Mechanics

Review

-Algebra

-Unit Conversion

-Vectors

-Dimensional Analysis

Kinematics

(Motion without Cause)

Linear

Position: $\vec{r} = \langle x, y \rangle$

Average Quantities

$$\bar{\vec{V}} = \frac{\Delta \vec{r}}{\Delta t}, \quad \bar{\vec{a}} = \frac{\Delta \vec{V}}{\Delta t}$$

Vector Math

Graphical

X

Area Slope

V

Area Slope

a

Instantaneous – Kinematic

$$\Delta x = V_i \Delta t + \frac{1}{2} a \Delta t^2$$

$$V_f = V_i + a\Delta t$$

$$V_f{}^2 = V_i^2 + 2a\Delta x$$

Problem Solving

- Break up each stage/object/dimension/ + same # of eq's each
- Find same # eq's as unknowns

Rotational

Position: θ (in radians)

Average Quantities

$$\bar{\omega} = \frac{\Delta\theta}{\Delta t}, \quad \bar{\alpha} = \frac{\Delta\omega}{\Delta t}$$

Sign Conventions:

C.W. (-)

C.C.W (+)

Graphical

$\Delta\theta$

Area Slope

ω

Area Slope

α

Instantaneous — Kinematic

$$\Delta\theta = \omega_i \Delta t + {}^1\!/_2\, \alpha \Delta t^2$$

$$\omega_f = \omega_i + \alpha \Delta t$$

$$\omega_f{}^2 = \omega_i{}^2 + 2\alpha\Delta\theta$$

Problem Solving Similar to Linear

Linear ⟷ Rotational

Tangential Components:

Acceleration $\alpha = \dfrac{a_t}{r}$

Velocity $\omega = \dfrac{v_t}{r}$

Radial Component of Acceleration:

$$a_r = \omega^2 r$$

Acceleration

Forces

(motion with cause)

Linear (Point Particle)

Newton's Laws:

1) Inertia

2) $\sum \vec{F}_{ext} = M_{sys} a_{sys}$

($\sum \vec{F}, a, \Delta\vec{V}$ all point in the same direction)

3) Force pairs ($\vec{F}_{12} = -\vec{F}_{21}$)

Uniform Circular Motion (Point Particle):

$$\hat{r}, \hat{t}, \hat{y}$$

$$\langle \sum \vec{F}_r, \sum \vec{F}_t, \sum \vec{F}_y \rangle = m\langle a_r, 0, 0\rangle$$

$$a_r = \frac{v^2}{r}$$

Friction - Opposes Relative Motion Between

$$\left|\vec{F}^{f,s,\max}\right| = \mu_s \left|\vec{F}^N\right|$$

$$\left|\vec{F}^{f,k}\right| = \mu_k \left|\vec{F}^N\right|$$

Methods:

- Identify Systems
- FBD
- Align Cord w/ acceleration
- Constraints
- Apply 2nd Law

Universal Gravity:

$$F^G = \frac{GM_1 M_2}{r_{12}^2}$$

Orbits:

$$\sum F_r = m\frac{V^2}{r}$$

Angular (Rigid Body)

Torque:

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$|\vec{\tau}| = |\vec{r}||\vec{F}|\sin(\theta) = r\perp|\vec{F}|$$

Methods:

- Extended FBD (eFBD) for Representation of Angular Motion
- FBD Linear Motion
- Apply 2nd Law

Moment of Inertia (Point Particle):

$$I = mr^2$$

$\vec{p} = m\vec{v}$

Forces Over Time

$K = \dfrac{1}{2} mV^2$

Forces over Distances

Momentum

Linear

Definition $\vec{p} = m\vec{v}$

2nd Law Revisited $\Sigma\bar{\vec{F}} = \dfrac{\Delta\vec{P}}{\Delta t}$

Impulse (J) Momentum Theorem:

$$\Delta\vec{P} = \Sigma\bar{\vec{F}}\Delta t \equiv \vec{J}$$

$$\underbrace{\Sigma\bar{\vec{F}}_{ext}\Delta t}_{\text{Area under force over time curve}}$$

Conservation of Momentum

if $\Sigma\vec{F}_{ext.}\Delta t \approx 0$ then $\Sigma\vec{P}_i = \Sigma\vec{P}_f$

Angular

Define:

$$\vec{L} = \vec{r} \times \vec{P}$$

$$\vec{L} = I\vec{\omega}$$

$\Sigma\tau = \dfrac{\Delta L}{\Delta t}$, if $\Sigma\tau\Delta t = 0$

Conservation of Angular Momentum:

$$\Delta L = 0$$

$$\Sigma L_i = \Sigma L_f$$

w/ $\Sigma\tau = I\alpha = I\dfrac{\Delta\omega}{\Delta t}$

$K_{lin} = P^2/2m$

$K_{rot} = L^2/2I$

Energy

Work-Energy Theorem

$$\Sigma K_i + \Sigma W = \Sigma K_f, K_{ein} = \frac{1}{2}mv2$$

$$\vec{W} = \vec{F} \bullet \Delta\vec{r}$$

$$= |\vec{F}||\Delta\vec{r}|\cos(\theta) = F_{II}|\Delta\vec{r}|$$

Plots & Work

$$W = \underbrace{F_x\Delta x}_{} + F_y\Delta y + F_z\Delta z$$

Area under F_x as a function of Position Curve

Conservative vs. Non-Conservative

- Conservative work is independent of path and only depends on location, e.g. gravity and springs. $W = W_c + W_{nc}$
- Create Potential Energy function of position. $W_c = -\Delta U$
- and $\boxed{W = -\Delta U + W_{nc}}$

Conservation of Energy

Gravitational Potential Energy

$$U_{Gravity} = mgh$$

where:

- k=spring constant
- x=displacement of the spring from equilibrium

$$\Sigma E_i + \Sigma W_{nc} = \Sigma E_f$$

where, $E = K + U$

Elastic Potential Energy

$$U_{spring} = \frac{1}{2}kx^2$$

where:

- k=spring constant
- x=displacement of the spring from equilibrium

Add up all the initial kinetic and potential energy, then add to that the net external non-conservative work, and set that equal to all the final kinetic and potential energy.

Rotational

Rotational Energy

$$K = K_{ein} + K_{rot}$$

$$K_{rot} = \frac{1}{2} I\omega^2$$