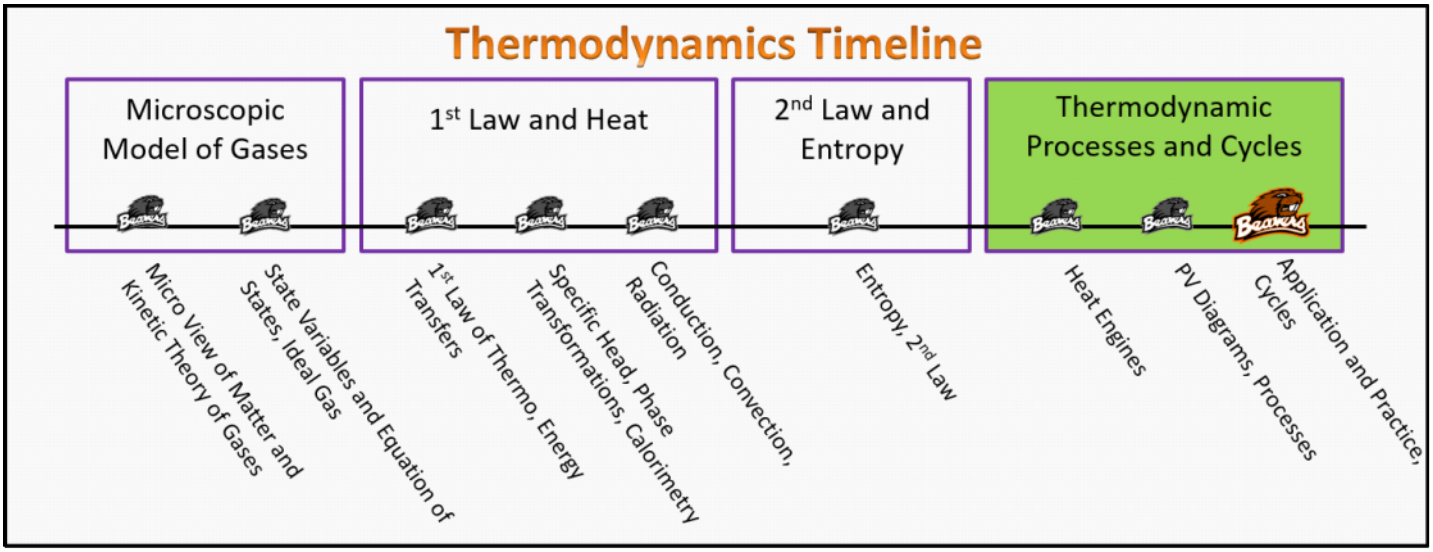
PC.2.L3 - Solutions

Thursday, February 7, 2019 1:16 PM

Thermodynamics
Foundation Stage (PC.2.L3)

Lecture 3
Application and Practice, Cycles



Textbook Chapters (* Calculus version)

- **BoxSand** :: KC videos (Thermodynamic Cycles)
- **Knight** (College Physics : A strategic approach 3rd) :: N/A
- ***Knight** (Physics for Scientists and Engineers 4th) :: 21.2 ; 21.3 ; 21.4 ; 21.6
- **Giancoli** (Physics Principles with Applications 7th) :: N/A

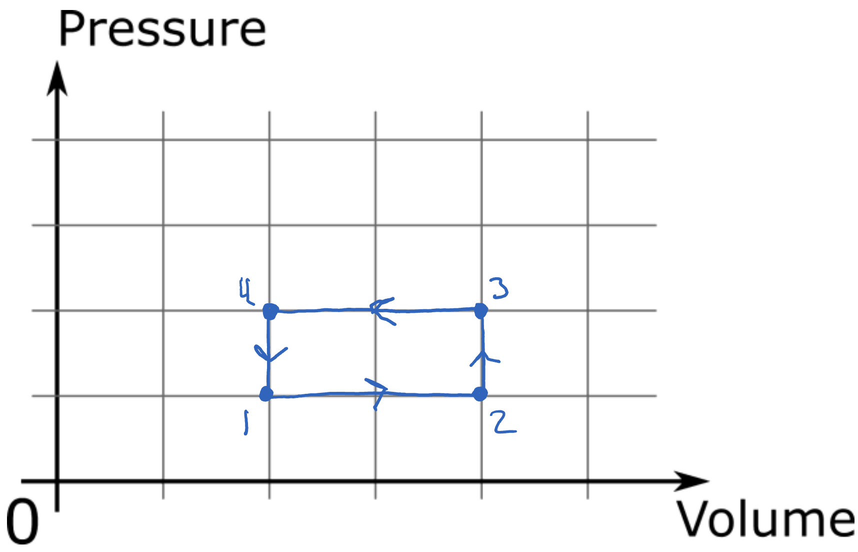
Warm up

PC.2.L3-1:

Description: Sketch the curves on a PV diagram given the type of process.

Learning Objectives: [?] - Can you identify the objectives from the previous lecture, and this lecture, that this question is relevant to?

Problem Statement: On the PV diagram below, sketch a curve that represents an ideal gas taken through an isobaric process that doubles the volume from one equilibrium state to another equilibrium state. After the gas is at this new equilibrium state, then sketch the curve that represents taking the gas thorough an isochoric doubling of pressure to a third equilibrium state. Finally, the gas is taken back to its original equilibrium state via one more isobaric compression and isochoric decrease in temperature.



Selected Learning Objectives

1. Coming soon to a lecture template near you.

Key Terms

- Thermodynamic cycle
- Efficiency

Key Equations

Key Concepts

- Coming soon to a lecture template near you.

Questions

Act I: Isochoric

PC.2.L3-2:

Description: Use a PV diagram to determine net work for a cycle. Determine efficiency given net heat in. (4 minutes + 2 minutes + 2 minutes + 3 minutes + 3 minutes)

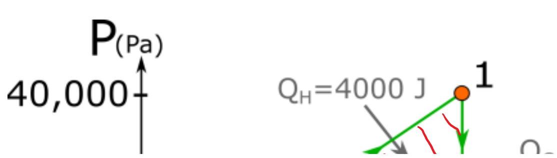
Learning Objectives: [1, 12, 13]

Problem Statement: Consider the PV diagram shown below.

(a) What are the units of the area enclosed by the cycle? Hint: Pressure is force per area.

$[P \times V] = \left(\frac{N}{m^2}\right)(m^3) = Nm$

(1) 1,000 N·m



force per area.

$[P \times V] = \left(\frac{N}{m^2}\right)(m^3) = Nm$

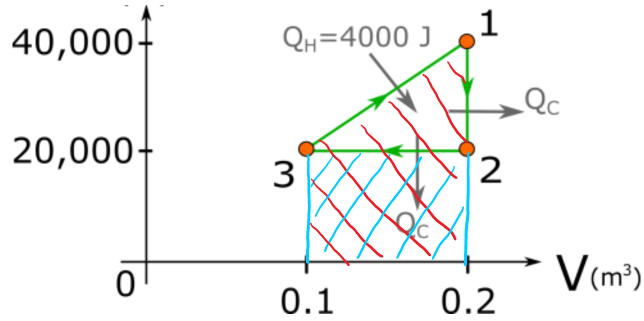
(1) 1,000 N·m ✓
(2) 1,000 J
(3) 1,000 kg·m²/s²

$Nm = J = kg\frac{m}{s^2}m$

(b) What is the sign of the net work?

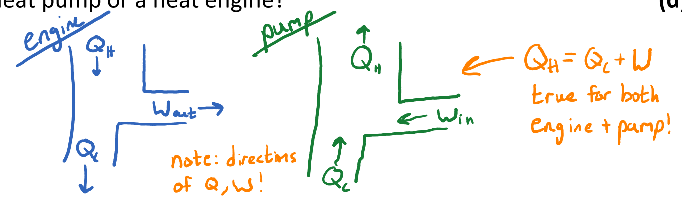
(1) Positive
(2) Negative
(3) Zero

$W_{1\to2} = 0$ (isochoric)
$W_{3\to1}$ = negative
$W_{2\to3}$ = positive
$|W_{3\to1}| > |W_{2\to3}|$ ⇒ negative net work



(c) Is this a heat pump or a heat engine?

negative net work ⇒ engine



(d) What is the efficiency of this cycle?

$$\text{eff.} = \frac{\text{usable } E \text{ out}}{E_{in}} \Rightarrow \frac{W_{out}}{Q_H} = \frac{1000\,J}{4000\,J} = 0.25 = 25\%$$

(e) Is there more total heat entering or exiting the system in a complete cycle?

$Q_H = Q_c + W_{out}$ ⇒ more enters

See diagrams for part (c)

PC.2.L3-3:

Description: Identify the most commonly used physics when analyzing a cycle problem. (3 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: As an up-and-coming thermal engineer you decide to get a few tattoos regarding thermodynamic cycles. Which of the following equations/concepts would you get on your sleeve if you plan to attend Thermo U?

✓ (1) $\Delta E^{TH} = 3/2\, N\, k_B\, \Delta T$ ← ideal monatomic gas
✓ (2) $\Delta E^{TH} = W + Q$ ← 1st law of thermo
✓ (3) $P\,V = N\,k_B\,T$ ← Ideal gas law
✓ (4) **Work** = ± **area** under PV curve
(5) $Q/\Delta t = k\,A\,\Delta T / L$ ← conduction
(6) $Q/\Delta t = e\,\sigma\,A\,T^4$ ← radiation

PC.2.L3-4:

Description: Cycle problem. (30 minutes)

Learning Objectives: [1, 12, 13]

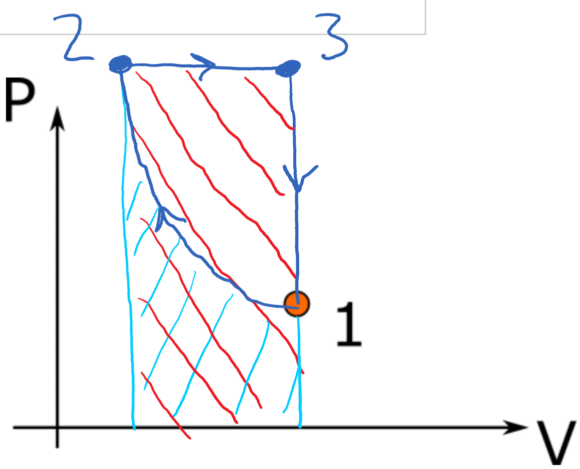
Problem Statement: 25 moles of an ideal monatomic gas undergoes a thermodynamic cycle consisting of three processes:

Process **1** → **2** :: Compression with **P V** = Constant, from P_1 = 100 kPa , V_1 = 1.6 m3 to V_2 = 0.2 m3.

Process **2** → **3** :: Constant pressure to $V_3 = V_1$.

Process **3** → **1** :: Constant volume with $E_1 - E_3$ = -1680 kJ.

(a) Use the **PV** to the right to sketch this cycle.



(b) Is this a power or refrigeration cycle?

power

$W_{2\to3}$, $W_{1\to2}$ } net negative work

CW ⇒ negative work!

$W_{3\to1} = 0$

(c) Given P_1, V_1, and **n**, what is T_1?

$PV = nRT$

$T_1 = \frac{(100{,}000)(1.6)}{(25)(8.31)} = 770\ K$

n = 25 moles	1	2	3
P (kPa)	100	800	800
V (m^3)	1.6	0.2	1.6
T (K)	770	770	6160

(d) What is the pressure at equilibrium state **2**?

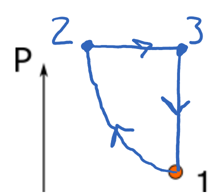
$PV = \underbrace{nRT}_{const} \Rightarrow V\downarrow_8 \Rightarrow P\uparrow^8 \Rightarrow 800{,}000\ Pa$

(e) What is the temperature at equilibrium state **3**?

$PV = nRT \Rightarrow \underbrace{\frac{P}{nR}}_{const} = \frac{T}{V} \quad V\uparrow^8 \Rightarrow T\uparrow^8 \Rightarrow 6{,}160\ K$

(f) Which of the following quantities are zero?

	1 → 2	2 → 3	3 → 1	Complete Cycle
ΔE^{TH} (kJ)	0	1680	-1680	0
W (kJ)		-1,120	0	
Q (kJ)		[illegible]	[illegible]	



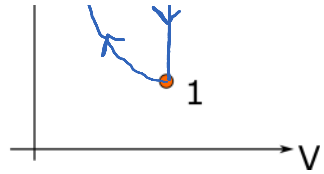
(1) $\Delta E^{TH}_{1\ to\ 2}$ ← $\Delta T = 0$

(2) $\Delta E^{TH}_{2\ to\ 3}$ ...

quantities are zero?

(1) $\Delta E^{TH}_{1\ to\ 2}$ ← $\Delta T = 0$
(2) $\Delta E^{TH}_{2\ to\ 3}$ +
(3) $\Delta E^{TH}_{3\ to\ 1}$ −
(4) ΔE^{TH}_{Total} ← $\Delta T = 0$
(5) $W_{1\ to\ 2}$ +
(6) $W_{2\ to\ 3}$ −
(7) $W_{3\ to\ 1}$ ← $\Delta V = 0$
(8) W_{Total} −
(9) $Q_{1\ to\ 2}$
(10) $Q_{2\ to\ 3}$
(11) $Q_{3\ to\ 1}$
(12) Q_{Total}

ΔE^{TH} (kJ)	0	1680	-1680	0
W (kJ)		−1,120	0	
Q (kJ)		+2800	−1680	



(g) What is the change in thermal energy from **2** to **3**?

$0 = 0 + W_{2\to3} - 1680 \Rightarrow W_{2\to3} = +1680$

(h) What is the heat transfer from **3** to **1**?

$\Delta E^{th} = W + Q$

$-1680\,J = 0 + Q_{3\to1} \Rightarrow Q_{31} = -1680\,J$

(i) Which function would you use to calculate the work from **2** to **3**?

(1) **P** Δ**V**
(2) - **P** Δ**V**
(3) **n R T** ln(**V**f/**V**i)
(4) - **n R T** ln(**V**f/**V**i)
(5) **P V** ln(**V**f/**V**i)
(6) - **P V** ln(**V**f/**V**i)

$= -(800{,}000)(1.6 - 0.2)$

$= -1{,}120{,}000\,J$

(j) Is the work from **2** to **3** positive or negative?

expansion ⇒ negative

(k) Calculate the heat from **2** to **3**.

$\Delta E_{th} = W + Q \Rightarrow 1680 = -1120 + Q$

$Q = 2800$

(l) Which function would you use to calculate the work from **1** to **2**?

area under isotherm $= -nRT\ln\left(\frac{V_f}{V_i}\right)$

(1) **P** Δ**V**
(2) - **P** Δ**V**
(3) **n R T** ln(**V**f/**V**i)
(4) - **n R T** ln(**V**f/**V**i)
(5) **P V** ln(**V**f/**V**i)
(6) - **P V** ln(**V**f/**V**i)

$V_f < V_i \Rightarrow \ln\left(\frac{V_f}{V_i}\right) < 0$

$\Rightarrow -nRT\ln\left(\frac{0.2}{1.6}\right)$ will be +

$= -(25)(8.31)(770)\ln\left(\frac{0.2}{1.6}\right)$

$= 333{,}000\,J$

	1 → 2	2 → 3	3 → 1	Complete Cycle
ΔE^{TH} (kJ)	0	1680	-1680	0
W (kJ)	333	−1,120	0	−787
Q (kJ)	−333	+2800	−1680	+787

(m) Is the work from **1** to **2** positive or negative?

Compression ⇒ positive

(n) Calculate the heat from **1** to **2**.

$\Delta E_{th} = W + Q$

$0 = +333 + Q_{12}$

$Q_{1-2} = -333$

(o) Finish the table of energies.

$\Delta E^{th}_{tot} = W_{tot} + Q_{tot}$ ✓

(p) Calculate the efficiency of this cycle.

$eff. = \frac{\Sigma W}{Q_H} = \frac{787\,kJ}{2800\,kJ} = 0.28$ or 28%

Conceptual questions for discussion

1. Coming soon.

Hints

PC.2.L3-1: No hints.

PC.2.L3-2: No hints.

PC.2.L3-3: No hints.

PC.2.L3-4: No hints.