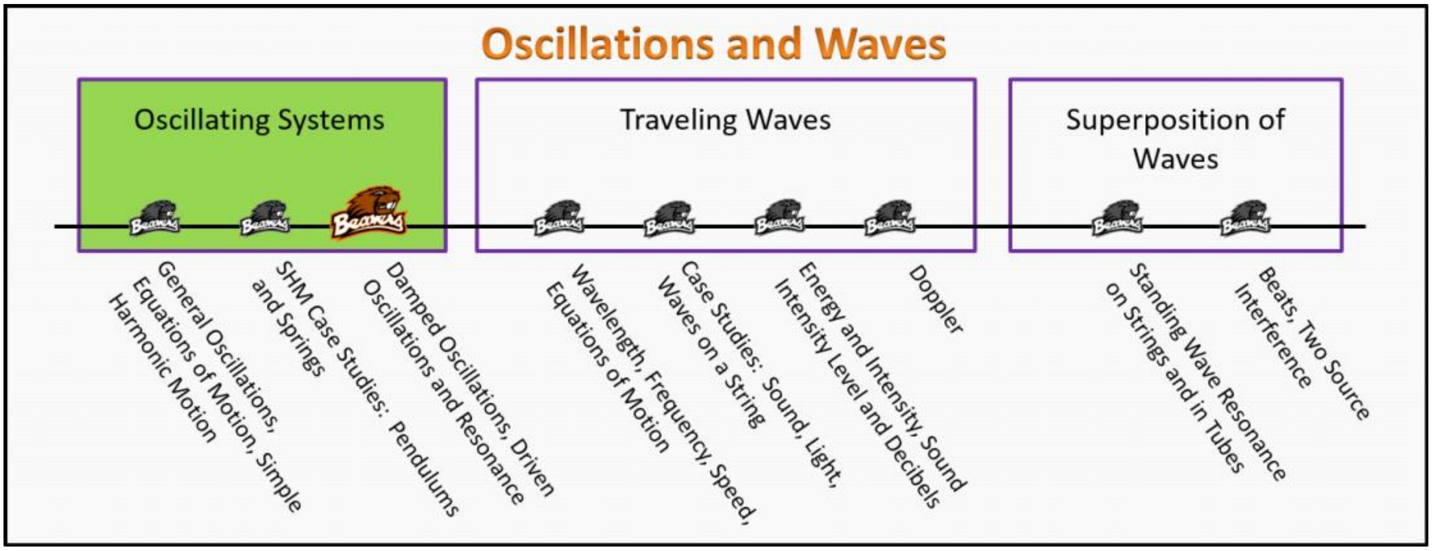
Oscillating Systems
Foundation Stage (OS.2.L3)

Lecture 3
Damped Oscillations, Driven Oscillations and Resonance



Textbook Chapters (* Calculus version)

- **BoxSand** :: KC videos (Damping)
- **Knight** (College Physics : A strategic approach 3rd) :: 14.6 ; 14.7
- ***Knight** (Physics for Scientists and Engineers 4th) :: 15.7 ; 15.8
- **Giancoli** (Physics Principles with Applications 7th) :: 11-5 ; 11-6

Warm up

OS.2.L3-1:

Description: Conceptual question about how a damped simple harmonic oscillators parameters are dependent on time.

Learning Objectives: [?] - Can you identify the objectives from the previous lecture, and this lecture, that this question is relevant to?

Problem Statement: Which of the following quantities are constant for a damped simple harmonic oscillator?

F (1) Amplitude
T (2) Period
T (3) Frequency
T (4) Angular frequency
F (5) Displacement from equilibrium
F (6) Speed
F (7) Acceleration

Selected Learning Objectives

1. Coming soon to a lecture template near you.

Key Terms

- Spring-mass oscillator
- Pendulum (i.e. simple pendulum)

Key Equations

Key Concepts

- Coming soon to a lecture template near you.

Questions

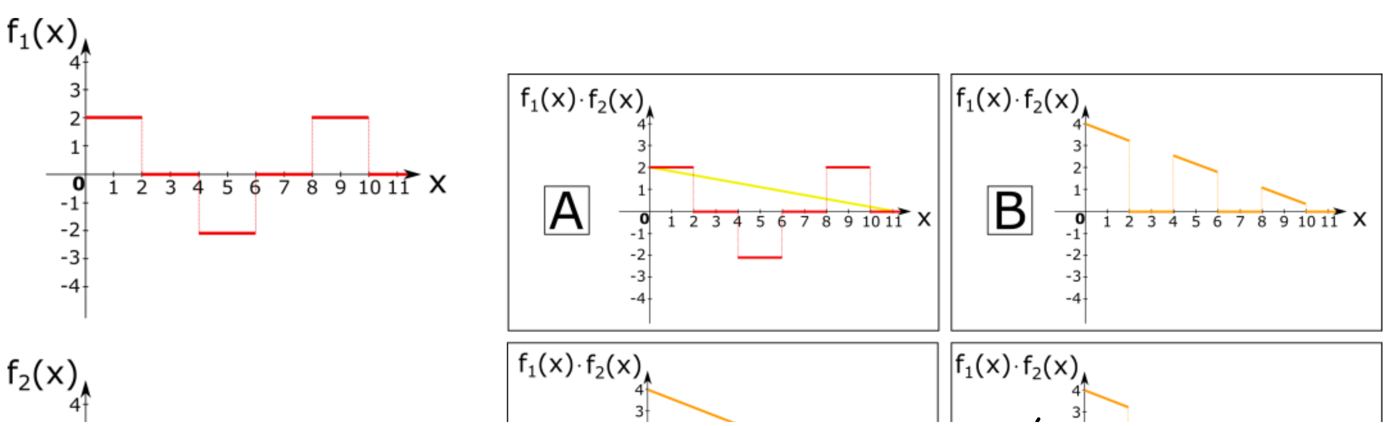
Act I: Initial Conditions and Modeling

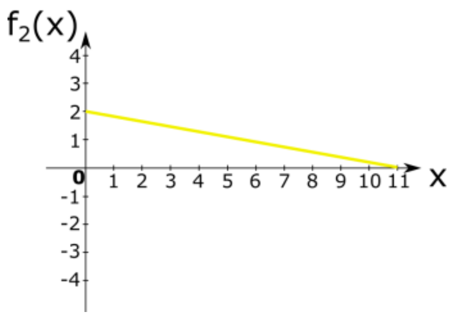
OS.2.L3-2:

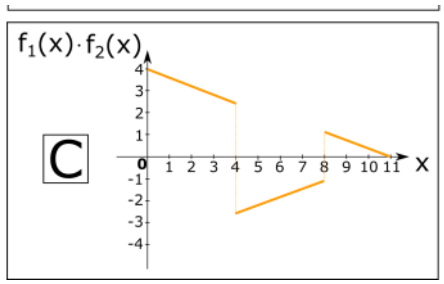
Description: Proportional reasoning for SHM with spring constant and frequency. Proportional reasoning for SHM with period and free-fall acceleration. Proportional reasoning for SHM with spring constant and mass and max speed. (2 minutes + 2 minutes + 5 minutes)

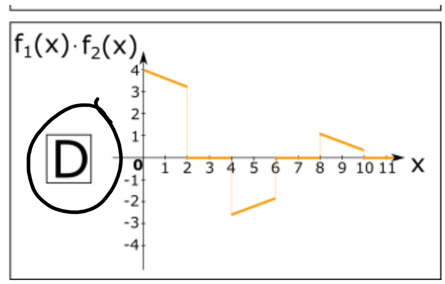
Learning Objectives: [1, 12, 13]

Problem Statement: Consider the graphical representation of two equations that are functions of **x**, **f_1(x)** and **f_2(x)**. Which graph correctly represents the graphical representation of the product of **f_1(x)** and **f_2(x)**, **f_1(x)·f_2(x)**?









<u>Example</u>

$f_1(x=3)=0$

$F_2(x=3) \approx 1.5$

So $f_1(x=3) \cdot F_2(x=3) = 0(1.5) = 0$

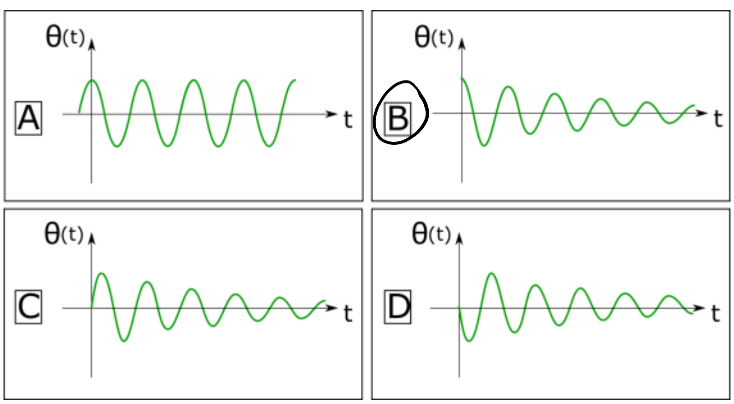
OS.2.L3-3:

Description: Conceptual question to compare features of spring-mass and pendulum systems. (4 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: Watch carefully the demo I am performing with a pendulum right now.

(a) Which graph best represents the angle as a function of time for the pendulum?



(b) Which mathematical representation best represents the angle of the pendulum as a function of time?

F (1) $\boldsymbol{\theta}(\mathbf{t}) = \boldsymbol{\theta}_{\mathbf{max}} \cos(\boldsymbol{\omega}\, \mathbf{t})$

F (2) $\boldsymbol{\theta}(\mathbf{t}) = -\boldsymbol{\theta}_{\mathbf{max}}\, \mathbf{t} \cos(\boldsymbol{\omega}\, \mathbf{t})$

F (3) $\boldsymbol{\theta}(\mathbf{t}) = \boldsymbol{\theta}_{\mathbf{max}}\, e^{t/\tau} \cos(\boldsymbol{\omega}\, \mathbf{t})$

F (3) $\boldsymbol{\theta}(\mathbf{t}) = \boldsymbol{\theta}_{\mathbf{max}}\, e^{t/\tau} \cos(\boldsymbol{\omega}\, \mathbf{t})$

T (4) $\boldsymbol{\theta}(\mathbf{t}) = \boldsymbol{\theta}_{\mathbf{max}}\, e^{-t/\tau} \cos(\boldsymbol{\omega}\, \mathbf{t})$

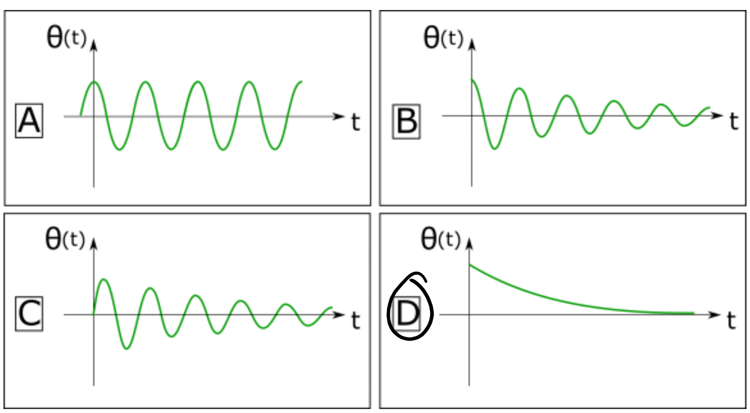
F (5) $\boldsymbol{\theta}(\mathbf{t}) = -\boldsymbol{\theta}_{\mathbf{max}}\, e^{-t/\tau} \cos(\boldsymbol{\omega}\, \mathbf{t})$

OS.2.L3-4:

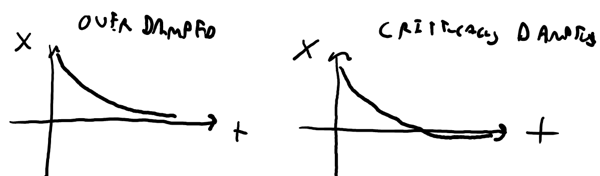
Description: Extract SHM quantities from a graphical representation of a spring-mass oscillator. (1 minutes + 3 minutes + 3 minutes + 3 minutes + 3 minutes)

Learning Objectives: [1, 12, 13]

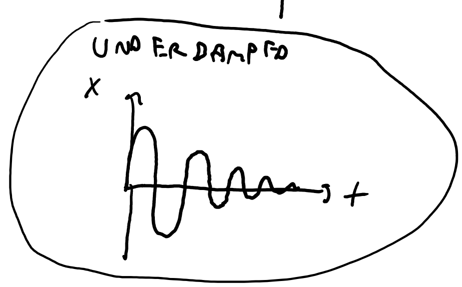
Problem Statement: An overdamped pendulum with lots of friction is released from rest at θ = 10°. Which of the following plots best represent the displacement from equilibrium of the pendulum's angle as a function of time?



$$\underbrace{\frac{\Delta^2 x}{\Delta t^2} \approx -\omega^2 x - \frac{c_1}{m}|\vec{v}|}_{\text{3 SOLUTIONS}}$$



WE ARE STUDYING THIS SOLUTION →



Act II: Amplitude function and Time Constant (τ)

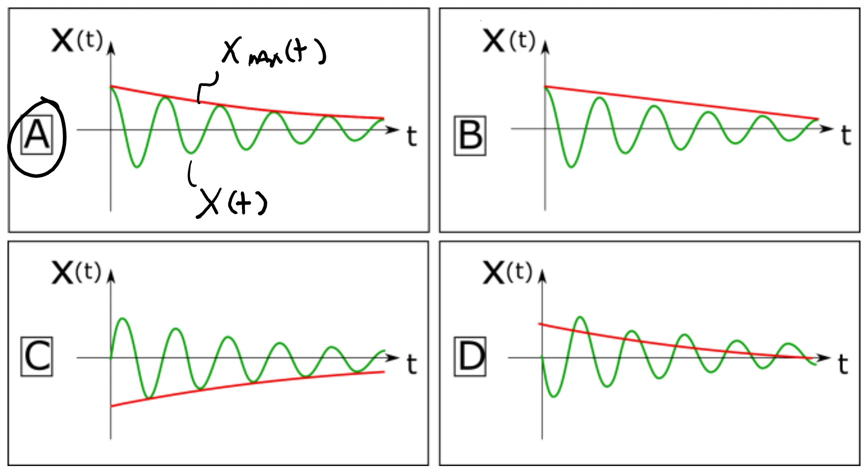
OS.2.L3-5:

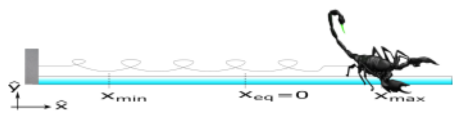
Description: Extract SHM quantities from a mathematical representation of a spring-mass oscillator. (1 minute + 2 minutes + 1 minute + 3 minutes + 2 minutes + 2 minutes + 2 minutes).

Learning Objectives: [1, 12, 13]

Problem Statement: Scott the Scorpion is attached to an ideal spring with non-negligible air resistance and released from rest at **x** = 10 cm.

(a) Which of the following graphs best represent the position *and* amplitude function for Scott?

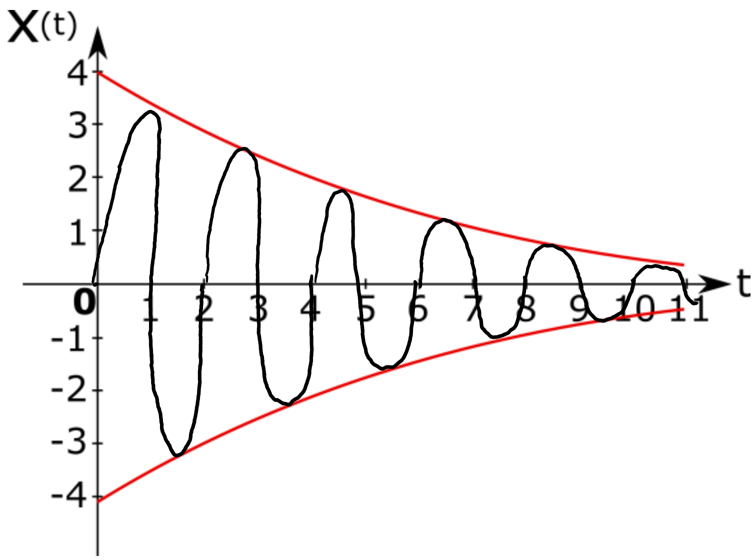




$$X(t) = X_{max}(t)\,\sin(\omega t)$$

$$X_{max}(t) = X_{max}(t=0)\, e^{-t/\tau}$$

(b) Below is a graph representing the displacement from an equilibrium position vs time. The plot has the amplitude function, and it's negative, already drawn. If at **t** = 0 the oscillator was at **x** = 0, sketch a plausible position as a function of time graph that has the same amplitude function if the period of oscillation is 2 seconds.



OS.2.L3-6:

Description: Conceptual question about period of pendulum oscillator and changing mass. (3 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: Consider the plot below representing the displacement from equilibrium of a Silly named animal on an ideal spring.

(a) What is the displacement of the oscillator at **t** = 0s?

$X(t=0) = \boxed{3\ m}$

(b) What is the amplitude at **t** = 0 s?

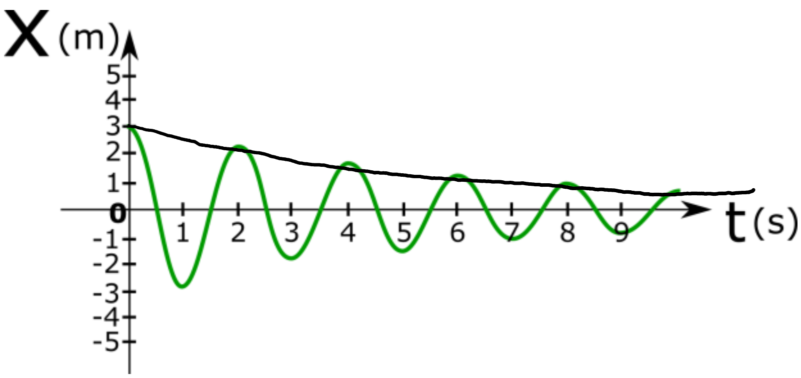
$X_{max}(t=0) = \boxed{3\ m}$

(c) What is the displacement at **t** = 2.5 s?

$X(t=2.5) = \boxed{0\ m}$

(d) What is the amplitude at **t** = 2.5 s?

$X_{max}(t=2.5) \approx \boxed{2\ m}$



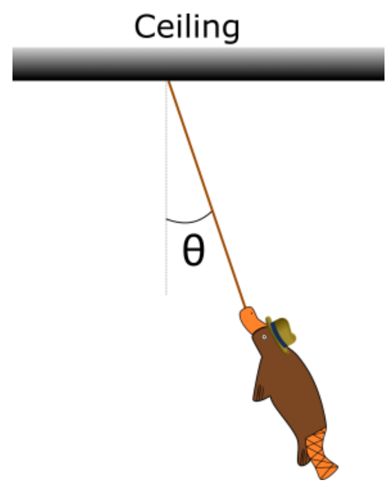
OS.2.L3-7:

Description: Conceptual question about period for pendulum and changing mass distribution. (3 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: Perry the Platypus pendulum starts from rest at θ = 8° and it is noticed that after 20 seconds his amplitude has reduced to 2°. What is the time constant, τ, for this damped simple harmonic oscillator?

(1) 0.25 s
(2) -1.39 1/s
(3) 4 1/s
(4) 14.4 s
(5) 20 s
(6) 24.8 s



$$X_{max}(t) = X_{max}(t=0)\, e^{-t/\tau}$$

$$2^\circ = 8^\circ e^{-20s/\tau}$$

$$\frac{2}{8} = e^{-20s/\tau}$$

$$\ln\left(\frac{2}{8}\right) = \ln\left(e^{-20s/\tau}\right)$$

$$\ln\left(\frac{2}{8}\right) = -\frac{20\,s}{\tau}$$

$$\tau = \frac{-20\,s}{\ln\left(\frac{2}{8}\right)} \approx \boxed{14.4\,s}$$

$$X_{max}(t) = 8^\circ e^{-t/14.4}$$

OS.2.L3-8:

Description: Ranking question for frequency of pendulums with different mass and lengths. (4 minutes)

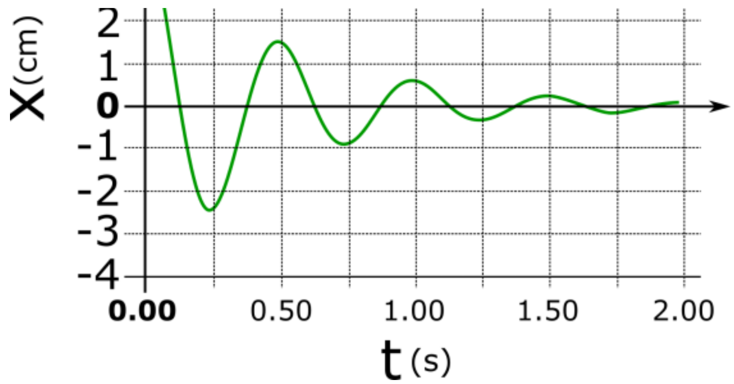
Learning Objectives: [1, 12, 13]

Problem Statement: Below is a plot of a car's vertical displacement from equilibrium after it went over a bump on the road.

(a) Estimate the period of oscillation?

(1) 0.25 s
(2) 0.5 s
(3) 0.75 s

(4) 1.0 s



(b) Estimate the time constant for this oscillation.

$$X(t) = X_{max}(t=0)\, e^{-t/\tau}$$

$$1.5\text{ cm} = 4\text{ cm}\, e^{-0.5/\tau}$$

$$\ln\left(\frac{1.5}{4}\right) = \frac{-0.5}{\tau}$$

$$\tau = \frac{-0.5}{\ln\left(\frac{1.5}{4}\right)} \approx \boxed{0.510\text{ s}}$$

OS.2.L3-9:

Description: Activity to find g using a pendulum. (20 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: A small earthquake starts a slight vibrating back and forth motion of a lamppost. The amplitude of the vibration of the top of the lamppost the moment the earthquake stops is 6.5 cm, then 8.0 seconds later the amplitude is 1.8 cm.

(a) During which time intervals is the lamppost a driven oscillator?

(1) From the moment the earth quake starts to when the quake stops.
(2) From the moment the earth quake starts to when the oscillations of the lamppost stop.
(3) From the moment the quake stops to when the oscillations of the lamppost stop.

(b) During which time intervals is the lamppost a damped oscillator?

(1) From the moment the earth quake starts to when the quake stops.
(2) From the moment the earth quake starts to when the oscillations of the lamppost stop.
(3) From the moment the quake stops to when the oscillations of the lamppost stop.

(c) A small earthquake starts a slight vibrating back and forth motion of a lamppost. The amplitude of the vibration of the top of the lamppost the moment the earthquake stops is 6.5 cm, then 8.0 seconds later the amplitude is 1.8 cm. The post vibrates with a period of 0.15 seconds. **Find the time constant for this damped oscillation.**

(1) 0.161 s
(2) 0.278 s
(3) -1.28 s
(3) 3.61 s
(4) 6.23 s
(5) 20 s

$$X_{max}(t) = X_{max}(t=0)\, e^{-t/\tau}$$

$$1.8\text{ cm} = 6.5\text{ cm}\, e^{-8/\tau}$$

$$\ln(1.8/6.5) = -8/\tau$$

$$\boxed{\tau = 6.23\text{ s}}$$

(5) 20 s

$$\boxed{\tau \approx 6.23\ s}$$

(d) The lamppost is at its maximum displacement when the quake stops. What is the displacement from equilibrium at t = 8.3 seconds after the quake as stopped?

(1) 0.0 cm
(2) -0.50 cm
(3) -0.858 cm
(4) 1.72 cm

$$X(t) = 6.5\, e^{-t/6.23} \cos\left(\frac{2\pi}{.15} t\right)$$

$$X(t=8.3) = 6.5\, e^{-8.3/6.23} \cos\left(\frac{2\pi}{.15}(8.3)\right)$$

$$\approx \boxed{-0.858\ cm}$$

Act III: Driven Damped Oscillators

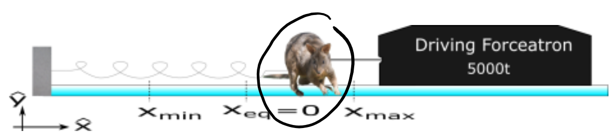
OS.2.L3-10:

Description: Extract SHM quantities from a mathematical representation of a spring-mass oscillator. (1 minute + 2 minutes + 1 minute + 3 minutes + 2 minutes + 2 minutes + 2 minutes).

Learning Objectives: [1, 12, 13]

Problem Statement: Padmé the pademelon is attached to an ideal spring with non-negligible air resistance and a driving force. It is observed that Padmé's motion can be characterized by an angular frequency of **ω** = 5 rad/s.

(a) With Padmé and the spring as a system, which of the following mathematical representations is a correct application of Newton's 2nd law in the x-direction?



(1) $\left|\vec{F}^{driving}\right| + \left|\vec{F}^{s}\right| + \left|\vec{F}^{g}\right| + \left|\vec{F}^{Drag}\right| + \left|\vec{F}^{N}\right| = m\, a_x$

(2) $\left|\vec{F}^{driving}\right| + \left|\vec{F}^{s}\right| + \left|\vec{F}^{Drag}\right| = 0$

(3) $\left|\vec{F}^{driving}\right| - \left|\vec{F}^{s}\right| - \left|\vec{F}^{Drag}\right| = m\, a_x$

(4) $\left|\vec{F}^{driving}\right| + \left|\vec{F}^{s}\right| + \left|\vec{F}^{Drag}\right| = m\, a_x$

$$\Sigma F_x = m a_x$$

$$F_x^{s} + F_x^{DRAG} + F_x^{DRIVE} = m a_x$$

(b) Which of the following functional forms of the driving force would result in a sustained oscillation?

(1) $\left|\vec{\mathbf{F}}^{\mathbf{dr}}\right| = 10\ \mathrm{N}$

(2) $\left|\vec{\mathbf{F}}^{\mathbf{dr}}\right| = 5\ \mathbf{t}$

(3) $\left|\vec{\mathbf{F}}^{\mathbf{dr}}\right| = \mathbf{F_o}\ \sin(\ \boldsymbol{\omega}_{\mathbf{dr}}\ \mathbf{t})$

(4) $\left|\vec{\mathbf{F}}^{\mathbf{dr}}\right| = -5\ \mathbf{t}$

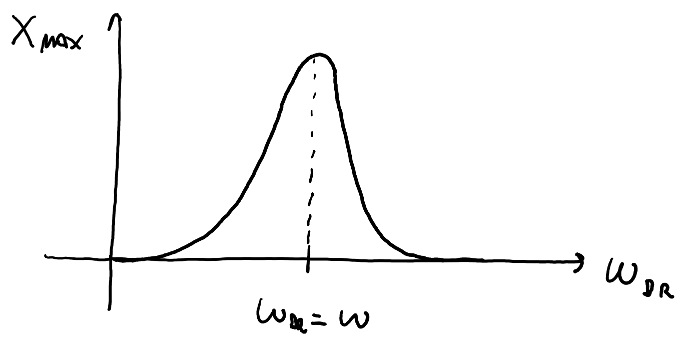
(c) Padmé the pademelon is attached to an ideal spring with non-negligible air resistance and a driving force. It is observed that Padmé's motion can be characterized by an angular frequency of $\boldsymbol{\omega}$ = 5 rad/s. **Which of the following driving force angular frequencies, $\boldsymbol{\omega}_{\mathbf{dr}}$, would result in the largest amplitude sustained oscillation?**

(1) $\boldsymbol{\omega}_{\mathbf{dr}} =$ 2.5 rad/s

(2) $\boldsymbol{\omega}_{\mathbf{dr}} =$ 5 rad/s

(3) $\boldsymbol{\omega}_{\mathbf{dr}} =$ 10 rad/s

(4) $\boldsymbol{\omega}_{\mathbf{dr}} =$ 25 rad/s



Conceptual questions for discussion

1. Coming soon to a lecture template near you.

Hints

OS.2.L3-1: No hints.

OS.2.L3-2: No hints.

OS.2.L3-3: No hints.

OS.2.L3-4: No hints.

OS.2.L3-5: No hints.

OS.2.L3-6: No hints.

OS.2.L3-7: No hints.

OS.2.L3-8: No hints.

OS.2.L3-9: No hints.

OS.2.L3-10: No hints.