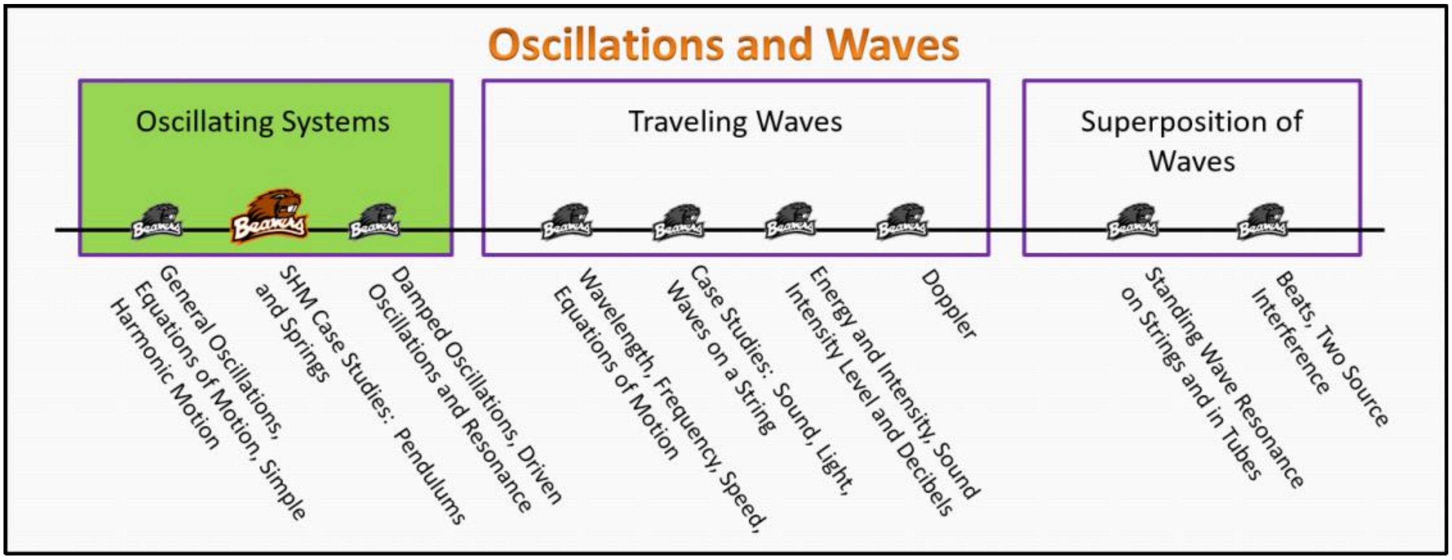
OS.2.L2 - Solutions

Sunday, March 3, 2019 8:18 PM

Oscillating Systems
Foundation Stage (OS.2.L2)

Lecture 2
SHM Case Studies: Pendulums and Spring



Textbook Chapters (* Calculus version)

- **BoxSand** :: KC videos (Springs & Pendulums)
- **Knight** (College Physics : A strategic approach 3rd) :: 14.2 ; 14.4 ; 14.5
- ***Knight** (Physics for Scientists and Engineers 4th) :: 15.4 ; 15.5 ; 15.6
- **Giancoli** (Physics Principles with Applications 7th) :: 11-1 ; 11-2 ; 11-3

Warm up

OS.2.L2-1:

Description: Conceptual question about how frequency is related to the oscillating system's parameters.

Learning Objectives: [?] - Can you identify the objectives from the previous lecture, and this lecture, that this question is relevant to?

Problem Statement: Congratulations! You've been selected to be the first person to go to Mars. Unfortunately NASA is on a budget and can't afford a stopwatch, so you will have to take with you a cheap spring-mass system and a pendulum.

(a) Which SHO could you use to keep track of time while on the flight from Earth to Mars in deep space?

SPRING-MASS

$$T = 2\pi\sqrt{\frac{M}{K}}$$

PENDULUM

$$T = 2\pi\sqrt{\frac{l}{g}}$$

(b) Which SHO could you use to keep track of time while on the surface of mars?

EITHER. SO MIGHT AS WELL USE BOTH.

$T = 2\pi\sqrt{\frac{m}{k}}$

$T = 2\pi\sqrt{\frac{\ell}{g}}$

NO g IN DEEP SPACE.

Selected Learning Objectives

1. Coming soon to a lecture template near you.

Key Terms

- Spring-mass oscillator
- Pendulum (i.e. simple pendulum)

Key Equations

Springs	Pendulums
$\omega = \sqrt{\frac{k}{m}}$	$\omega = \sqrt{\frac{g}{\ell}}$
$v_{max} = \omega\, x_{max}$	$\Omega_{max} = \omega\, \theta_{max}$
$a_{max} = \omega\, v_{max} = \omega^2\, x_{max}$	$\alpha_{max} = \omega\, \Omega_{max} = \omega^2\, \theta_{max}$
$E_{total} = \frac{1}{2}\, k\, x_{max}^2 = \frac{1}{2}\, m\, v_{max}^2$	$E_{total} = \frac{1}{2}\, m\, g\, \ell\, \theta_{max}^2 = \frac{1}{2}\, m\, \ell^2\, \Omega_{max}^2$

Key Concepts

- Coming soon to a lecture template near you.

Questions

Act I: Comparing Features of Spring-Mass Oscillators and Pendulums

OS.2.L2-2:

Description: Proportional reasoning for SHM with spring constant and frequency. Proportional reasoning for SHM with period and free-fall acceleration. Proportional reasoning for SHM with spring constant and mass and max speed. (2 minutes + 2 minutes + 5 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: Did someone say proportional reasoning?

(a) If the frequency of a spring-mass system is doubled and the mass remains constant, by how much has the spring constant changed?

(a) If the frequency of a spring-mass system is doubled and the mass remains constant, by how much has the spring constant changed?

(1) Decreased by a factor of 4
(2) Decreased by a factor of 2
(3) Remained the same
(4) Increased by a factor of 2
(5) Increased by a factor of 4

$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

$f \propto \sqrt{k}$

$k \propto f^2$

IF $f \to 2f$
THEN $k \to 4k$

(b) If we take a pendulum to planet Zorgon 3, the period doubles. What is the free-fall acceleration on Zorgon 3?

(1) 1/4 **g**
(2) 1/2 **g**
(3) **g**
(4) 2 **g**
(5) 4 **g**

$l = \text{const}$

$T = 2\pi\sqrt{\frac{l}{g}}$

$T \propto \frac{1}{\sqrt{g}}$

$g \propto \frac{1}{T^2}$

IF $T \to 2T$
THEN $g \to \frac{1}{4}g$

(c) A spring-mass system is switched out with one that has a spring constant that is 4 times as large, and a mass that is a one fourth as large. The amplitude of the oscillation is also decreased by a factor of 2. By what factor has the maximum speed changed in the oscillation?

(1) 1/4
(2) 1/2
(3) 1
(4) 2
(5) 4

$\omega = \sqrt{\frac{k}{m}}$

$V_{max} = \omega X_{max}$

$V_{max} = \sqrt{\frac{k}{m}} X_{max}$

$k \to 4k$
$m \to \frac{1}{4}m$
$X_{max} \to \frac{1}{2}X_{max}$

$V_{max} \to \sqrt{\frac{4}{\left(\frac{1}{4}\right)}}\ \frac{1}{2} V_{max}$

$\to 4 \cdot \frac{1}{2}$

$\to 2$

OS.2.L2-3:

Description: Conceptual question to compare features of spring-mass and pendulum systems. (4 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: Which of the following is the same for both spring-mass oscillators and pendulums with small amplitudes?

SHO

T (1) Frequency is independent of amplitude.
T (2) Frequency is independent of energy.
F (3) Frequency is dependent on mass.
F (4) Frequency is dependent on the free-fall acceleration **g**.
F (5) Both amplitudes are measured in meters.

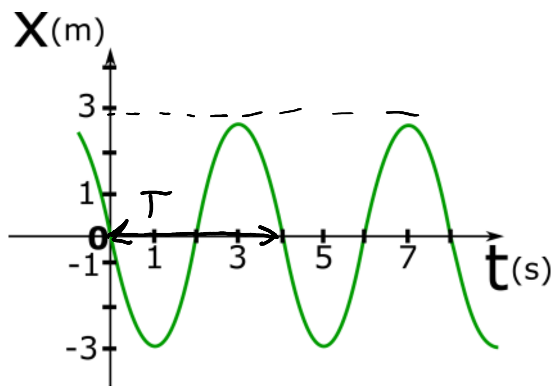
Act II: Spring-Mass Oscillator

OS.2.L2-4:

Description: Extract SHM quantities from a graphical representation of a spring-mass oscillator. (1 minutes + 3 minutes + 3 minutes + 3 minutes + 3 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: A plot of displacement from equilibrium vs time for a spring-mass system is shown below. The mass attached to the ideal spring has a mass of 7 kg.



(a) What is the period of oscillation?

$$T = 4 \text{ SEC.}$$

(b) What is the maximum speed the mass reaches?

$$V_{MAX} = \omega X_{MAX} = \frac{2\pi}{T} X_{MAX} = \frac{2\pi}{4}(3) \text{ m/s} = 4.71 \text{ m/s}$$

(c) What is the spring constant of the spring?

$$\omega = \sqrt{\frac{K}{m}} \qquad \frac{2\pi}{T} = \sqrt{\frac{K}{m}} \rightarrow \frac{2\pi}{4} = \sqrt{\frac{K}{7}}$$

$$K = 17.3 \text{ N/m}$$

(d) What is the total energy of this system?

$$E_{TOTAL} = \frac{1}{2} m V_{MAX}^2 \overset{OR}{=} \frac{1}{2} K X_{MAX}^2$$

$$= \frac{1}{2}(7)(4.71)^2 \overset{OR}{=} \frac{1}{2}(17.3)(3)^2$$

$$= 77.6 \text{ J} \overset{OR}{=} 77.8 \text{ J}$$

ROUNDING ERRORS

(e) Construct the mathematical equation that represents the displacement of this system as a function of time. USE SI units.

$$X(t) = -3 \text{ m} \sin\left(\frac{\pi}{2} t\right) \text{ METERS}$$

OS.2.L2-5:

Description: Extract SHM quantities from a mathematical representation of a spring-mass oscillator. (1 minute + 2 minutes + 1 minute + 3 minutes + 2 minutes + 2 minutes + 2 minutes).

Learning Objectives: [1, 12, 13]

Problem Statement: A classmate of yours was studying the motion of a spring-mass oscillator with a spring that has a spring constant of 25 N/m. Your classmate constructed an equation in SI units that represented the displacement of the mass as seen below. From this equation, determine the following quantities.

$$\mathbf{x}(t) = \frac{\pi}{18}\cos\left(\frac{\pi}{2}\,\mathbf{t}\right)$$

(a) Amplitude.

$\frac{\pi}{18}$ RAD $\simeq$ $\boxed{0.175 \text{ RAD}}$

OR

$\frac{\pi}{18}$ RAD $\times \frac{180^\circ}{\pi \text{ RAD}} = \boxed{10^\circ}$

(b) Period.

$\omega = \frac{\pi}{2}$

$\frac{2\pi}{T} = \frac{\pi}{2}$

$\boxed{T = 4 \text{ SEC}}$

(c) Frequency.

$f = \frac{1}{T}$

$\boxed{f = 0.25 \text{ Hz}}$

(d) Mass of the oscillator.

$\omega = \sqrt{\frac{k}{m}}$

$2\pi f = \sqrt{\frac{k}{m}}$

$m = \frac{K}{4\pi^2 f^2} = \frac{25}{4\pi^2 (0.25)^2} = \boxed{10.1 \text{ kg}}$

(e) Maximum speed of the mass.

$V_{max} = \omega X_{max}$

$= 2\pi f X_{max}$

$= 2\pi (0.25)\left(\frac{\pi}{18}\right)$

$= \boxed{0.274 \text{ m/s}}$

(f) Maximum acceleration of the mass.

$a_{max} = \omega^2 X_{max}$

$= 4\pi^2 f^2 X_{max}$

$= 4\pi^2 (0.25)^2 \left(\frac{\pi}{18}\right)$

$= \boxed{0.431 \text{ m/s}^2}$

(g) Total energy of oscillation.

$\frac{1}{2} k X_{max}^2 = \boxed{0.381 \text{ J}}$

OR

$\frac{1}{2} m V_{max}^2 = \boxed{0.381 \text{ J}}$

Act II: Pendulums

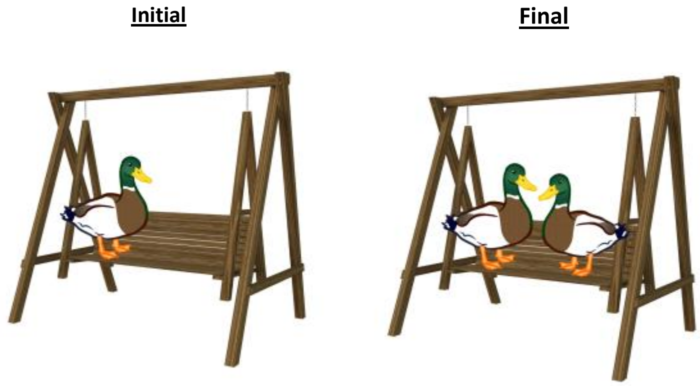
OS.2.L2-6:

Description: Conceptual question about period of pendulum oscillator and changing mass. (3 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: After losing the big game to the beavers, a sad duck is on a swing and it is observed that the period is 3.0 seconds. A second duck jumps on the swing in efforts to comfort it's friend. With the two ducks on the swing, the period is

(1) 6.0 s.
(2) greater than 3.0 s but not necessarily 6.0 s.
(3) 3.0 s.
(4) less than 3.0 s but not necessarily 1.5 s.
(5) 1.5 s.



$T = 2\pi\sqrt{\frac{l}{g}}$ ↑ no mass

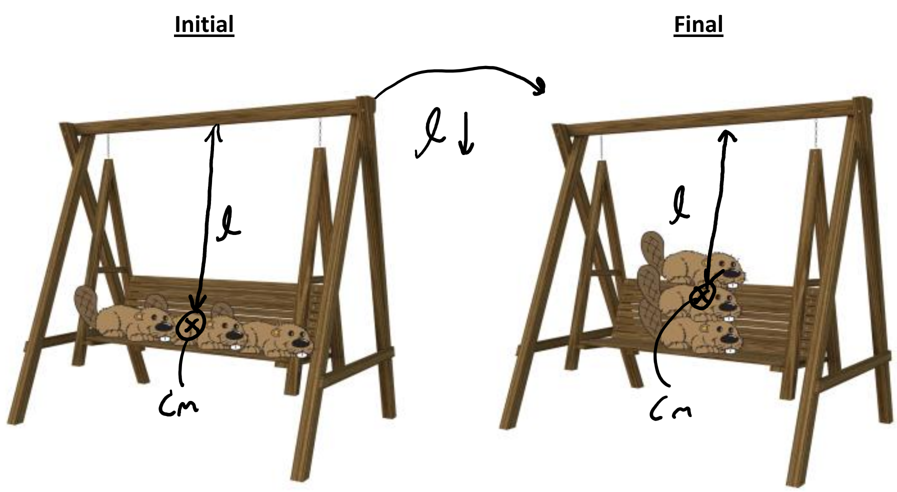
OS.2.L2-7:

Description: Conceptual question about period for pendulum and changing mass distribution. (3 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: After hearing the news about the big game against the ducks, three happy beavers initially on a swing side by side decide to do a beaver pile. The new natural frequency of the swing is

(1) greater.
(2) less.
(3) the same.



$f = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$

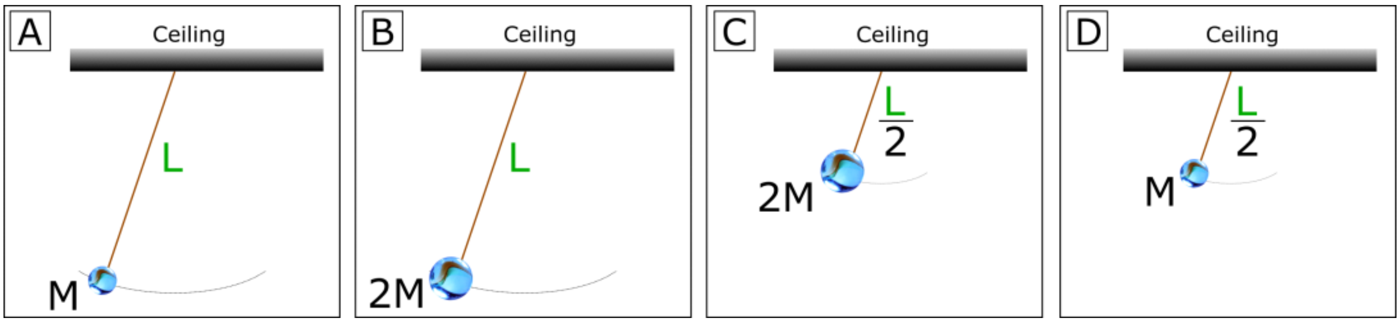
$f \propto \frac{1}{\sqrt{l}}$ If $l \downarrow$ then $f \uparrow$

OS.2.L2-8:

Description: Ranking question for frequency of pendulums with different mass and lengths. (4 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: A simple pendulum shown in case **A** consists of a mass **M** attached to a massless string of length **L**. If the mass is pulled to one side a small distance and released, it will swing back and forth undergoing SHM. Cases **B**, **C**, and **D** are variations of this system. Rank the oscillation frequency of each case.



f $\quad$ f $\quad$ $\sqrt{2}f$ $\quad$ $\sqrt{2}f$

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$$

$$f \propto \frac{1}{\sqrt{l}}$$

$$f_A = f_B < f_C = f_D$$

OS.2.L2-9:

Description: Activity to find g using a pendulum. (20 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: Each group should have a piece of string and a mass.

(a) Use *only* these two items (i.e. no cell phones, no rulers, etc..) to *estimate* **g**, the free-fall acceleration in units of m/s^2.

$$T = 2\pi\sqrt{\frac{l}{g}} \longrightarrow g = \frac{l}{T^2 4\pi^2}$$

l ← MEASURE

T ← MEASURE

$4\pi^2$ ← ESTIMATE $4 \times 3 \times 3$

(b) Use these items, and anything else at your disposal to estimate **g**. Do not change your answer until prompted.

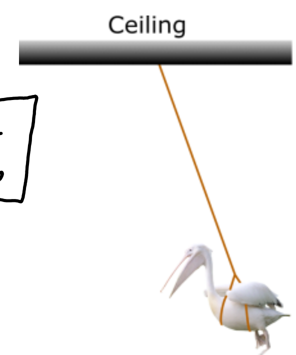
OS.2.L2-10:

Description: Write-out question for a pendulum. (6 minutes + 6 minutes + 5 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: Penelope the pelican is 4.0 kg and strapped to a harness via a light 3-m-long string that forms a simple pendulum. The pelican pendulum system is placed in an evacuated air chamber; don't panic, Penelope is wearing a high tech space suit.

(a) If the pelican pendulum system is released from an angle of 12°, how many oscillations are made in one day?



$$f = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{9.8}{3}}$$

$$f \approx 0.2876554072 \ \frac{1}{s}$$

$$\frac{0.2876554072 \text{ osc}}{1 \text{ s}} \times \frac{60 \text{ s}}{1 \text{ min}} \times \frac{60 \text{ min}}{1 \text{ hr}} \times \frac{24 \text{ hr}}{1 \text{ Day}} \approx \boxed{24853 \ \frac{\text{osc}}{\text{Day}}}$$

(b) The 4.0 kg pelican pendulum system (3-m-long string) is then transported (via pelican express) to the moon, where gravity is one sixth the value on Earth. If it is raised up to 10° and released, how long will it take to undergo the same number of oscillations as it did on Earth in one day?

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$$

$$f \propto \sqrt{g}$$

$$g \rightarrow \frac{1}{6}g$$

$$\text{so } f \rightarrow \frac{1}{\sqrt{6}}f$$

$$f \approx \frac{1}{\sqrt{6}}(0.2876554072) \text{ Hz}$$

$$f \approx 0.117438282 \ \frac{1}{s}$$

$$T = \frac{1}{f}$$

$$T = 8.515361372 \text{ s}$$

$$\frac{8.515361372 \text{ s}}{1 \text{ osc}} \times 24853 \text{ osc} \approx \boxed{211632 \text{ s}}$$

OR $\boxed{2.5 \text{ Days}}$

(c) What length could you make the string on the moon to have the answers to part (a) and (b) be equal?

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$$F = \frac{1}{2\pi}\sqrt{\frac{g}{\ell}}$$

SINCE $g \rightarrow \frac{1}{6}g$

FOR F TO BE CONST

ℓ WOULD HAVE TO GO TO $\rightarrow \frac{1}{6}\ell$

SO $\ell_{moon} = \frac{1}{6}(3)m = \boxed{\frac{1}{2}m}$

Conceptual questions for discussion

1. Coming soon to a lecture template near you.

Hints

OS.2.L2-1: No hints.

OS.2.L2-2: No hints.

OS.2.L2-3: No hints.

OS.2.L2-4: No hints.

OS.2.L2-5: No hints.

OS.2.L2-6: No hints.

OS.2.L2-7: No hints.

OS.2.L2-8: No hints.

OS.2.L2-9: No hints.

OS.2.L2-10: No hints.