Week 2 Challenge Homework Solutions

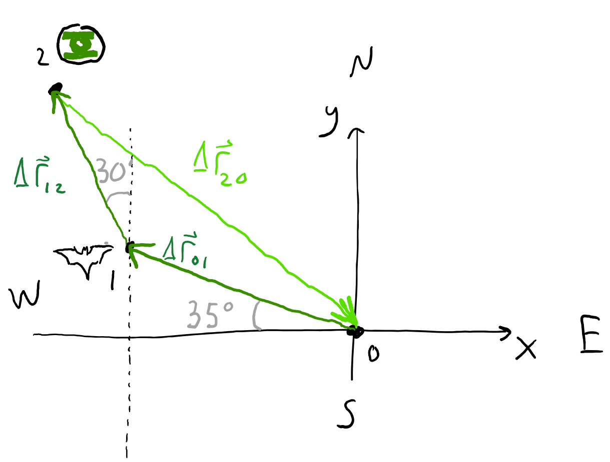
During the middle of a family picnic, Barry Allen received a message that his friends Bruce and Hal needed to be saved. Barry promised his wife Iris that he would be back in exactly 5 minutes. From that that picnic location, Barry runs at a speed of 600 m/s for 2 minutes at a heading of 35° north of west to save Bruce. He then changed his heading to 30° west of north, slows down to 400 m/s and runs for 1 minute to save Hal. (The changes in speed are essentially instantaneous and not part of solving this problem).

(a) (a) Draw a physical representation of the displacement during Barry's full trip.

(b) (b) Use the *Related Quantities* sense-making technique to compare Barry's total distance traveled to the magnitude of his displacement.

(c) (c) What average velocity (magnitude and direction) does Barry need to return back to the picnic in order to keep his promise to Iris?

(a) PHYSICAL REPRESENTATION



* NOTE THAT DISPLACEMENT VECTORS ADD TO $\vec{0}$ ..... $\Delta\vec{r}_{01} + \Delta\vec{r}_{12} + \Delta\vec{r}_{20} = \vec{0}$

A COMMON MISTAKE IS TO THINK THAT VELOCITY VECTORS ADD TO $\vec{0}$, BUT THIS IS NOT NECISSARILY TRUE.

* TOTAL TIME .... $\Delta t_{01} + \Delta t_{12} + \Delta t_{20} = 5$ MINS (✓2 MIN, ✓1 MIN, ?)

$2 + 1 + \Delta t_{20} = 5$

$\Delta t_{20} = 2$ MINS OR 120 SEC.

(c) $\Delta\vec{r}_{01} + \Delta\vec{r}_{12} + \Delta\vec{r}_{20} = \vec{0}$

BUT... $\bar{\vec{v}} = \frac{\Delta\vec{r}}{\Delta t}$

AND SINCE ALL VELOCITIES ARE CONSTANT .... $\bar{\vec{v}} = \vec{v}$

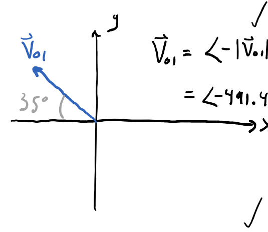
THUS.... $\vec{v} = \frac{\Delta\vec{r}}{\Delta t}$ ......OR...... $\Delta\vec{r} = \vec{v}\Delta t$

$\vec{v}_{01}\Delta t_{01} + \vec{v}_{12}\Delta t_{12} + \vec{v}_{20}\Delta t_{20} = \vec{0}$

Instructor Guide:

1. Encourage students to work in terms of proper notation, delta r's.
2. Remind students that north, south, east, west are not a coordinate; must define positive and negative directions.
3. Recall that displacement is related to average velocity via time.

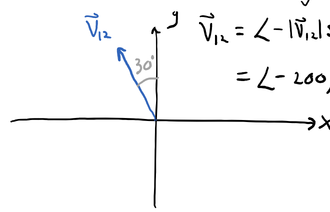
$$\vec{V}_{01}\Delta t_{01} + \vec{V}_{12}\Delta t_{12} + \vec{V}_{20}\Delta t_{20} = \vec{0}$$



$\vec{V}_{01} = \langle -|\vec{V}_{01}|\cos 35^\circ, |\vec{V}_{01}|\sin 35^\circ \rangle$ (600 m/s)

$= \langle -491.4912, 344.1459 \rangle$ m/s

$$120\langle -491.4912, 344.1459\rangle + 60\langle -200, 346.4102\rangle + 120\langle V_{02x}, V_{02y}\rangle = \langle 0,0\rangle$$



$\vec{V}_{12} = \langle -|\vec{V}_{12}|\sin 30, |\vec{V}_{12}|\cos 30 \rangle$ (400 m/s)

$= \langle -200, 346.4102 \rangle$ m/s

x |

$-58978.944 - 12000 + 120V_{02x} = 0$

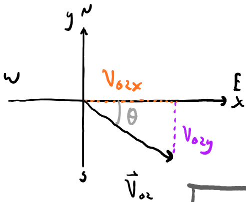
$V_{02x} = 591$ m/s

y |

$41297.508 + 20784.612 + 120V_{02y} = 0$

$V_{02y} = -517$ m/s

$\vec{V}_{02} = \langle 591, -517 \rangle$ m/s



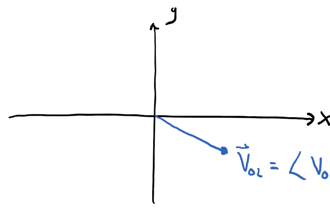
$|\vec{V}_{02}| = \sqrt{591^2 + 517^2}$

$|\vec{V}_{02}| = 785$ m/s

$\tan\theta = \dfrac{V_{02y}}{V_{02x}}$

$\theta = \tan^{-1}\left(\dfrac{517}{591}\right) \approx 41.2^\circ$

$\vec{V}_{02} = 785$ m/s @ 41.2° S of E



$\vec{V}_{02} = \langle V_{02x}, V_{02y} \rangle$ (? ?)

(b)

TOTAL DIST TRAVELLED $= |\Delta\vec{r}_{01}| + |\Delta\vec{r}_{12}| + |\Delta\vec{r}_{23}|$

$= (600\text{ m/s})(120\text{ s}) + (400\text{ m/s})(60\text{ s}) + (785\text{ m/s})(120\text{ s})$

$= 72000\text{ m} + 24000\text{ m} + 94200\text{ m}$

TOTAL DIST TRAVELLED ≈ 190000 m

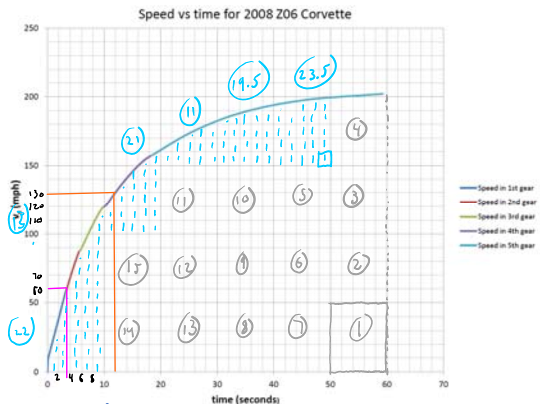
SO TOTAL DIST. TRAVELLED $> |\Delta\vec{r}_{03}| = 0$ m

THIS MAKES SENSE B/C BARRY RAN AROUND BUT RETURNED TO HIS STARTING LOCATION

Question 2

The graph shows the speed as a function of time for a corvette as it accelerates from rest to its top speed of about 202 mph on a straight horizontal road.

(a) (a) About how many seconds does it take this corvette to go from 0 mph to 60 mph.
(b) (b) About what percentage of the total time does the corvette spend in each gear?
(c) (c) Estimate the average acceleration in 1st gear.
(d) (d) What is the speed of the corvette at 11 seconds?
(e) (e) Estimate the total distance the corvette travels from its starting location to the location when it reaches the top speed (end of curve).
(f) (f) Use the *Order of Magnitude* sense-making technique to check your answer to part (e).



Instructor Guide:

1. Encourage students to use the boxes given to estimate the area. It's tedious but should not take more than 3 minutes to count all of the boxes.
2. Suggest that students constantly look at the axis and what quantity they are trying to find. For example, part a is just read off the graph, but often students will try to do a slope or area for this because we've been doing slopes and areas a lot because they are the "new" "more-challenging" operations compared to reading a value off the graph.
3. When using order of magnitude, suggest to convert total distance to miles if meters is unfamiliar.

a) 0 → 60 MPH IN ABOUT 3.5 SECONDS

b) $t_{total} \approx 60$ SEC

$t_{1st} \approx 3.5$ SEC

$t_{2nd} \approx (5.5 - 3.5) \approx 2$ SEC

$t_{3rd} \approx (10 - 5.5) \approx 4.5$ SEC

$t_{4th} \approx (19.5 - 10) \approx 9.5$ SEC

$t_{5th} \approx (60 - 19.5) \approx 40.5$ SEC

$\% t_{1st} = \frac{3.5}{60} \times 100 \approx 5.83\%$

$\% t_{2nd} = \frac{2}{60} \times 100 \approx 3.3\%$

$\% t_{3rd} = \frac{4.5}{60} \times 100 \approx 7.5\%$

$\% t_{4th} = \frac{9.5}{60} \times 100 \approx 15.8\%$

$\% t_{5th} = \frac{40.5}{60} \approx 67.5\%$

c) $\bar{a}_{x,1st} = \frac{\Delta V_x}{\Delta t} = \frac{26.82 \text{ m/s}}{(3.5 \text{ s})}$

SI: $60 \frac{mi}{hr} \times \frac{1 hr}{60 min} \times \frac{1 min}{60 sec} \times \frac{5280 ft}{1 mi} \times \frac{12 in}{1 ft} \times \frac{2.54 cm}{1 in} \times \frac{1 m}{100 cm} = 26.82 \text{ m/s}$

$\bar{a}_{x,1st} = 7.66 \text{ m/s}^2 \approx 0.78 \text{ g's}$

d) @ t = 11 SEC ... $V_x \approx 130$ mph

e) 1 Box = (2 sec)(10 mph) = (2 sec)(4.704 m/s) = 8.9408 m

1 Box = (10 sec)(50 mph) = (10 sec)(22.352 m/s) = 223.52 m

AREA = $\Delta X_{0,60} \approx$ 15 Boxes × $\frac{223.52 m}{1 Box}$ + 110 Boxes × $\frac{8.9408 m}{1 Box}$

≈ 4336.3 m

$\Delta X_{0,60} \approx 4340$ m or ≈ 2.69 MILES

f) TOOK CAR ~ 3 MI TO GO FROM REST TO TOP SPEED; SO ~10^0 MI. THIS SEEMS REASONABLE, I'D EXPECT IT WOULD TAKE LESS THAN 10^1 MILES FOR A CAR TO REACH ITS TOP SPEED :)