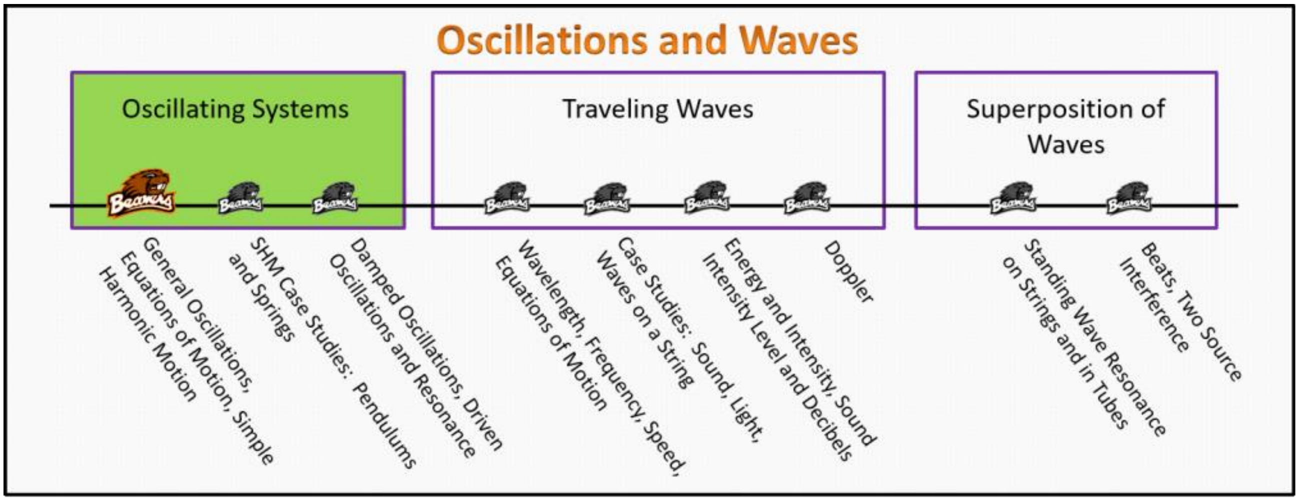
OS.2.L1 - Solutions

Sunday, March 3, 2019 8:17 PM

Oscillating Systems
Foundation Stage (OS.2.L1)

Lecture 1
General Oscillations, Equations of Motion, Simple Harmonic Motion



Textbook Chapters (* Calculus version)

- **BoxSand** :: KC videos (Simple Harmonic Oscillators)
- **Knight** (College Physics : A strategic approach 3rd) :: 14.1 ; 14.2 ; 14.3 ; 14.4
- ***Knight** (Physics for Scientists and Engineers 4th) :: 15.1 ; 15.2 ; 15.3 ; 15.4
- **Giancoli** (Physics Principles with Applications 7th) :: 11-1 ; 11-2 ; 11-3

Warm up

OS.2.L1-1:

Description: Identify oscillations in the world around you and discuss their the features.

Learning Objectives: [?] - Can you identify the objectives from the previous lecture, and this lecture, that this question is relevant to?

Problem Statement: Oscillations are perhaps the most prevalent phenomenon observed, and the concepts of oscillations are used to model a wide range of systems such as molecular bonds, biological systems, quantum systems, etc…

(a) List some of the oscillations you observe around you right now.

(b) All oscillators have a set of common features listed below:

- A quantity that repeats.
- An equilibrium position.
- A disturbance from the equilibrium position.
- A restoring force.

- A potential well.

Pick one of the oscillations from part (a) and discuss with your neighbors how the features listed above describe the oscillator you picked.

Selected Learning Objectives

1. Coming soon to a lecture template near you.

Key Terms

- Oscillation
- Equation of motion
- Equilibrium location (a.k.a. equilibrium position, equilibrium point, etc..)
- Disturbance from equilibrium
- Restoring force
- Potential energy well
- Period
- Frequency
- Angular frequency
- Simple Harmonic Motion (SHM)
- Simple Harmonic Oscillator (SHO)
- Initial conditions

Key Equations

Key Concepts

- Coming soon to a lecture template near you.

Questions

Act I: General Features of Oscillators

OS.2.L1-2:

Description: Conceptual question about features of oscillators. (3 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: Which of the following is/are necessary to make an object oscillate?

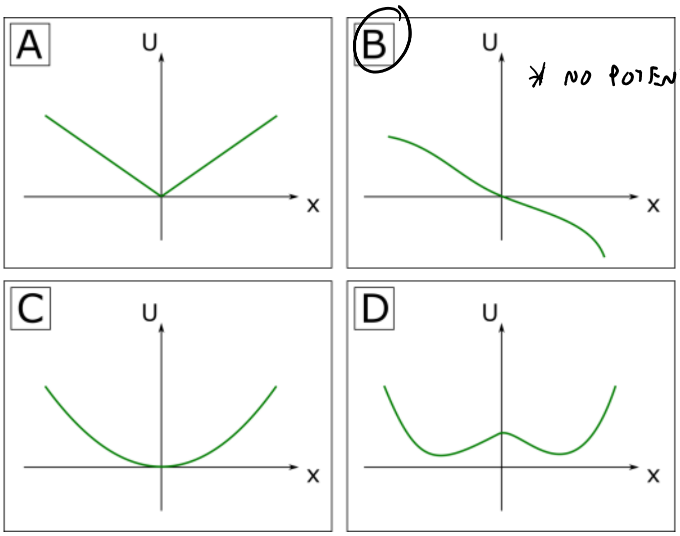
T (1) A stable equilibrium.
F (2) An unstable equilibrium.
T (3) Little or no friction.
T (4) A disturbance.

OS.2.L1-3:

Description: Given graphs of potential energy vs position identify non-oscillatory motion. (3 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: Which of these graphs of potential energy vs position can NOT give rise to oscillatory motion?

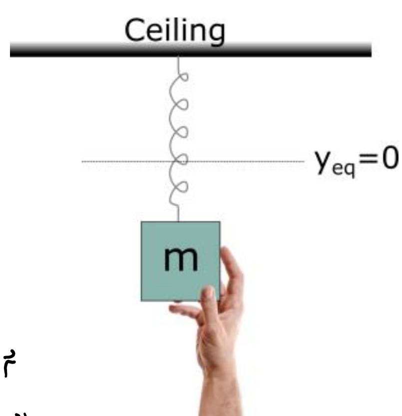


OS.2.L1-4:

Description: Conceptual question about features of oscillators. (2 minutes + 2 minutes + 2 minutes + 2 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: An object hangs motionless from a spring. Consider pulling the object down very slowly.



(a) Does the gravitational potential energy increase or decrease?

$U^g = mgy$ $y\downarrow$ so $U^g\downarrow$

(b) Does the spring potential energy increase or decrease?

$U^s = \frac{1}{2}ky^2$ $y\uparrow$ so $U^s\uparrow$

(c) Is there any external work?

(1) No
(2) Yes and it's positive.
(3) Yes and it's negative.

$\vec{F}\downarrow \; \downarrow\Delta\vec{r}$ $W^F = \vec{F}\cdot\Delta\vec{r}$

$= |\vec{F}||\Delta\vec{r}|\cos\theta$

$\theta = 0$ so $\cos\theta = +1$

(d) Does the overall energy of the spring-mass-Earth system increase or decrease or stay the same?

$\theta = 0$ so $\cos\theta = +1$

(d) Does the overall energy of the spring-mass-Earth system increase or decrease or stay the same?

(1) Increase.
(2) Decrease.
(3) Stays the same.

$W^{EXT} = \Delta U^{g} + \Delta U^{s}$

$+ \quad - \quad +$

* WHERE DID THIS ENERGY COME FROM? .... W^{EXT}

OR IF PERSON IS IN SYSTEM THEN E^{CHEM}

Act II: Features Specific to SHO

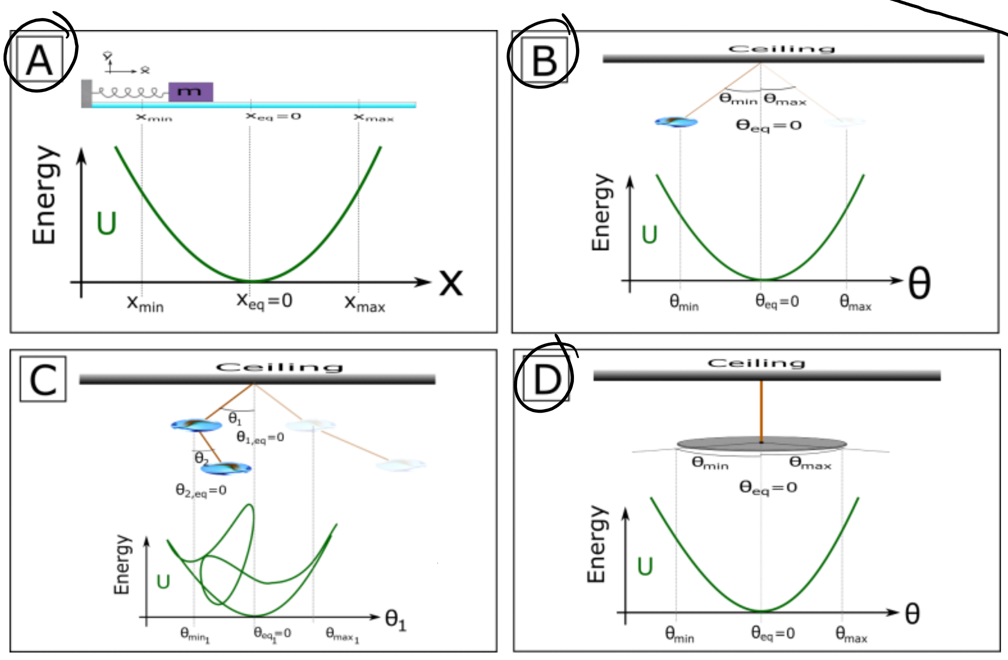
OS.2.L1-5:

Description: Given graphs of potential energy vs position identify simple harmonic oscillators. (3 minutes).

Learning Objectives: [1, 12, 13]

Problem Statement: Consider the four systems shown below and their associated potential energy functions as a function of position for small displacements. Which of the systems can be modeled as a simple harmonic oscillator?

QUADRATIC P.E.



OS.2.L1-6:

Description: Given force equations as a function of displacement from equilibrium, identify which would give rise to simple harmonic motion. (3 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: Consider the below equations for a force that a spring applies when it is stretched from its equilibrium position, where **x** represents the displacement from the equilibrium position. Which springs would give rise to simple harmonic motion if a mass is attached?

(1) $\mathbf{F}_x = 5\,\mathbf{x}$
(2) $\mathbf{F}_x = -7\,\mathbf{x}$
(3) $\mathbf{F}_x = 8\,x - 0.01\,\mathbf{x}^2$
(4) $\mathbf{F}_x = -\,\mathbf{x} + 0.01\,\mathbf{x}^2$

LINEAR RESTORING FORCE

IF X IS VERY SMALL THEN THIS IS BASICALLY SHM

OS.2.L1-7:

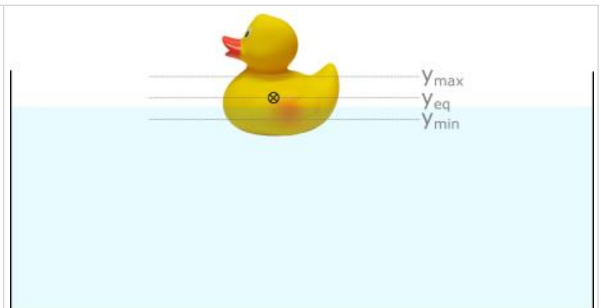
Description: Conceptual question about period for simple harmonic motion. (2 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: Consider displacing Duckbert the rubber ducky by a small amount in the vertical direction such that he undergoes SHM with an amplitude of 1.0 cm and a period of 0.75 seconds. If the amplitude of oscillation is tripled and Duckbert still undergoes SHM, what is the new period of oscillation?

(1) Greater than 0.75 seconds.
(2) Less than 0.75 seconds.
(3) Equal to 0.75 seconds.

FOR SHM, PERIOD IS INDEPENDENT OF AMPLITUDE



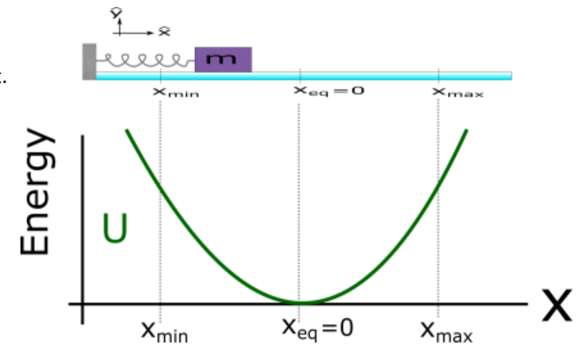
OS.2.L1-8:

Description: Conceptual question about equilibrium position for simple harmonic motion. (4 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: For an oscillation undergoing SHM, the equilibrium position is

F (1) the point at which the velocity of the oscillator is zero.
T (2) the point at which there is no net force on the oscillator.
F (3) the point at which the oscillator is at a maximum displacement.
F (4) the point at which the oscillator has a maximum acceleration.



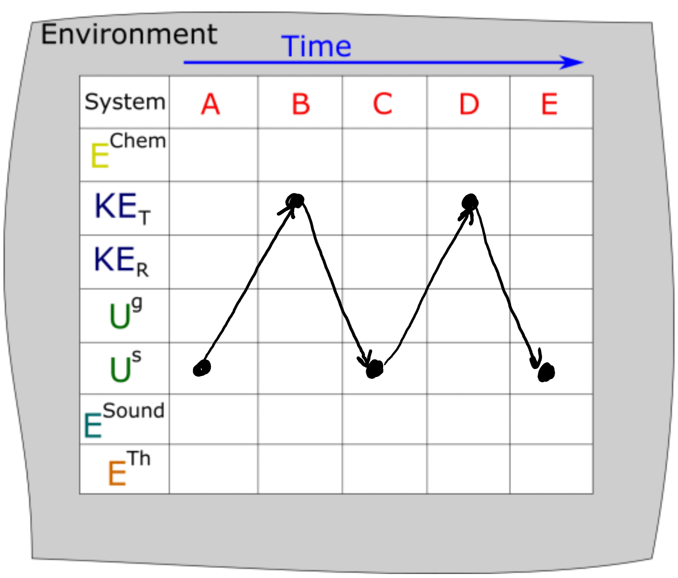
OS.2.L1-9:

Description: Complete an energy flow diagram for a SHO. (3 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: Complete the energy flow diagram that would best represent a spring-mass oscillator undergoing SHM with no air resistance or friction. The snapshots are taken at the following locations:
A : The mass is at the maximum negative displacement;
B : The mass is moving to the right and at the equilibrium location;
C : The max is at the maximum displacement;
D : The mass is moving to the left and at the equilibrium location;
E : The mass is at the maximum negative displacement.

System = spring + mass



OS.2.L1-10:

Description: Conceptual question relating energy and amplitude of a simple harmonic oscillator. (3 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: Which of the following statements are true for a SHO?

F (1) If the amplitude of the oscillation is doubled, the period is quadrupled.
F (2) If the energy of the oscillator is halved, the period of the oscillator is also halved.
T (3) If the amplitude of the oscillator is halved, the energy is decreased by a factor of 4.
T (4) If the amplitude of the oscillator is increased by a factor of 3, the period of the motion is not affected.

$U \propto x^2$

IF $x \to \frac{1}{2}x$

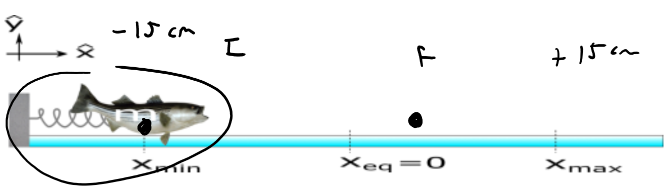
THEN $U \to \frac{1}{4}U$

OS.2.L1-11:

Description: Energy analysis for a SHO. (6 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: A 1.0 kg bass is pulled 15 cm in the negative direction from the equilibrium position and released from rest on a frictionless surface. If the spring constant of the spring is 5 N/m, how fast will the bass be moving when it travels through the location at positive 5.0 cm? (Hint: use your energy analysis skills from PH201)



SYSTEM m + SPRING

$KE_{Ti} + U^s_i + W_{ext} = KE_{Tf} + U^s_f$

$U^s_i = KE_{Tf} + U^s_f$

$\frac{1}{2}kx_i^2 = \frac{1}{2}mv_f^2 + \frac{1}{2}kx_f^2$

Act III: Initial Conditions and Modeling

OS.2.L1-12:

Description: Conceptual question about angular frequency vs angular velocity. (4 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: Angular frequency vs angular velocity?

(a) Consider the gif playing now. In UCM, how do the magnitudes of the angular frequency and angular velocity relate?

(1) The magnitude of angular frequency is greater than that of the angular velocity.
(2) The magnitude of angular frequency is less than that of the angular velocity.
(3) The magnitude of angular frequency is equal to that of the angular velocity.

(b) What is different about angular frequency and angular velocity?

F (1) Nothing is different, they are the same.
T (2) Angular frequency is a scalar, while angular velocity is a vector.
F (3) Angular frequency is a vector, while angular velocity is a scalar.
T (4) The angular frequency is a constant value for SHM, while angular velocity can be a function time for SHM.

ANGULAR FREQUENCY ≠ ANGULAR VELOCITY

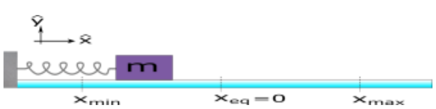
OS.2.L1-13:

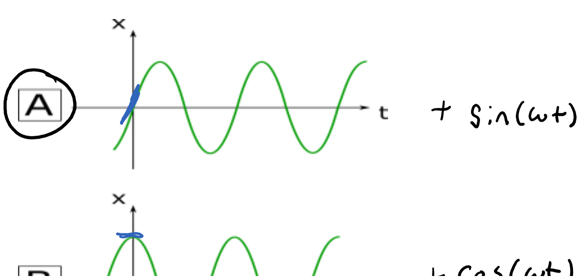
Description: Given a mass-spring system and initial conditions via written description and physical representation, identify the correct graphical representation. Find sign of velocities and accelerations at specific points. (3 minutes + 5 minutes + 5 minutes + 3 minutes).

Learning Objectives: [1, 12, 13]

Problem Statement: A **t** = 0 seconds an mass passes through the equilibrium position moving to the right (+ x direction).

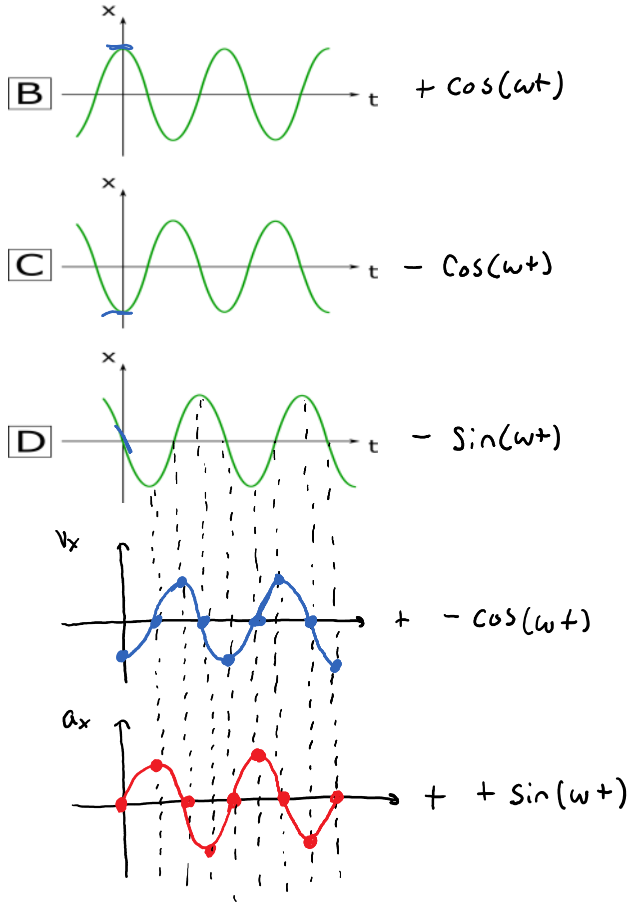
(a) Which of the following plots could represent the mass' position as a function of time?





(b) Is the velocity at t = 0 s positive, negative, or zero for each case?

v_A	v_B	v_C	v_D
+	0	0	−



v_A	v_B	v_C	v_D
+	0	0	−

(c) Is the acceleration at t = 0 positive, negative, or zero for each case?

a_A	a_B	a_C	a_D
0	−	+	0

* 3 METHODS

1) FROM $X(t)$ SKETCH $V(t)$ THEN $a(t)$

2) LOOK AT PHYSICAL REP.

@ $x = 0$ $\Sigma\vec{F} = \vec{0}$ SO $\vec{a} = \vec{0}$

@ X_{max} $\Sigma\vec{F}$ ← SO $\vec{a}$ (−)

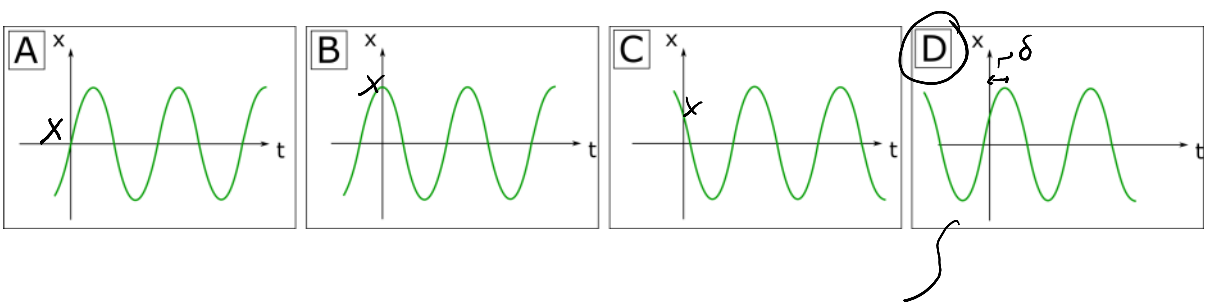
3) CONCAVITY OF $X(t)$

IF $X(t)$ ∪ THEN a (+)

IF $X(t)$ ∩ THEN a (−)

IF $X(t)$ / THEN a (0)

(d) Suppose at t = 0 s the mass was already in the positive x direction and moving to the right. Which of the following plots could represent the object's position as a function of time?



$$X(t) = X_{max}\sin(\omega t - \delta)$$

PHASE

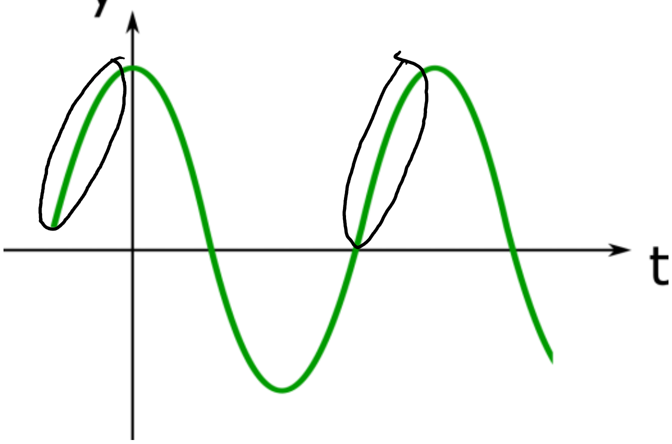
OS.2.L1-14:

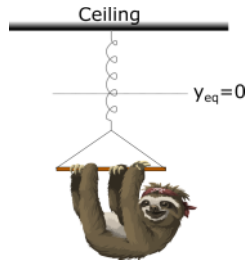
Description: Given a position vs time graph for SHM, identify locations for a given velocity and acceleration signs. (4 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: Sloan the Sloth is suspended from a spring oscillating up and down as indicated in the graph below. Select a point on the graph where $\mathbf{v}_z$ is positive and $\mathbf{a}_z$ is negative.

SLOPE + CONCAVITY ∩





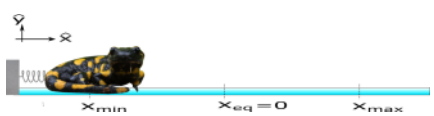
OS.2.L1-15:

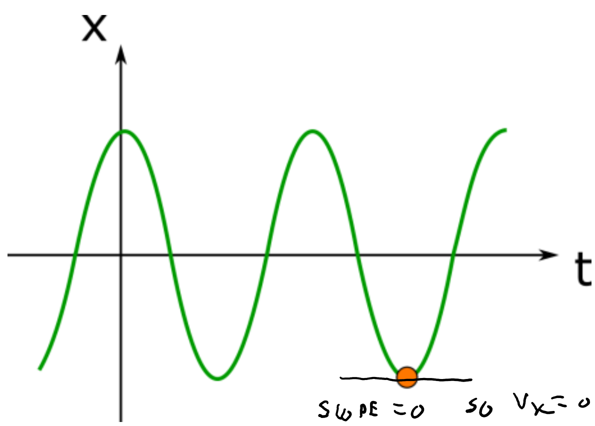
Description: Given a graph of position vs time for SHM and a specific location, determine direction for velocity and net force. (4 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: Sally the Salamander holds on to a negligible mass spring. The position of Sally as a function of time is plotted below. What can you say about the velocity and the force acting on Sally at the instant indicated by the orange dot?

T (1) Velocity is zero ; force is to the right.
F (2) Velocity is zero ; force is to the left.
F (3) Velocity is negative ; force is to the right.
F (4) Velocity is negative ; force is to the left.
F (5) Velocity is positive ; force is to the right.
F (6) Velocity is positive ; force is to the left.





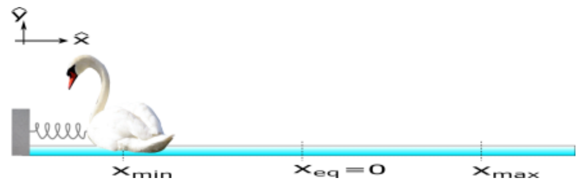
@ max DISP. ΣF = max & a = max

OS.2.L1-16:

Description: Given initial conditions via written and physical representations, choose the correct graphical and mathematical representation for position, velocity, and acceleration. (3 minutes + 1 minute + 2 minutes + 3 minutes + 4 minutes + ...)

Learning Objectives: [1, 12, 13]

Problem Statement: Sean the Swan is resting on a frictionless surface and attached to a spring which is initially pulled back 15 cm and released from rest as seen in the image below. Sean proceeds to make 20 oscillations in 30 seconds.



(a) What is the period of oscillation?

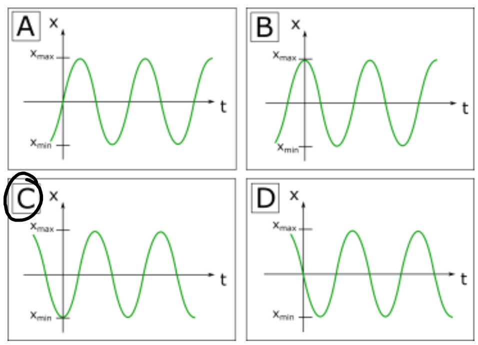
$T = \frac{\text{TIME}}{1 \text{ cycle}} = \frac{30\text{ s}}{20} = \boxed{\frac{3}{2}\text{ s}}$

(b) What is the frequency of oscillation?

$f = \frac{1}{T} = \boxed{\frac{2}{3}\ \frac{1}{s}}$

Sean the Swan is resting on a frictionless surface and attached to a spring which is initially pulled back 15 cm and released from rest as seen in the image below. Sean proceeds to make 20 oscillations in 30 seconds.

(c) Which of the following plots could represent Sean's position as a function of time?



(d) Which of the following equations could represent Sean's position as a function of time?

(1) $\mathbf{x}(t) = \mathbf{x_{max}} \sin(\boldsymbol{\omega}\, \mathbf{t})$

(2) $\mathbf{x}(t) = -\mathbf{x_{max}} \sin(\boldsymbol{\omega}\, \mathbf{t})$

(3) $\mathbf{x}(t) = \mathbf{x_{max}} \cos(\boldsymbol{\omega}\, \mathbf{t})$

(4) $\mathbf{x}(t) = -\mathbf{x_{max}} \cos(\boldsymbol{\omega}\, \mathbf{t})$

$= -15\text{ cm} \cos\left(\frac{2\pi}{(\frac{3}{2})} t\right)$

$= -15\text{ cm} \cos\left(\frac{4}{3}\pi t\right)$

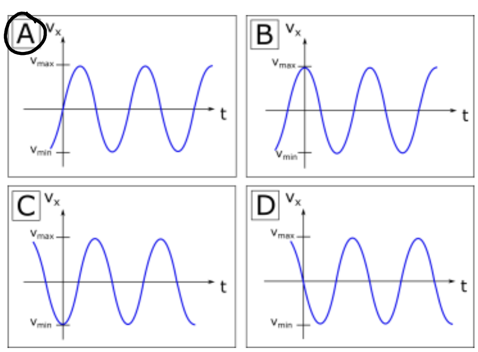
(e) What is Sean's position at t = 0.80 s?

$X(t=0.8) = -15\text{ cm} \cos\left(\frac{4}{3}\pi (0.8)\right)$

$\boxed{X(t=0.8) \approx 14.7\text{ cm}}$

Sean the Swan is resting on a frictionless surface and attached to a spring which is initially pulled back 15 cm and released from rest as seen in the image below. Sean proceeds to make 20 oscillations in 30 seconds.

(f) Which of the following plots could represent Sean's velocity as a function of time?



(g) Which of the following equations could represent Sean's velocity as a function of time?

(1) $\mathbf{v}_x(t) = \mathbf{v_{max}} \sin(\boldsymbol{\omega}\, \mathbf{t})$

(2) $\mathbf{v}_x(t) = -\mathbf{v_{max}} \sin(\boldsymbol{\omega}\, \mathbf{t})$

(3) $\mathbf{v}_x(t) = \mathbf{v_{max}} \cos(\boldsymbol{\omega}\, \mathbf{t})$

(4) $\mathbf{v}_x(t) = -\mathbf{v_{max}} \cos(\boldsymbol{\omega}\, \mathbf{t})$

$= \omega X_{max} \sin(\omega t)$

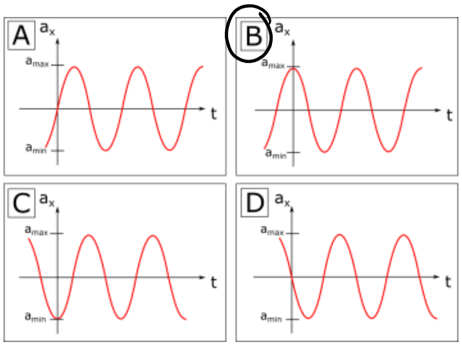
(h) What is Sean's velocity at t = 0.80 s? (Hint: $\mathbf{v_{max}} = \mathbf{x_{max}}\,\boldsymbol{\omega}$)

$V(t=0.8\text{ s}) = \left(\frac{4}{3}\pi\right)(15\text{ cm}) \sin\left(\frac{4}{3}\pi (0.8)\right)$

$\boxed{V(t=0.8\text{ s}) \approx -61.5\text{ cm/s}}$

Sean the Swan is resting on a frictionless surface and attached to a spring which is initially pulled back 15 cm and released from rest as seen in the image below. Sean proceeds to make 20 oscillations in 30 seconds.

(i) Which of the following plots could represent Sean's acceleration as a function of time?



(d) Which of the following equations could represent Sean's acceleration as a function of time?

(1) $\mathbf{a_x}(t) = \mathbf{a_{max}} \sin(\boldsymbol{\omega}\, \mathbf{t})$

(2) $\mathbf{a_x}(t) = -\mathbf{a_{max}} \sin(\boldsymbol{\omega}\, \mathbf{t})$

(3) $\mathbf{a_x}(t) = \mathbf{a_{max}} \cos(\boldsymbol{\omega}\, \mathbf{t})$

(4) $\mathbf{a_x}(t) = -\mathbf{a_{max}} \cos(\boldsymbol{\omega}\, \mathbf{t})$

$= \omega V_{max} \cos(\omega t)$

$= \omega^2 X_{max} \cos(\omega t)$

(e) What is Sean's acceleration at t = 0.80 s? (Hint: $\mathbf{a_{max}} = \mathbf{v_{max}}\, \boldsymbol{\omega}$)

$a_x(t=0.8) = \left(\frac{4}{3}\pi\right)^2 (15\,cm) \cos\left(\frac{4}{3}\pi (0.8)\right)$

$a_x(t=0.8) \approx -257\ cm/s^2$

Conceptual questions for discussion

1. Coming soon to a lecture template near you.

Hints

OS.2.L1-1: No hints.

OS.2.L1-2: No hints.

OS.2.L1-3: No hints.

OS.2.L1-4: No hints.

OS.2.L1-5: No hints.

OS.2.L1-6: No hints.

OS.2.L1-7: No hints.

OS.2.L1-8: No hints.

OS.2.L1-9: No hints.

OS.2.L1-10: No hints.

OS.2.L1-11: No hints.

OS.2.L1-12: No hints.

OS.2.L1-13: No hints.

OS.2.L1-14: No hints.

OS.2.L1-15: No hints.

OS.2.L1-16: No hints.