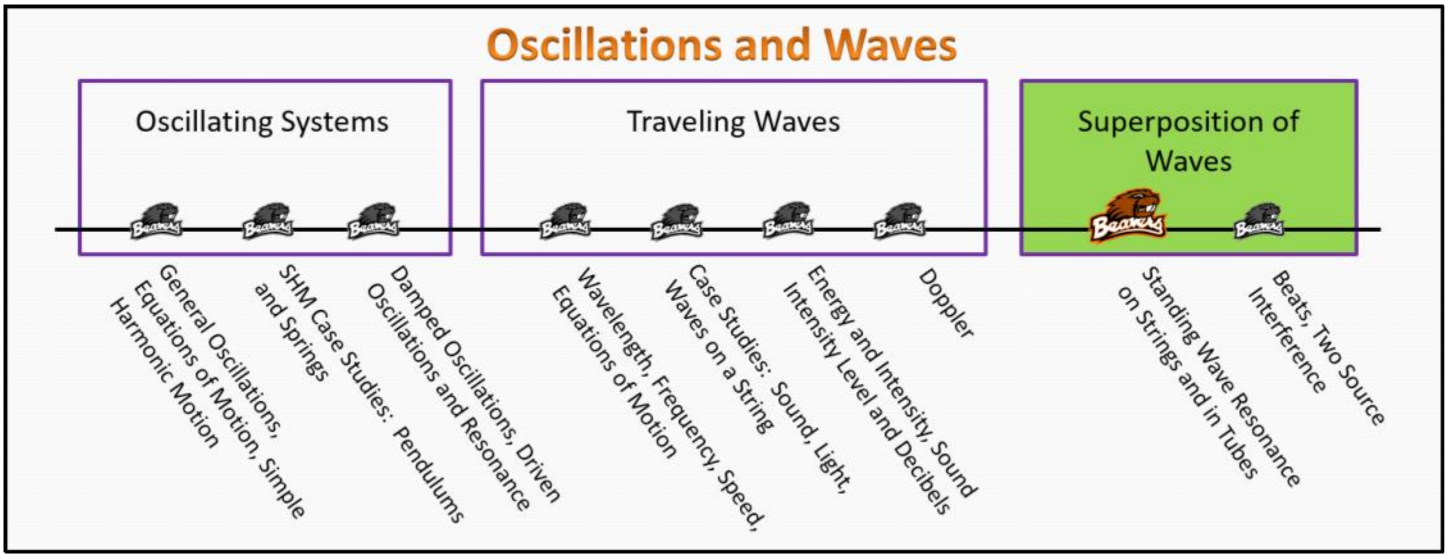
SW.2.L1 - Solutions

Friday, March 15, 2019 7:14 AM

Superposition of Waves
Foundation Stage (SW.2.L1)

Lecture 1
Standing Wave Resonance on Strings and in Tubes



Textbook Chapters (* Calculus version)

- **BoxSand** :: KC videos (Doppler Shift)
- **Knight** (College Physics : A strategic approach 3rd) :: 15.7
- ***Knight** (Physics for Scientists and Engineers 4th) :: 16.9
- **Giancoli** (Physics Principles with Applications 7th) :: 12-7 ; 12-8 ; 12-9

Warm up

SW.2.L1-1:

Description: Conceptual question connecting wave fronts and motion of source.

Learning Objectives: [?] - Can you identify the objectives from the previous lecture, and this lecture, that this question is relevant to?

Problem Statement: The picture below is a top view of ripple waves made by a water bug on the surface of the water. From the wave pattern, we can see that the bug has been moving

Selected Learning Objectives

1. Coming soon to a lecture template near you.

Key Terms

- Power
- Intensity
- Threshold of human hearing intensity
- Sound intensity level
- Decibels

Key Equations

Key Concepts

- Coming soon to a lecture template near you.

Questions

Act I: Superposition of Waves

SW.2.L1-2:

Description: Conceptual question about how observed frequency is affected by relative motion. (5 minutes)

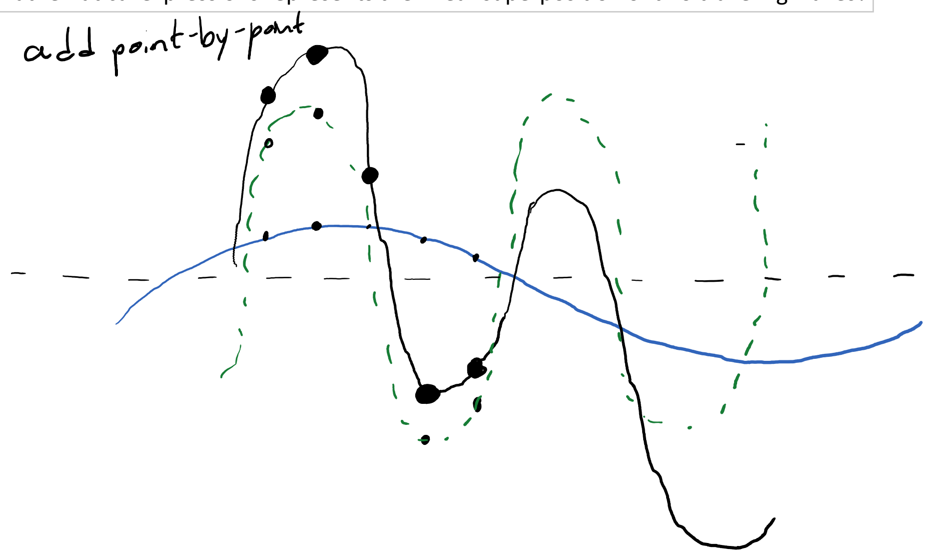
Learning Objectives: [1, 12, 13]

Problem Statement: Which of the following mathematical expressions represents the linear superposition of two traveling waves?

(1) $D(x,t) = D_1(x,t) + D_2(x,t)$

(2) $D(x,t) = \left(D_1(x,t)\right)^2 + \left(D_2(x,t)\right)^2$

(3) $D(x,t) = \sqrt{D_1(x,t) + D_2(x,t)}$



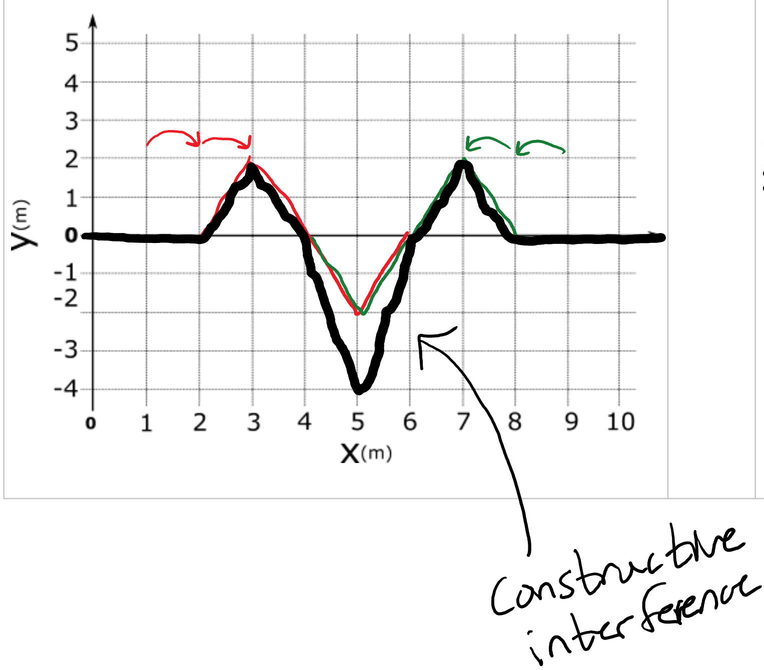
SW.2.L1-3:

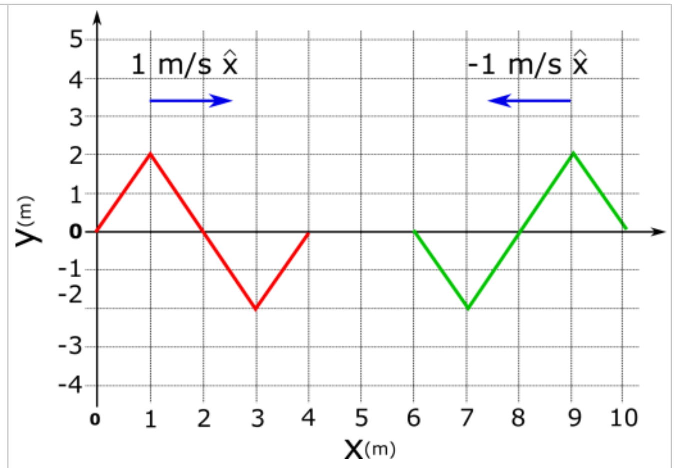
Description: Conceptual question about how observed frequency is affected by relative motion. (3 minutes + 4 minutes)

Learning Objectives: [1, 12, 13]

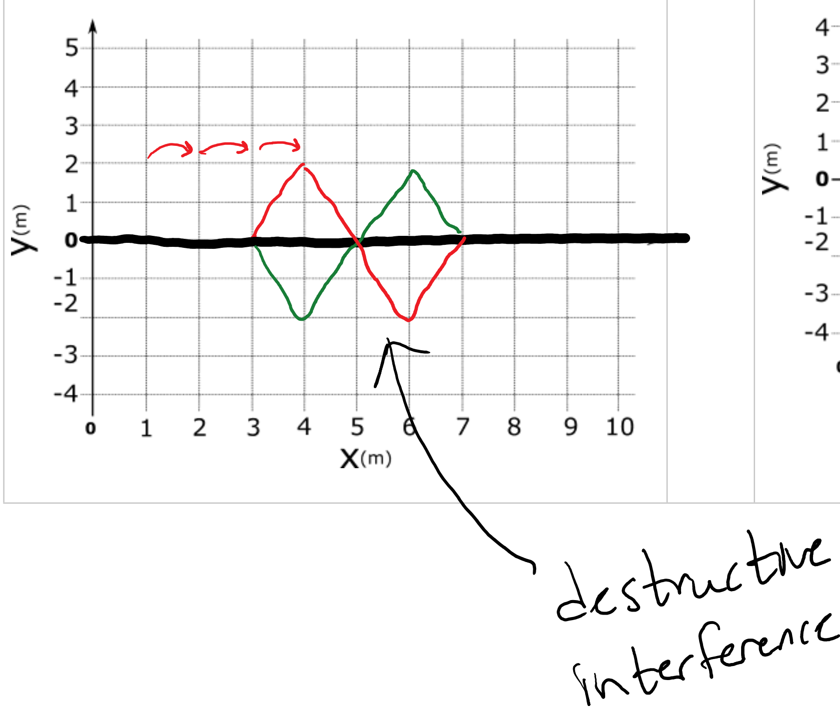
Problem Statement: Two traveling waves show below are traveling at 1 m/s in opposite directions.

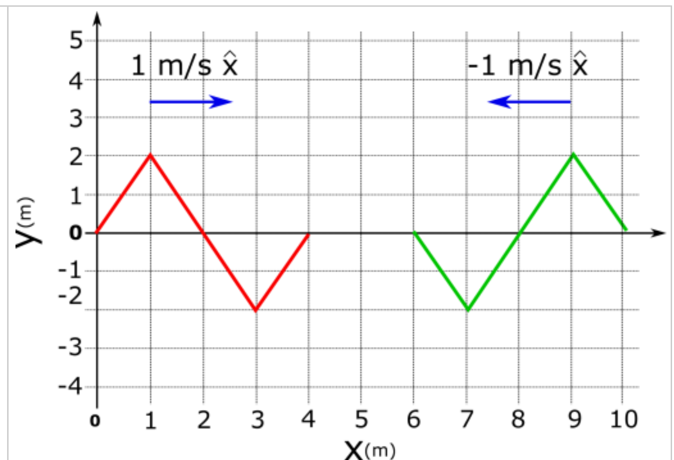
(a) Sketch the shape of the wave at **t** = 2 seconds.



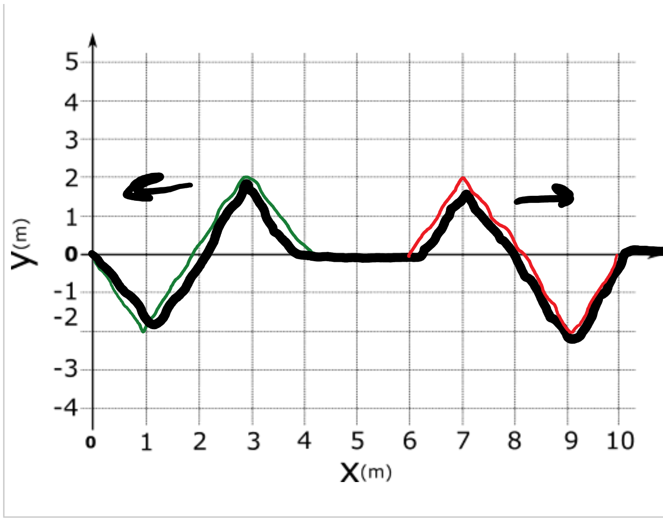


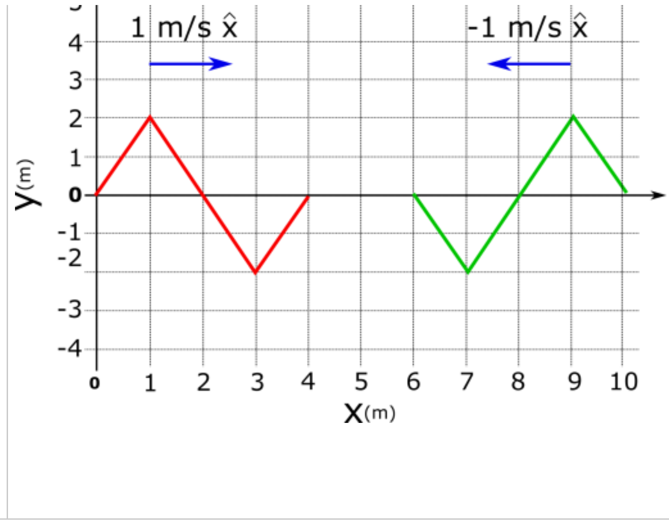
(a) Sketch the shape of the wave at **t** = 3 seconds.





(c) Sketch the shape of the wave at **t** = 6 seconds.





Act II: Representations of Standing Wave Patterns

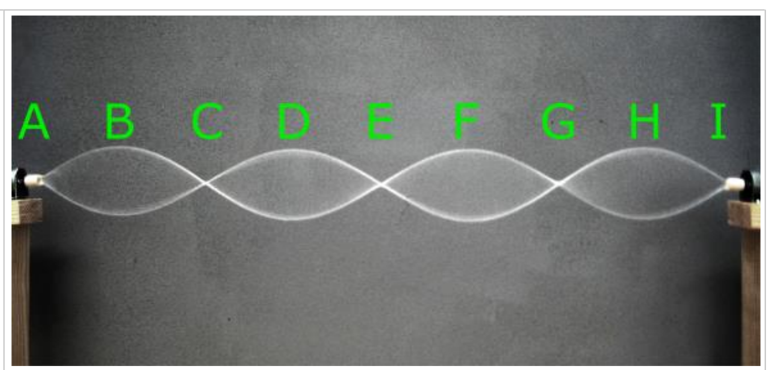
SW.2.L1-4:

Description: Conceptual question about how observed frequency is affected by relative motion. (4 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: A standing wave pattern for a string on a symmetric apparatus is shown below.

(a) Which locations on the string are oscillating with the largest amplitude?

(1) A (2) B (3) C (4) D (5) E	(6) F (7) G (8) H (9) I	

Anti-nodes

(b) Which locations on the string are oscillating with the smallest amplitude?

A C E G I

(b) Which locations on the string are oscillating with the smallest amplitude?

A, C, E, G, I

nodes

SW.2.L1-5:

Description: Conceptual question about how observed frequency is affected by wind. (3 minutes).

Learning Objectives: [1, 12, 13]

Problem Statement: Consider the 5 snapshots in time of a standing wave pattern on a symmetric apparatus. Match each snapshot with the time in the cycle given what **t** = 0 looks like shown below.

(1) **t** = **T**/8

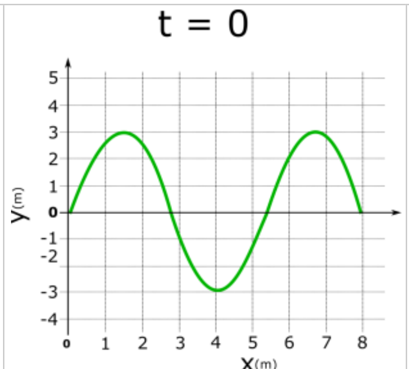
(2) **t** = **T**/4

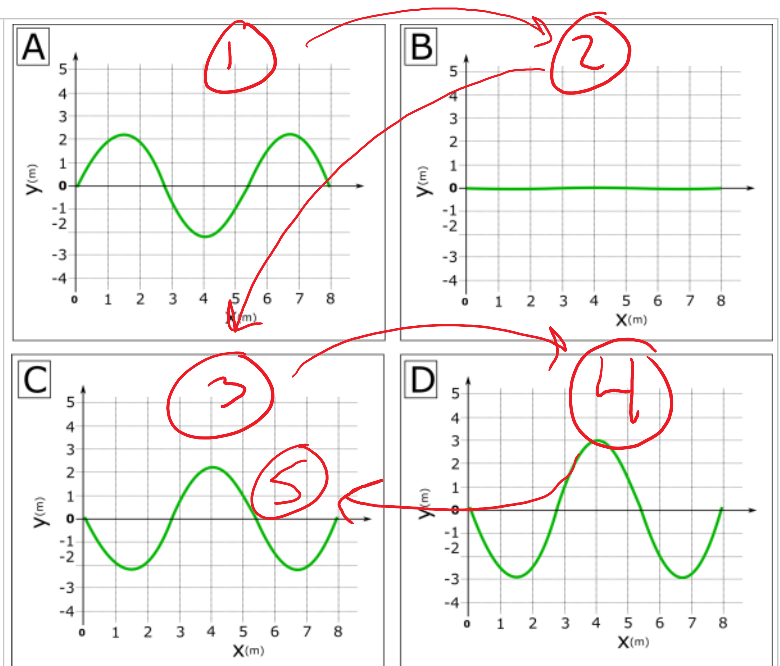
(3) **t** = 3**T**/8

(4) **t** = **T**/2

(5) t = 5T/8

⋮



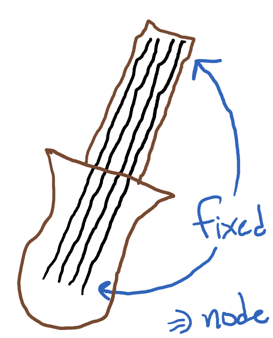


SW.2.L1-6:

Description: Calculate observed frequency for moving source and stationary observer. (2 minutes + 2 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: Consider the standing wave patterns show below.

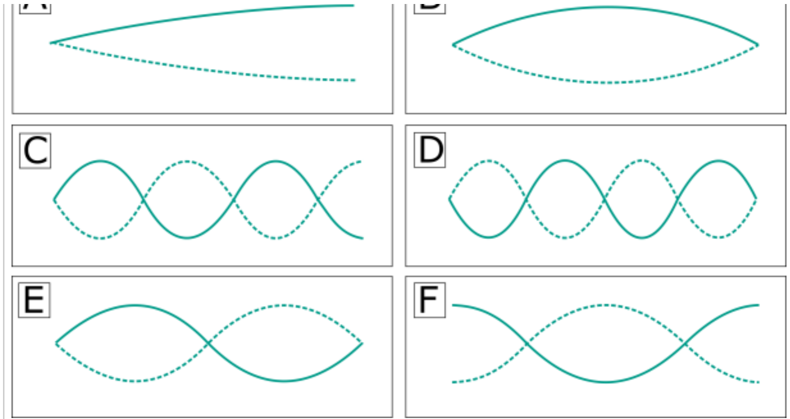


(a) Which are possible if the standing wave patterns are

A

B

representing the resonant modes of a guitar string.



B, D, E   ends are fixed ⇒ nodes

(b) Which mode will produce the lowest frequency if the images are of the same guitar string?

B

(c) Label each possible standing wave pattern with what harmonic it is.

B → 1st
E → 2nd
D → 4th

Symmetric solutions b/c boundaries are symmetric (fixed-fixed)

SW.2.L1-7:

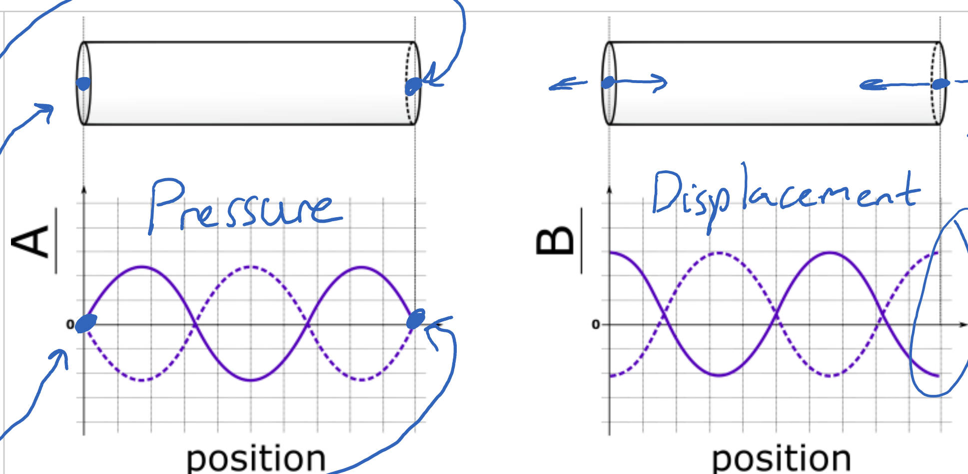
Description: Calculate observed frequency for moving observer and stationary source. (2 minutes + 2 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: Below is an open-open tube which is resonating in its 3rd harmonic. Also shown are two plots of either displacement vs position or pressure vs position. Label the axes of the plots with either displacement or pressure.

(1) **A** = Displacement
B = Pressure

(2) **A** = Pressure
B = Displacement



Pressure   Displacement

P ~ atm & const ⇒ node!

ends are pressure nodes

ends are displacement anti-node

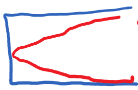
SW.2.L1-8:

Description: Calculate observed frequency for moving source and moving observer. (3 minutes + 2 minutes)

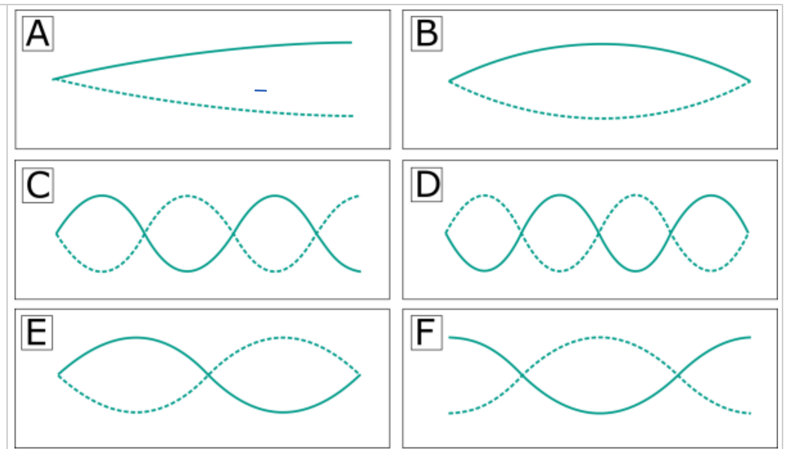
Learning Objectives: [1, 12, 13]

Problem Statement: Consider the standing wave patterns show below.

(a) Which are possible if the standing wave patterns are representing the resonant modes for sound in an open-closed tube?



← displacement wave   A, C



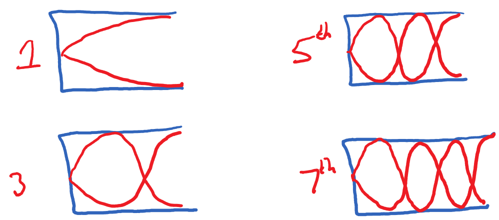
(b) Which mode will produce the lowest frequency?

A

(c) Label each possible standing wave pattern with what harmonic it is.

A → 1st

C → 7th



← anti-symmetric solutions b/c boundaries are anti-symmetric (open-closed)

Act III: Mathematical Modeling of Standing Wave Resonance

SW.2.L1-9:

Description: Calculate observed frequency for moving source and moving observer. (3 minutes + 2 minutes)

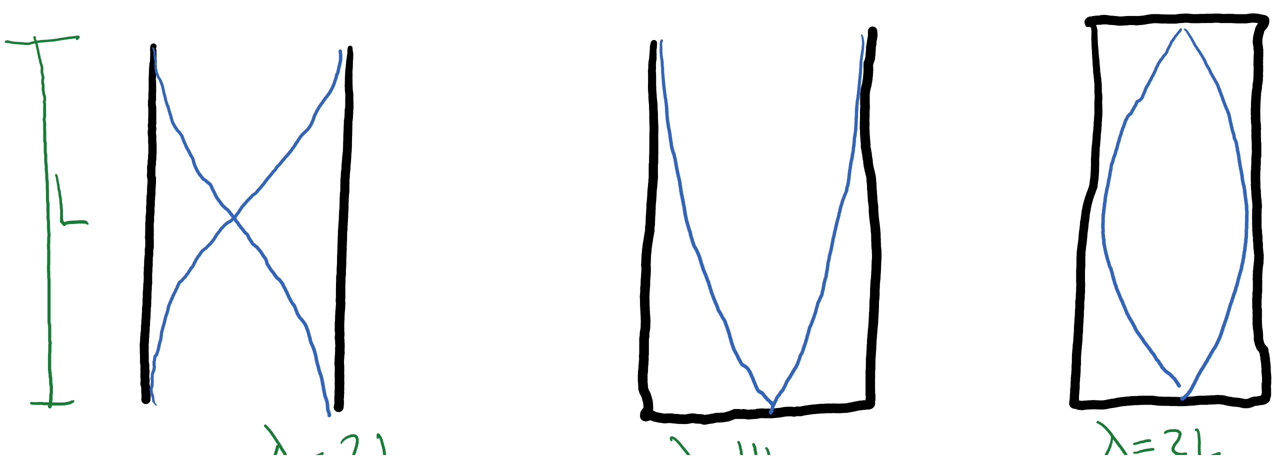
Learning Objectives: [1, 12, 13]

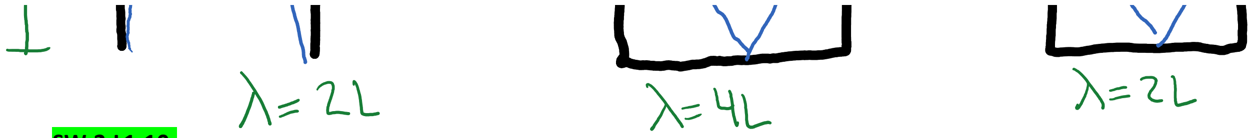
Problem Statement: You wish to make an organ for your home but are limited on the length of the tubes by the height of your ceiling. Which configuration will give you the lowest possible frequency for the same given length?

$v = f\lambda$

$f = \frac{v}{\lambda} \Rightarrow \lambda\uparrow\ f\downarrow$

(1) ~~Open-open~~
(2) Open-closed
(3) ~~Closed-closed~~
(4) All will have the same fundamental frequency.





SW.2.L1-10:

Description: Calculate observed frequency for stationary source and moving observer with wind. (3 minutes + 3 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: Below is a table will 4 resonant modes for a standing wave on a string in a symmetric apparatus with length **L**.

(a) Fill out the wavelength column in terms of the length **L**.

(b) Can you determine a mathematical model that could be used to predict all resonant mode wavelengths?

$\lambda = \frac{2L}{m}$

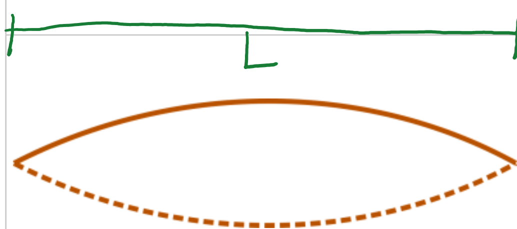
(c) Fill out the frequencies in terms of the speed of the wave **v**, and the length of the resonator **L**.

$v = f\lambda$

$f = \frac{v}{\lambda}$

(d) Can you determine a mathematical model that could be used to predict all resonant mode frequencies?

$f = m\frac{v}{2L}$

SWR Diagram	Mode	Wavelength (λ)	Frequency (f)
	1	$\frac{2L}{1}$	$\frac{1v}{2L}$
	2	$\frac{2L}{2}$	$\frac{2v}{2L}$
	3	$\frac{2L}{3}$	$\frac{3v}{2L}$
	4	$\frac{2L}{4}$	$\frac{4v}{2L}$

SW.2.L1-11:

Description: Calculate observed frequency for moving source and moving observer with reflection. (1 minute + 4 minutes + 1 minute + 5 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: Below is a table will 4 resonant modes for a standing wave on a string in an anti-symmetric apparatus with length **L**.

(a) Fill out the wavelength column in terms of the length **L.**

(b) Can you determine a mathematical model that could be used to predict all resonant mode wavelengths?

$\lambda = \frac{4L}{m}$

(c) Fill out the frequencies in terms of the speed of the wave **v**, and the length of the resonator **L.**

$f = \frac{v}{\lambda}$

(d) Can you determine a mathematical model that could be used to predict all resonant mode frequencies?

$f = \frac{mv}{4L}$

but only

$m = 1, 3, 5, 7, \ldots$

odd modes!

SWR Diagram	Mode	Wavelength (λ)	Frequency (f)
	**1**	$\frac{4L}{1}$	$\frac{1v}{4L}$
	**3**	$\frac{4L}{3}$	$\frac{3v}{4L}$
	**5**	$\frac{4L}{5}$	$\frac{5v}{4L}$
	**7**	$\frac{4L}{7}$	$\frac{7v}{4L}$

SW.2.L1-12:

Description: Conceptual question about observed wavelength given relative motion of light source and observer. (3 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: An anti-symmetric system that has a fundamental frequency of 100 Hz.

(a) Which of the following are harmonics?

(1) 200 Hz
(2) 300 Hz
(3) 400 Hz
(4) 500 Hz
(5) 600 Hz
(6) 700 Hz

odd sol'ns

(b) Fill out which resonant mode the allowed frequencies are.

Frequency	Resonant mode
200 Hz	
300 Hz	3
400 Hz	
500 Hz	5
600 Hz	
700 Hz	7

Fundamental ⇒ 1st

SW.2.L1-13:

Description: Conceptual question about what quantities can be found via Doppler shift with light from galaxies. (3 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: A guitar string of total length 1.5 meters and mass of 2.1 g is fixed between two points 0.640 meters apart. The tension is set to 88.1 N.

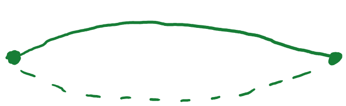
(a) When the string is plucked, what resonant mode does the string vibrate in?

(1) 1st harmonic
(2) 2nd harmonic
(3) 3rd harmonic

(b) What is the linear mass density of the string?

(1) 0.0014 kg/m
(2) 0.00328 kg/m
(3) 1.4 kg/m
(4) 3.28 kg/m

$\mu = \frac{mass}{length} = \frac{0.0021\ kg}{1.5\ m}$



(c) What is the frequency of the sound?

(1) 82 Hz
(2) 110 Hz
(3) 128 Hz
(4) 147 Hz
(5) 196 Hz
(6) 247 Hz
(7) 330 Hz

$v = f\lambda$

$f = \frac{v}{\lambda}$

$v = \sqrt{\frac{F}{\mu}}$

$\lambda = 2L = 2 \times 0.64m$

(d) What will the wavelength of the 3rd harmonic be?

(1) 0.213 m
(2) 0.427 m
(3) 0.5 m
(4) 0.85 m
(5) 1 m
(6) 2 m

$\lambda = \frac{2L}{n}$

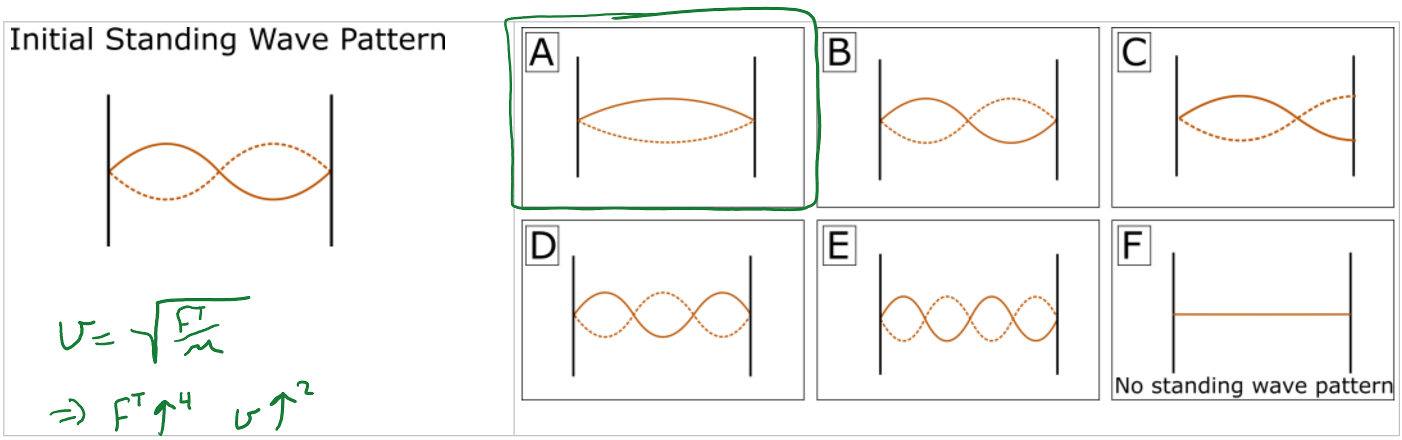
$\lambda_3 = \frac{2L}{3} = \frac{2}{3}(0.64m)$

SW.2.L1-14:

Description: Conceptual question about observed frequency from light emitted by distant galaxy. (5 minutes + 5 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: A standing wave on a string vibrates as shown in the figure below. Which standing wave pattern is produced if the tension is quadrupled while the frequency and length of the string are held constant.



$v = \sqrt{\frac{F^T}{\mu}}$

$\Rightarrow F^T \uparrow 4 \quad v \uparrow 2$

$\lambda = \frac{v}{f} \Rightarrow \lambda \uparrow 2$

SW.2.L1-15:

Description: Conceptual question about observed frequency from light emitted by distant galaxy. (5 minutes + 5 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: Two identical strings are fixed on both ends to the same apparatus. The string **A** is vibrating in its fundamental mode and it is observed that string **B** begins to vibrate at its third harmonic, driven by string **A**.

(a) Which of the following physical parameters are the same for both strings?

(1) Frequency
~~(2) Wavelength~~
~~(3) Speed~~
(4) Tone

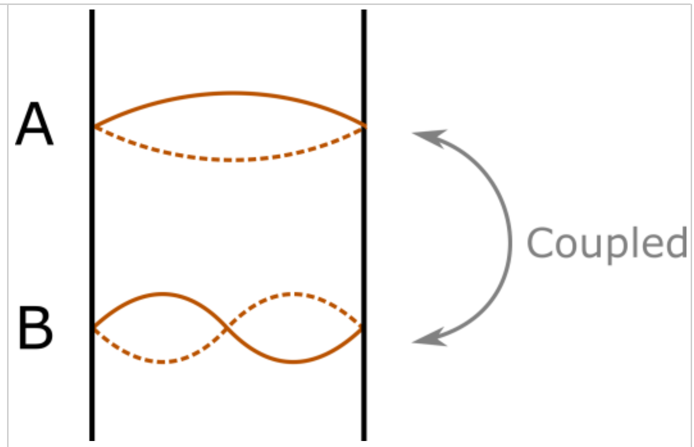
(b) What is the ratio of the tension of string **B** to that of string **A**?

$v = f\lambda \qquad f_A = f_B \qquad \lambda_B = \frac{1}{2}\lambda_A$

$\frac{v_B}{v_A} = \frac{f_B \lambda_B}{f_A \lambda_A} = \frac{1}{2}$

$v^2 = \frac{F^T}{\mu}, \quad \mu_A = \mu_B$

$\frac{F_B^T}{F_A^T} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$



Conceptual questions for discussion

1. Coming soon to a lecture template near you.

Hints

SW.2.L1-1: No hints.

SW.2.L1-2: No hints.

SW.2.L1-3: No hints.

SW.2.L1-4: No hints.

SW.2.L1-5: No hints.

SW.2.L1-6: No hints.

SW.2.L1-7: No hints.

SW.2.L1-8: No hints.

SW.2.L1-9: No hints.

SW.2.L1-10: No hints.

SW.2.L1-11: No hints.

SW.2.L1-12: No hints.

SW.2.L1-13: No hints.

SW.2.L1-14: No hints.

SW.2.L1-15: No hints.

SW.2.L1-16: No hints.

SW.2.L1-17: No hints.