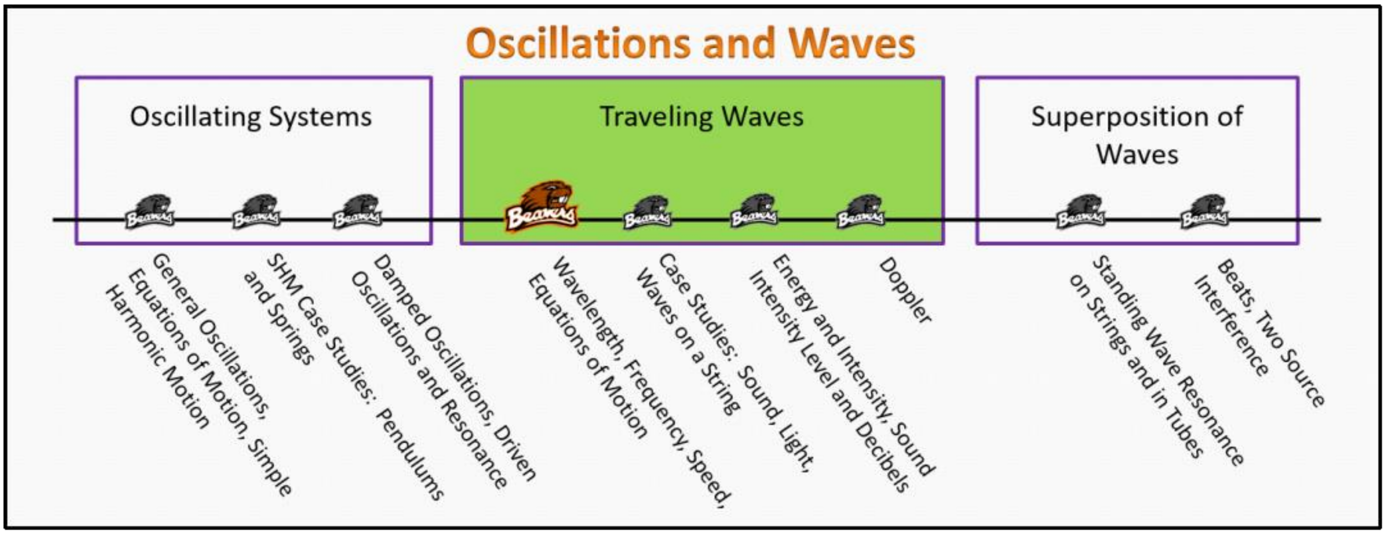
TW.2.L1 - Solutions

Sunday, March 3, 2019 8:03 PM

Traveling Waves
Foundation Stage (TW.2.L1)

Lecture 1
Wavelength, Frequency, Speed, Equations of Motion



Textbook Chapters (* Calculus version)

- **BoxSand** :: KC videos (Traveling Waves)
- **Knight** (College Physics : A strategic approach 3rd) :: 14.1 ; 14.2 ; 14.3 ; 14.4
- ***Knight** (Physics for Scientists and Engineers 4th) :: 15.1 ; 15.2 ; 15.3 ; 15.4
- **Giancoli** (Physics Principles with Applications 7th) :: 11-1 ; 11-2 ; 11-3

Warm up

TW.2.L1-1:

Description: Using a graphing device, plot a sin function and determine its motion based on adding and subtracting a constant in the argument.

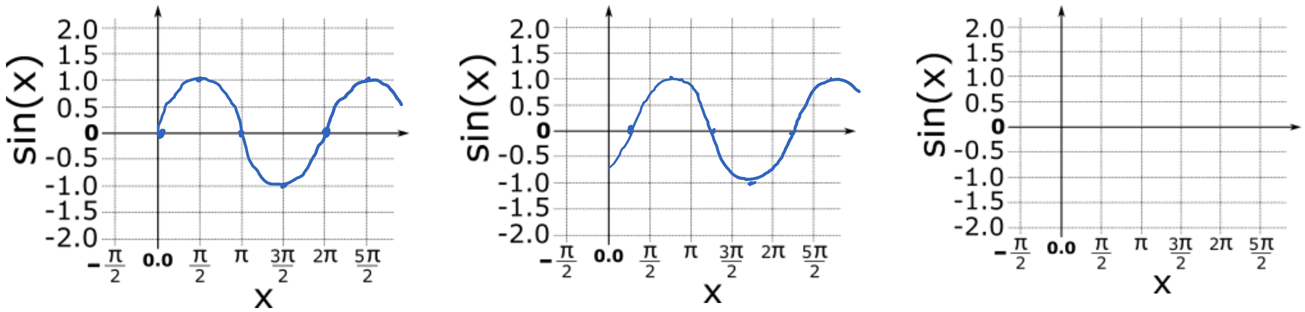
Learning Objectives: [?] - Can you identify the objectives from the previous lecture, and this lecture, that this question is relevant to?

Problem Statement: Using your favorite graphing device (e.g. graphing calculator, WolframAlpha, Mathematica, MATLAB, etc..) sketch the following plots on the given graphs below from **x** = -π/2 and **x** = 2 π.

(a) sin(x)

(b) sin(x - π/~~8~~ 4)

(c) sin(x + π/8)



(d) What can you conclude about the direction of motion of the sin function if you subtract or add a value in the argument?

Sin (x - something) moves the wave to the right

Selected Learning Objectives

1. Coming soon to a lecture template near you.

Key Terms

- Mechanical wave
- Electromagnetic wave
- Matter wave
- Gravity wave
- Transverse wave
- Longitudinal wave
- Combination of transverse and longitudinal waves
- Wavelength
- Wave number
- Wave speed
- Oscillator speed
- Snapshot graph
- History graph

Key Equations

Key Concepts

- Coming soon to a lecture template near you.

Questions

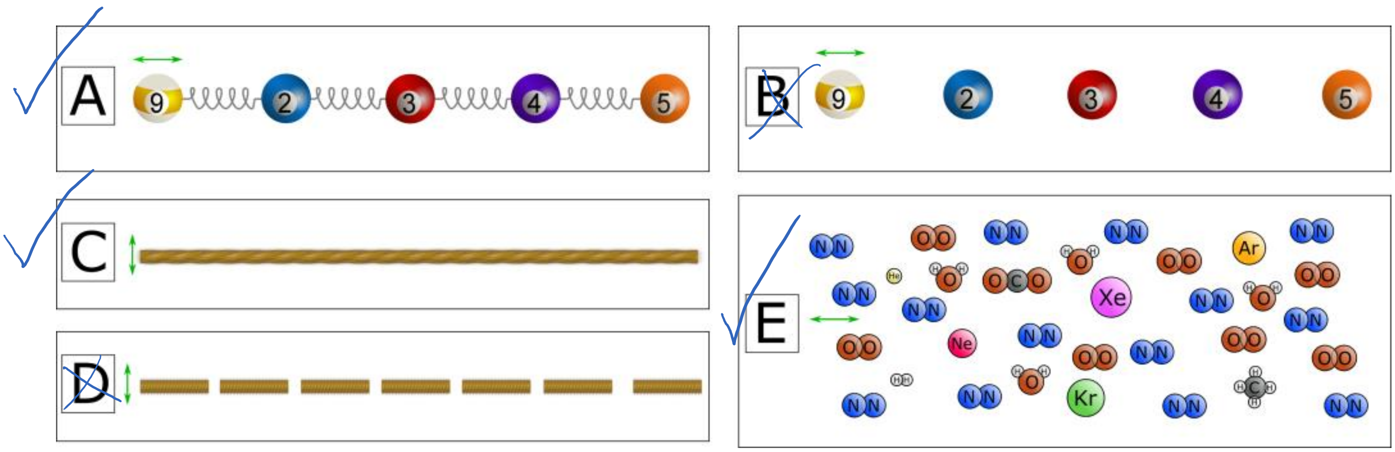
Act I: Classifying Waves

TW.2.L1-2:

Description: Conceptual question about features of oscillators. (3 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: Which system would sustain a traveling wave if you wiggled the left most end?



TW.2.L1-3:

Description: Given graphs of potential energy vs position identify non-oscillatory motion. (3 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: Which of the following are necessarily true about traveling waves?

(1) Traveling waves transport energy from one location to another.
(2) Traveling waves are a collection of many oscillators.
(3) Traveling waves transport matter from one location to another.
(4) Traveling waves have a velocity that is dependent on the medium though which the wave travels.
(5) Traveling waves have a frequency determined by the source of the wave.

TW.2.L1-4:

Description: Conceptual question about features of oscillators. (2 minutes + 2 minutes + 2 minutes + 2 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: Match the wave phenomena with the wave type.

(a) Crowd at sporting event.
(b) Diffraction pattern of accelerated elections through two small slits.
(c) HAM Radio waves.
(d) A ripple of space-time.

(1) Gravity wave.
(2) Electromagnetic wave.
(3) Matter wave.
(4) Mechanical wave.

TW.2.L1-5:

Description: Given graphs of potential energy vs position identify simple harmonic oscillators. (3 minutes).

Learning Objectives: [1, 12, 13]

Problem Statement: Match the wave phenomena with the mode of the wave.

(a) Crowd at sporting event.
(b) Earthquake wave.
(c) Water wave.

(1) Transverse wave.
(2) Longitudinal wave.
(3) Combination of transverse and longitudinal.

Act II: Characteristics of Waves

TW.2.L1-6:

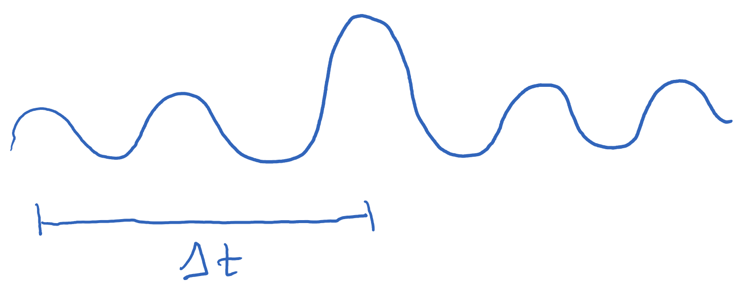
Description: Given force equations as a function of displacement from equilibrium, identify which would give rise to simple harmonic motion. (3 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: A friend shows you a picture of a series of periodic ripple on the surface of a pond from a stone thrown into the pond.

(a) Your friend tells you the picture was taken 2.3 seconds after the source began emitting the wave. From this picture what can you determine?

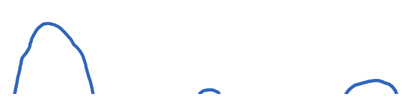
(1) Wave length.
(2) Period. ← $\Delta t/2$
(3) Frequency. ← $\frac{1}{T} = \frac{2}{\Delta t}$
(4) Wave speed.
(6) Amplitude.



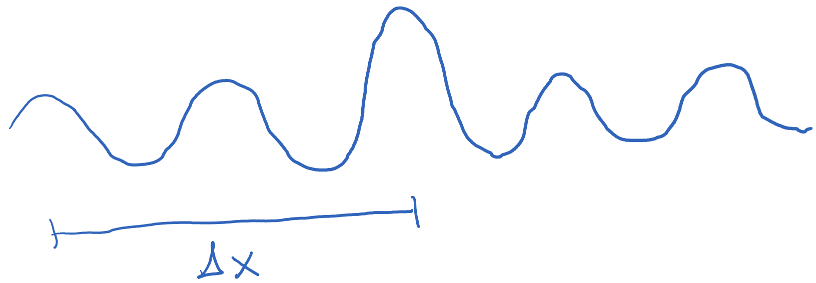
(b) What if your friend showed you the picture and instead only told you that the outermost ripple is 10.0 m from the source of the ripples. What could you determine then?

(1) Wave length. ← $\Delta x/2$
(2) Period.
(3) Frequency.
(4) Wave speed.
(6) Amplitude.

if you also know Δt from previous question, then



from previous question, then you can find these.



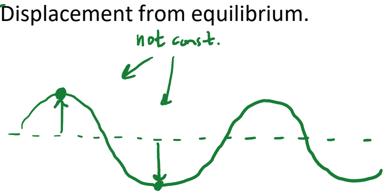
TW.2.L1-7:

Description: Conceptual question about period for simple harmonic motion. (2 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: With regards to traveling waves in one medium, which of the following are constants in both space and time?

(1) Wave speed. ← determined by medium
(2) Frequency. ← determined by source!
~~(3)~~ Oscillator speed.
(4) Wavelength. ← result of 1+2
~~(5)~~ Displacement from equilibrium.



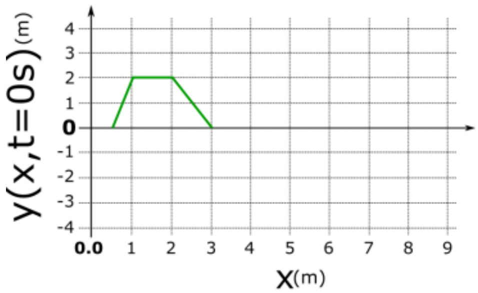
Act III: Equation Modeling

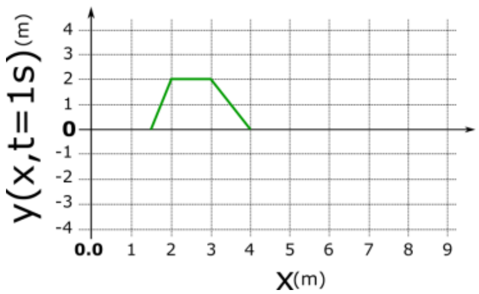
TW.2.L1-8:

Description: Conceptual question about equilibrium position for simple harmonic motion. (4 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: Consider the two snapshots of a wave pulse traveling along the x-direction. Assume the pulse does not change its shape in the time intervals we are interested in.





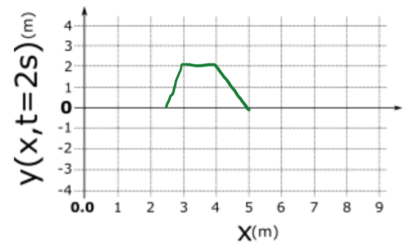
(a) What is the speed of the wave?

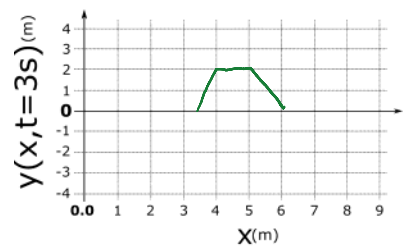
1 m/s

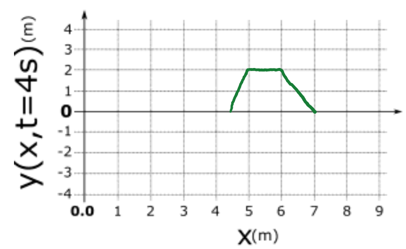
x=5m

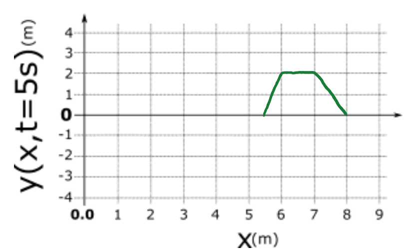
t (s)	y (m)
0	0
1	0
2	0
3	2
4	2
4.5	0
5	0

(b) Sketch the wave pulse at **t** = 2 s, 3 s, 4 s, and 5 s.

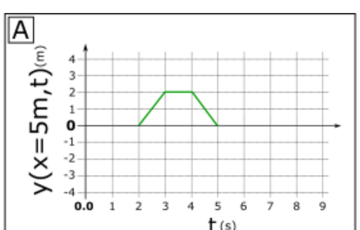


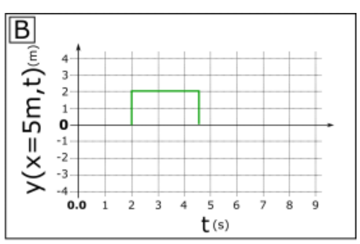


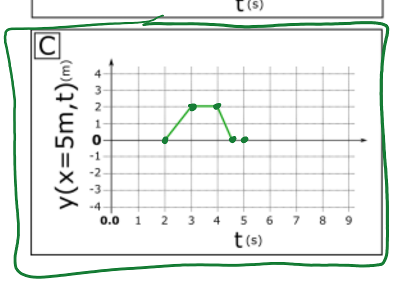


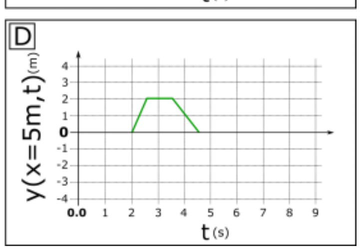


(c) Which of the following plots represents the history graph for the wave pulse at **x** = 5 meters?







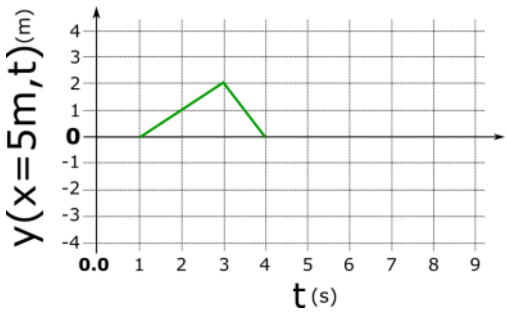


TW.2.L1-9:

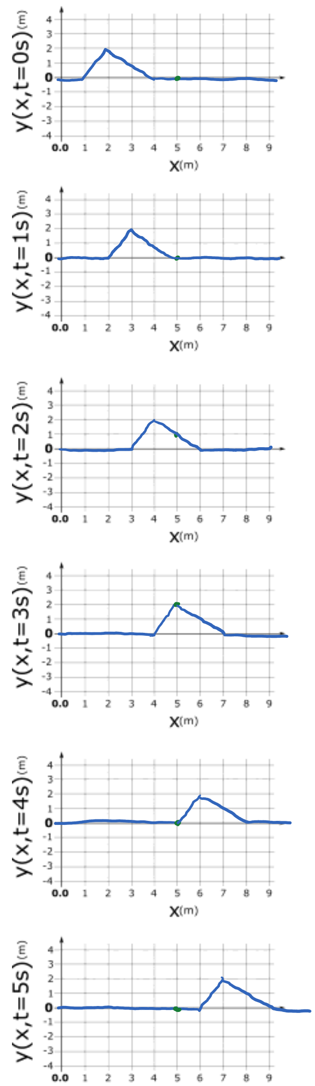
Description: Complete an energy flow diagram for a SHO. (3 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: Consider the history plot for a traveling wave pule that maintains its shape. The history plot was created at **x** = 5 m. Sketch snapshots of the wave pulse on the provided graphs given that the pulse has a speed of 1 m/s to the right.

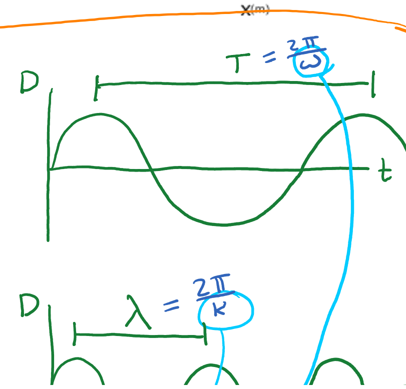


I drew the green dots on each graph → first, then t=2s picture

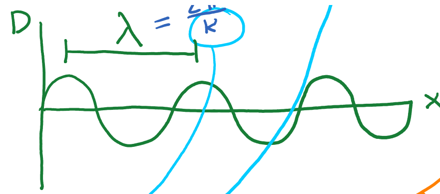


period (T) is time it takes for one oscillation.

wavelength (λ) is



Wavelength (λ) is distance for one oscillation



TW.2.L1-10:

Description: Conceptual question relating energy and amplitude of a simple harmonic oscillator. (3 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: A traveling wave is modeled by the equation **D**(**x**,**t**) = 45 sin(0.5 **x** - 2 **t**), where **D** and **x** are in meters, **t** is in seconds, and all other constants are in SI units.

(a) What is the amplitude of the wave?

45 m

(b) What is the period of the wave?

$\omega = 2 \frac{rad}{sec}$

$T = \frac{2\pi}{\omega} = \pi$ sec

(c) What is the wavelength of the wave?

$k = 0.5 \frac{rad}{meter}$ ← spatial frequency

$\lambda = \frac{2\pi}{k} = 4\pi$ meters

(d) What is the wave speed?

$v = \lambda f$

$= \lambda \frac{1}{T}$

$= (4\pi\ m)(\frac{1}{\pi}\ \frac{1}{s})$

$v = 4\ m/s$

(e) Does the wave travel in the positive x-direction or negative x-direction?

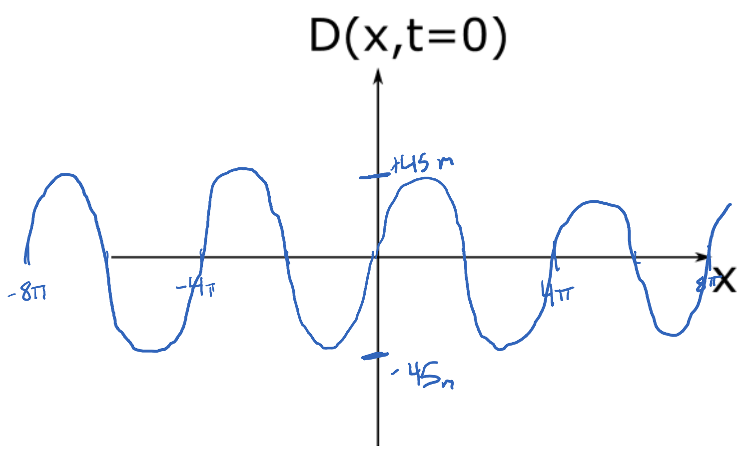
+x direction

(f) What mode of wave is this?

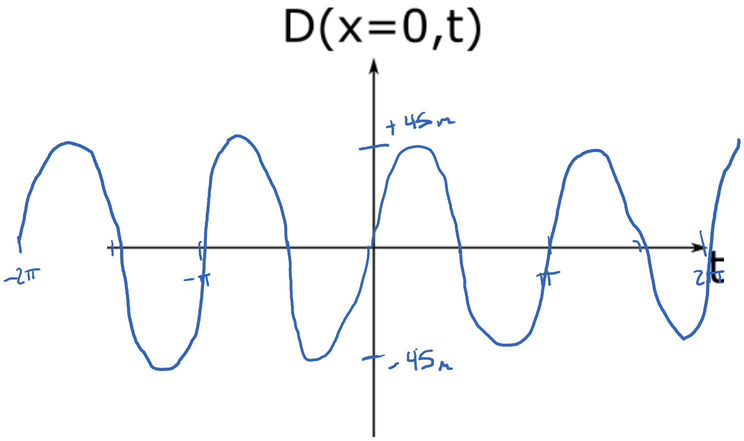
could be either transverse or long.!

(g) A traveling wave is modeled by the equation **D**(**x**,**t**) = 45 sin(0.5 **x** - 2 **t**), where **D** and **x** are in meters, **t** is in seconds, and all other constants are in SI units. **Sketch a snapshot of the wave at t = 0 s as a function of position. Include about two oscillations in both the positive and negative x region.**

$\Rightarrow D(x,0) = 45 \sin(0.5x)$



(h) A traveling wave is modeled by the equation **D**(**x**,**t**) = 45 sin(0.5 **x** - 2 **t**), where **D** and **x** are in meters, **t** is in seconds, and all other constants are in SI units. **Sketch the displacement from equilibrium of the wave at x = 0 as a function of time. Include about two oscillations in both the positive and negative t regions.**

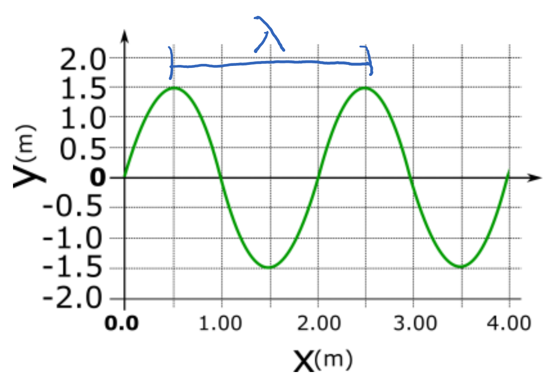


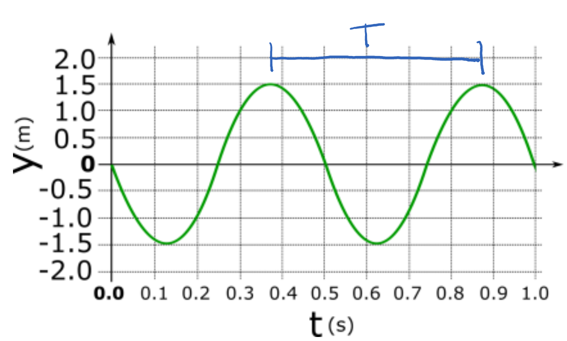
TW.2.L1-11:

Description: Energy analysis for a SHO. (6 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: A traveling wave is modeled by the graphical representations shown below, a snapshot and a history graph are shown below for **t** = 0 s and **x** = 0 s respectively.





(a) What is ~~the amplitude~~ of the wave?

$\lambda = 1.5$ m

(b) What is the period of the wave?

0.37 sec → 0.87 sec

⇒ $T = 0.5$ sec

(c) What is the wavelength of the wave?

0.5m → 2.5m $\lambda = 2$m

(d) What is the wave speed?

$v = \lambda f = \frac{\lambda}{T} = 4$ m/s

(e) Does the wave travel in the positive x-direction or negative x-direction?

can't tell

(f) What mode of wave is this?

$y(x,t)$ ⇒ transverse

(f) What mode of wave is this?

t (s)

$y(x,t) \Rightarrow$ transverse

(g) Which of the following equations is a correct mathematical representation for this traveling wave assuming all constants are SI units?

(1) **D**(**x**,**t**) = 1.5 sin(π **x** - 4 π **t**)

(2) **D**(**x**,**t**) = 1.5 sin(2 **x** - 0.5 **t**)

(3) **D**(**x**,**t**) = 1.5 cos(2 **x** - 0.5 **t**)

(4) **D**(**x**,**t**) = - 1.5 sin(- π **x** + 4 π **t**)

$k = \frac{2\pi}{\lambda} = \frac{2\pi}{2} = \pi \frac{rad}{m}$

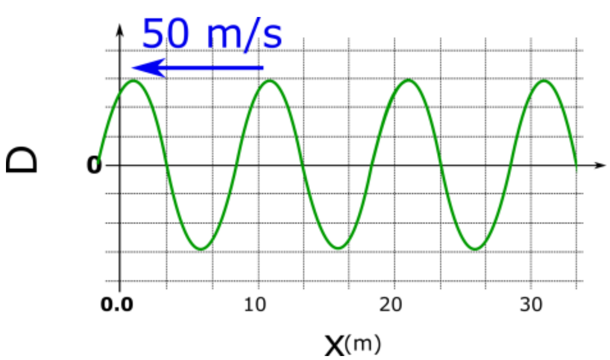
$\omega = \frac{2\pi}{T} = \frac{2\pi}{1/2} = 4\pi \frac{rad}{sec}$

TW.2.L1-12:

Description: Conceptual question about angular frequency vs angular velocity. (4 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: What is the frequency of this traveling wave?



$\lambda = 10m$

$v = 50 m/s$

$v = f\lambda$

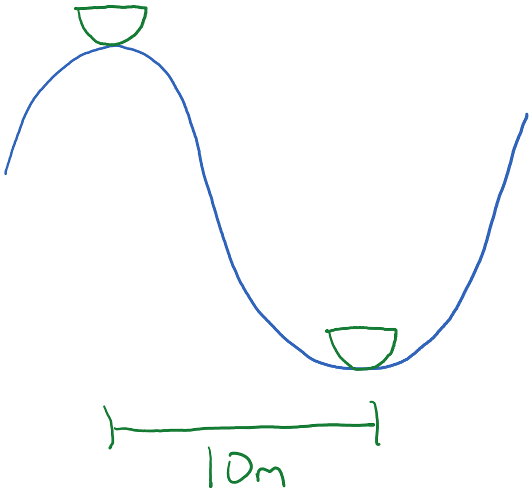
$f = \frac{v}{\lambda} = \frac{50 m/s}{10 m} = 5 hz$

TW.2.L1-13:

Description: Given a mass-spring system and initial conditions via written description and physical representation, identify the correct graphical representation. Find sign of velocities and accelerations at specific points. (3 minutes + 5 minutes + 5 minutes + 3 minutes).

Learning Objectives: [1, 12, 13]

Problem Statement: Two canoes are 10 m apart on a lake. Each bobs up and down with a period of 4.0 seconds. When one canoe is at its highest point, the other canoe is at its lowest point. Both canoes are always within a single cycle of the waves. Determine the speed of the waves.



$T = 4 \text{ sec}$

$\frac{1}{2}\lambda = 10 m$

$\Rightarrow \lambda = 20 m$

$v = \lambda f = \lambda \frac{1}{T} = 5 m/s$

Conceptual questions for discussion

1. Coming soon to a lecture template near you.

Hints

TW.2.L1-1: No hints.

TW.2.L1-2: No hints.

TW.2.L1-3: No hints.

TW.2.L1-4: No hints.

TW.2.L1-5: No hints.

TW.2.L1-6: No hints.

TW.2.L1-7: No hints.

TW.2.L1-8: No hints.

TW.2.L1-9: No hints.

TW.2.L1-10: No hints.

TW.2.L1-11: No hints.

TW.2.L1-12: No hints.

TW.2.L1-13: No hints.