

# Mechanics

## Linear

## Rotational

Position:  $\vec{r} = \langle x, y \rangle$

Position:  $\theta$  (in radians)

### Average Quantities

Vector Math

$$\bar{\vec{v}} = \frac{\Delta \vec{r}}{\Delta t}$$

$$\bar{\vec{a}} = \frac{\Delta \vec{v}}{\Delta t}$$

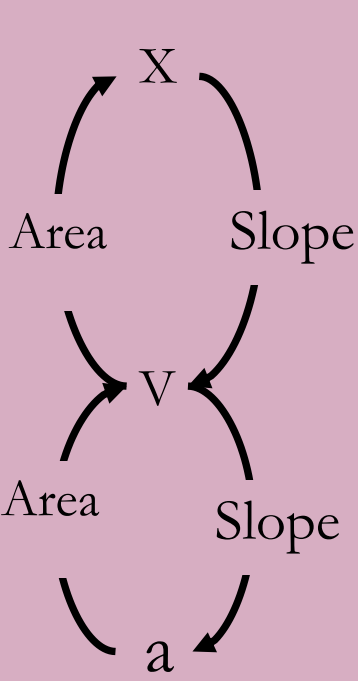
### Average Quantities

Sign Conventions:  
C.W. (-)  
C.C.W. (+)

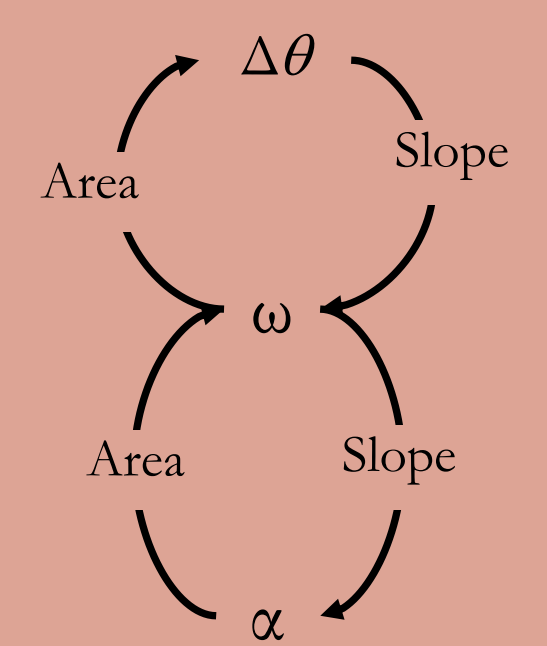
$$\bar{\omega} = \frac{\Delta \theta}{\Delta t}$$

$$\bar{\alpha} = \frac{\Delta \omega}{\Delta t}$$

### Graphical



### Graphical



## Kinematics

(Motion without Cause)

### Instantaneous - Kinematic

$$\Delta x = v_i \Delta t + \frac{1}{2} a \Delta t^2$$

$$v_f = v_i + a \Delta t$$

$$v_f^2 = v_i^2 + 2a \Delta x$$

### Problem Solving

- Break up each stage/object/dimension/ + same # of eq's each
- Find same # eq's as unknowns

### Instantaneous - Kinematic

$$\Delta \theta = \omega_i \Delta t + \frac{1}{2} \alpha \Delta t^2$$

$$\omega_f = \omega_i + \alpha \Delta t$$

$$\omega_f^2 = \omega_i^2 + 2\alpha \Delta \theta$$

- Problem Solving  
Similar to Linear

## Linear

## Rotational

### Tangential Components:

$$\alpha = \frac{a_t}{r}$$

Acceleration

$$\omega = \frac{v_t}{r}$$

Velocity

### Radial Component of Acceleration:

$$a_r = \omega^2 r$$

## Review

- Algebra
- Unit Conversion
- Vectors
- Dimensional Analysis

## Acceleration

## Linear (Point Particle)

### Newton's Laws:

- 1) Inertia
- 2)  $\sum \vec{F}_{ext} = M_{sys} a_{sys}$   
( $\sum \vec{F}, a, \Delta \vec{V}$  all point in the same direction)
- 3) Force pairs ( $\vec{F}_{12} = -\vec{F}_{21}$ )

### Uniform Circular Motion (Point Particle):

$$\langle \sum \vec{F}_r, \sum \vec{F}_t, \sum \vec{F}_y \rangle = m \langle a_r, 0, 0 \rangle$$

$$a_r = \frac{v^2}{r}$$

## Forces

(motion with cause)

### Universal Gravity:

$$F^G = \frac{GM_1 M_2}{r_{12}^2}$$

Orbits:

$$\sum \vec{F}_r = m \frac{v^2}{r}$$

### Methods:

- Identify Systems
- FBD
- Align Cord w/ acceleration
- Constraints
- Apply 2nd Law

### Friction - Opposes Relative Motion Between

$$|\vec{F}^{f,s,max}| = \mu_s |\vec{F}^N|$$

$$|\vec{F}^{f,k}| = \mu_k |\vec{F}^N|$$

### Torque:

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$|\vec{\tau}| = |\vec{r}| |\vec{F}| \sin(\theta) = r \perp |\vec{F}|$$

## Angular (Rigid Body)

### Methods:

- Extended FBD (eFBD) for Representation of Angular Motion
- FBD - Linear Motion
- Apply 2nd Law

Moment of Inertia (Point Particle):

$$I = mr^2$$

## Forces over Distances

$$\vec{p} = m\vec{v}$$

## Forces Over Time

$$K = \frac{1}{2} m v^2$$

## Linear

Definition  $\vec{p} = m\vec{v}$

2nd Law Revisited  $\sum \vec{F} = \frac{\Delta \vec{P}}{\Delta t}$

### Impulse (J) Momentum Theorem:

$$\Delta \vec{P} = \sum \vec{F} \Delta t \equiv \vec{J}$$

## Momentum

$$\sum \vec{F}_{ext} \Delta t$$

Area under force over time curve

### Conservation of Momentum

if  $\sum \vec{F}_{ext} \Delta t \approx 0$  then  $\sum \vec{P}_i = \sum \vec{P}_f$

$$K_{lin} = \frac{P^2}{2m}$$

$$K_{rot} = \frac{L^2}{2I}$$

## Angular

Define:

$$\vec{L} = \vec{r} \times \vec{P}$$

$$\vec{L} = I\vec{\omega}$$

$$\sum \tau = \frac{\Delta L}{\Delta t}, \text{ if } \sum \tau \Delta t = 0$$

Conservation of Angular Momentum:

$$\Delta L = 0$$

$$\sum L_i = \sum L_f$$

w/  $\sum \tau = I\alpha = I \frac{\Delta \omega}{\Delta t}$

## Energy

### Work-Energy Theorem

$$\sum K_i + \sum W = \sum K_f, K_{cin} = \frac{1}{2} m v^2$$

$$\vec{W} = \vec{F} \cdot \Delta \vec{r}$$

$$= |\vec{F}| |\Delta \vec{r}| \cos(\theta) = F_{||} |\Delta \vec{r}|$$

### Conservative vs. Non-Conservative

- Conservative work is independent of path and only depends on location, e.g. gravity and springs.  $W = W_c + W_{nc}$
- Create Potential Energy  $W_c = -\Delta U$  and function of position.  $W = -\Delta U + W_{nc}$

### Plots & Work

$$W = F_x \Delta x + F_y \Delta y + F_z \Delta z$$

Area under  $F_x$  as a function of Position Curve

## Conservation of Energy

### Gravitational Potential Energy

$$U_{Gravity} = mgh$$

- where:
- k=spring constant
  - x=displacement of the spring from equilibrium

$$\sum E_i + \sum W_{nc} = \sum E_f$$

where,  $E = K + U$

Add up all the initial kinetic and potential energy, then add to that the net external non-conservative work, and set that equal to all the final kinetic and potential energy.

### Elastic Potential Energy

$$U_{spring} = \frac{1}{2} kx^2$$

- where:
- k=spring constant
  - x=displacement of the spring from equilibrium

## Rotational

### Rotational Energy

$$K = K_{cin} + K_{not}$$

$$K_{not} = \frac{1}{2} I \omega^2$$