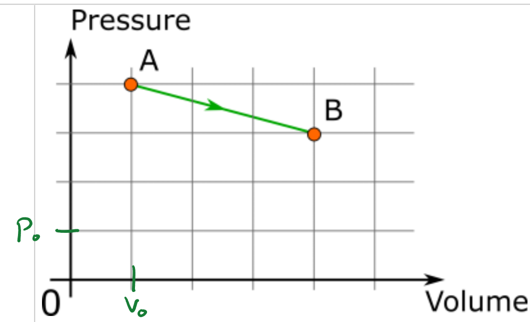
$$P_2 = nR\frac{T_2}{V_2} = nR\frac{3T_1}{2V_1} = \frac{3}{2}\left(nR\frac{T_1}{V_1}\right) = \frac{3}{2}P_1$$

MG.2.L1-6:

Description: Compare thermal energies of gases when in equilibrium. (4 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: An ideal gas trapped in a cylinder expands while is pressure drops. The starting point for this process is labeled **A** and the end point is labeled **B** in the graph of pressure versus volume shown below.



(a) Which of the following are correct with respect to what happens to the pressure?

(1) $\mathbf{P}_f = 1/2\ \mathbf{P}_i$
(2) $\mathbf{P}_f = 3/4\ \mathbf{P}_i$
(3) $\mathbf{P}_f = \mathbf{P}_i$
(4) $\mathbf{P}_f = 3\ \mathbf{P}_i$

$P_i = 4P_o$

$P_f = 3P_o$

(b) Which of the following are correct with respect to what happens to the volume?

(1) $\mathbf{V}_f = 1/2\ \mathbf{V}_i$
(2) $\mathbf{V}_f = 3/4\ \mathbf{V}_i$
(3) $\mathbf{V}_f = 3\ \mathbf{V}_i$
(4) $\mathbf{V}_f = 4\ \mathbf{V}_i$

$V_f = 4V_o$

$V_i = V_o$

(c) What is the final temperature in terms of the initial?

(1) $\mathbf{T}_f = 3/16\ \mathbf{T}_i$
(2) $\mathbf{T}_f = 3\ \mathbf{T}_i$
(4) $\mathbf{T}_f = 4\ \mathbf{T}_i$
(5) $\mathbf{T}_f = 16/3\ \mathbf{T}_i$

$\frac{T_f}{T_i} = \frac{P_f V_f}{P_i V_i} = \left(\frac{3}{4}\right)(4)$

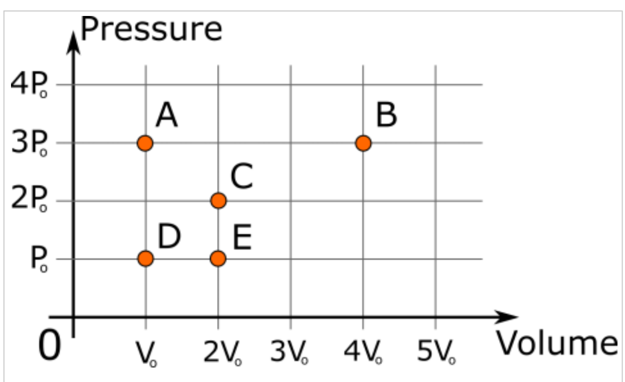
$T_f = 3T_i$

MG.2.L1-7:

Description: Compare temperature, average speed, kinetic energy, and thermal energy of gases in equilibrium. (8 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: Five points representing five different equilibrium states of one mole of an idea gas are labeled on the pressure-volume graph shown below. Rank the temperatures of the idea gas in each equilibrium state.



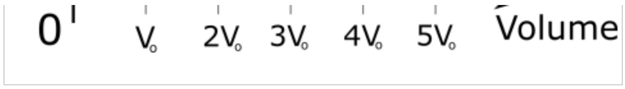
$T = PV/nR \Rightarrow \propto PV$

Smallest → D: $P_o V_o \Rightarrow P_o V_o \Rightarrow T_o$

E: $P_o(2V_o) \Rightarrow 2T_o$

A: $(3P_o)V_o \Rightarrow 3T_o$

C: $(2P_o)(2V_o) \Rightarrow 4T_o$



$C: (2P_0)(2V_0) \Rightarrow 4T_0$

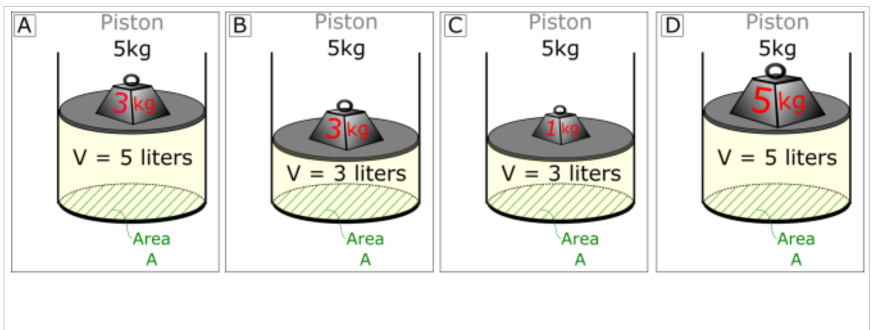
largest $\rightarrow B: (3P_0)(4V_0) \Rightarrow 12T_0$

MG.2.L1-8:

Description: Compare temperature, average speed, kinetic energy, and thermal energy of gases in equilibrium. (8 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: Cylinders with equal cross-sectional areas contain different volumes of an idea gas sealed in by pistons. There is a weight sitting on top of each piston. The gas is the same in all four cases and is at the same temperature. The pistons are free to move without friction. Rank the pressure of the gas in each cylinder.



Same areas A

$\Rightarrow PA =$ force holding weights up

$\Rightarrow P_C < P_A = P_B < P_D$

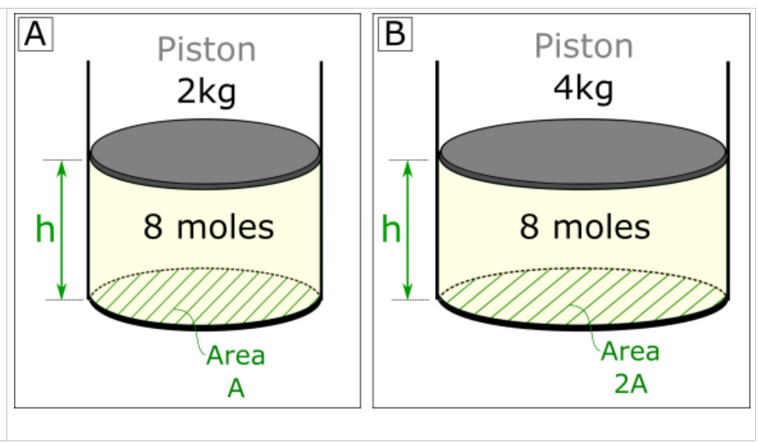
MG.2.L1-9:

Description: Compare temperature, average speed, kinetic energy, and thermal energy of gases in equilibrium. (8 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: Two cylinders are filled to the same height H with ideal gases. The gases are different, and the cross-sectional areas of the cylinders are different. Both cylinders have pistons that are free to move without friction. The temperature of the gas in cylinder A is ___________________ the temperature of the gas in cylinder B.

(1) greater than
(2) less than
(3) equal to



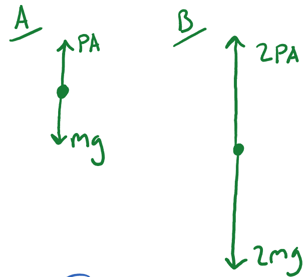
$(P_B A_B)_B = 2(P_A A_A)$ A $\uparrow PA$ B $\uparrow 2PA$

$n_A = n_B = 8$ moles $V = \frac{1}{2} V$

$(P_B A_B)_B = 2(P_A A_A)$

$P_B 2A = 2 P_A A$

$\Rightarrow P_A = P_B$



$n_A = n_B = 8_{moles}$ $V_A = \frac{1}{2} V_B$

$PV = nRT$

$\Rightarrow T = \frac{PV}{nR}$

$\frac{P_A V_A}{n_A R} \overset{?}{\lesseqgtr} \frac{P_B V_B}{n_B R}$, $V_B = 2V_A$

$T_A < T_B$

Conceptual questions for discussion

1. Stay tuned

Hints

MG.2.L1-1: Recall that average acceleration is the change in velocity divided by change in time. Also, a change in a vector quantity can be found by placing the initial and final vectors tail to tail, with the change pointing from the initial to the final.

MG.2.L1-2: Recall that the net external force causes an acceleration of the center of mass of the object, thus first determine which direction the acceleration of the puck is.

MG.2.L1-3: Use a vector operation diagram to help determine the direction of the average acceleration.

MG.2.L1-4: If an object is speeding up and moving in a circle, there must be a component of acceleration in the tangential direction. Discuss with your neighbors what "speeding up at a decreasing rate" means with regards to tangential acceleration.

MG.2.L1-5: No hints.

MG.2.L1-6: No hints.

MG.2.L1-7: No hints.