**Week 3 Challenge Homework Solutions**

Two bacteria are next to each other moving in the same direction. One moves with an initial velocity of 20 μm/s, accelerating at a rate of 5 μm/s². The other starts with a velocity of 60 μm/s and is decelerating at a rate of 2 μm/s². 1 μm= $10^{-6}$ m.

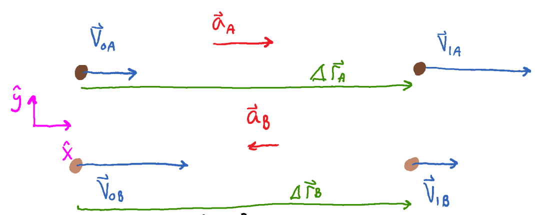
(a) (a) Find the time and position at which the bacteria meet again for an epic battle.

(b) (b) Use the *Related Quantities* sense-making technique to compare the position found in part (a) for both bacteria. How do you expect these positions to compare?

(c) (c) Sketch a plot of the position as a function of time for the two bacteria. Any important feature of the motion should be scaled to the time at which it happened.

**Instructor Guide:**

1. Drawing a physical representation is important here. This might help connect that the time and displacements are the same.
2. Encourage students to make a known and unknown table for both objects. It will help point out that both objects are exactly the same known and unknowns, thus in the mathematical representation the same equation will be used for both objects.




CONNECTIONS

* $\Delta t_A = \Delta t_B \equiv \Delta t$ — 1 EQN 2 UNKNS

* $\Delta X_A = \Delta X_B \equiv \Delta X$ — 1 EQN 2 UNKNS

OBJECT A SNAPSHOTS i 0 AND f 1

| K | UK |
|---|---|
| $V_{0xA} = 20\,\mu m/s$ | $\Delta X_A \equiv \Delta X$ |
| | $V_{1xA}$ |
| $a_{xA} = 5\,\mu m/s^2$ | $\Delta t_A \equiv \Delta t$ |

OBJECT B USE SNAPSHOTS i 0 AND f 1

| K | UK |
|---|---|
| $V_{0Bx} = 60\,\mu m/s$ | $\Delta X_B \equiv \Delta X$ |
| | $V_{1Bx}$ |
| $a_{Bx} = -2\,\mu m/s^2$ | $\Delta t_B \equiv \Delta t$ |

* $\Delta X_A = V_{0Ax}\Delta t_A + \frac{1}{2}a_{Ax}\Delta t_A^2$ — 1 EQN 2 UNKNS

* $\Delta X_B = V_{0B}\Delta t_B + \frac{1}{2}a_{Bx}\Delta t_B^2$ — 1 EQN 2 UNKNS

$$\Delta X = V_{0Ax}\Delta t + \frac{1}{2}a_{Ax}\Delta t^2 \qquad \Delta X = V_{0Bx}\Delta t + \frac{1}{2}a_{Bx}\Delta t^2$$

PHYSICS

MATH

$$V_{0Ax}\Delta t + \frac{1}{2}a_{Ax}\Delta t^2 = V_{0Bx}\Delta t + \frac{1}{2}a_{Bx}\Delta t^2$$

$$V_{0Ax} + \frac{1}{2}a_{Ax}\Delta t = V_{0Bx} + \frac{1}{2}a_{Bx}\Delta t$$

$$\frac{1}{2}a_{Ax}\Delta t - \frac{1}{2}a_{Bx}\Delta t = V_{0Bx} - V_{0Ax}$$

$$\frac{1}{2}\Delta t\,(a_{Ax} - a_{Bx}) = V_{0Bx} - V_{0Ax}$$

a) $$\Delta t = \frac{2(V_{0Bx} - V_{0Ax})}{(a_{Ax} - a_{Bx})} = \frac{2(60-20)}{(5-(-2))} = \frac{80}{7} = \boxed{11.4\,\mu s}$$

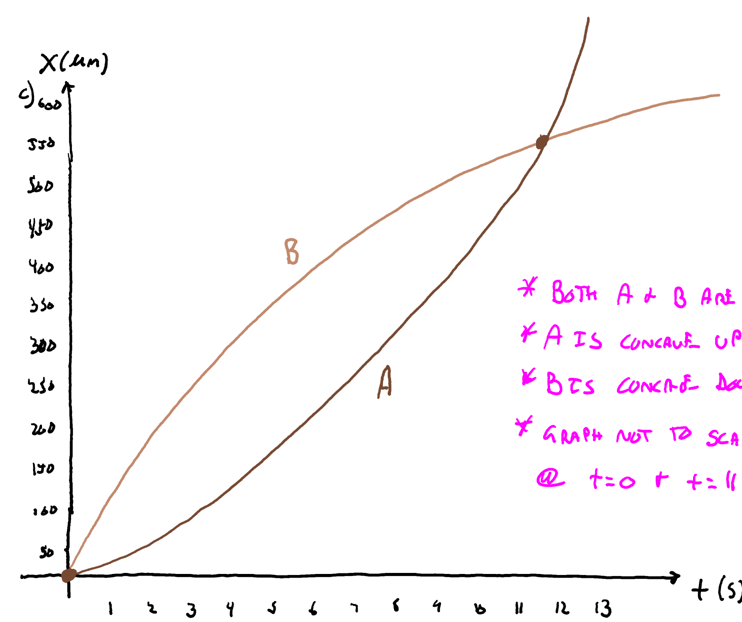
$$\Delta X = V_{0Ax}\Delta t + \frac{1}{2}a_{Ax}\Delta t^2 \xrightarrow{OR} \Delta X = V_{0Bx}\Delta t + \frac{1}{2}a_{Bx}\Delta t^2$$

$$\Delta X = (20)\left(\tfrac{80}{7}\right) + \tfrac{1}{2}(5)\left(\tfrac{80}{7}\right)^2 \qquad \Delta X = (60)\left(\tfrac{80}{7}\right) + \tfrac{1}{2}(-2)\left(\tfrac{80}{7}\right)^2$$

$$\Delta X = 555\,\mu m \longleftrightarrow \boxed{\Delta X = 555\,\mu m}$$

b) A IS MOVING SLOWER AND SPEEDING UP, WHILE B IS MOVING FASTER INITIALLY BUT SLOWING DOWN. SO IT MAKES SENSE THAT B WILL PASS A INITIALLY BUT A WILL CATCH BACK UP SO THEIR ΔX WOULD BE THE SAME.

c)

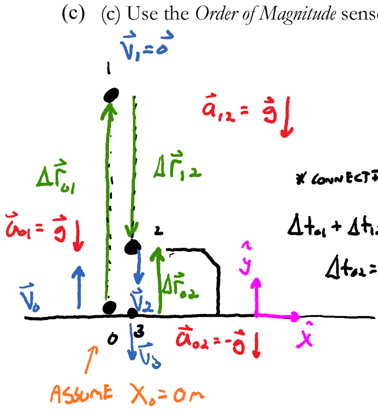



* BOTH A & B ARE QUADRATIC
* A IS CONCAVE UP
* B IS CONCAVE DOWN
* GRAPH NOT TO SCALE OTHER THAN @ t=0 & t=11.5 s

**Question 2**

A goalie kicks a soccer ball straight vertically into the air. It takes 5.00 s for the ball to reach its maximum height *and* come back down to the level of the crossbar. Assume the crossbar of a soccer goal is 2.44 m above the ground.

(a) (a) How fast was the ball originally moving when it was kicked?
(b) (b) How much longer would it take the ball to reach the ground?
(c) (c) Use the *Order of Magnitude* sense-making technique to verify your answer to part (a) and (b).




* CONNECTIONS

$\Delta t_{01} + \Delta t_{12} = 5\,s$

$\Delta t_{02} = 5\,s$

STAGE 0 → 1

| y: k | uk |
|---|---|
| $V_{1y} = 0$ | $\Delta y_{01}$ |
| $a_{01y} = -g$ | $V_{0y}$ |
| | $\Delta t_{01}$ |

STAGE 1 → 2

| y: k | uk |
|---|---|
| $V_{1y} = 0$ | $\Delta y_{12}$ |
| $a_{12y} = -g$ | $V_{2y}$ |
| | $\Delta t_{12}$ |

(a)

STAGE 0 → 2

| y: k | uk |
|---|---|
| $\Delta y_{02} = 2.44\,m$ | $V_{0y}$ |
| $a_{02y} = -g$ | $V_{2y}$ |
| $\Delta t_{02} = 5\,s$ | |

STAGE 0 → 2

$$\Delta X = V_{ix}\Delta t + \tfrac{1}{2}a_x \Delta t^2$$

$$\Delta y_{02} = V_{0y}\Delta t_{02} + \tfrac{1}{2}a_{02y}\Delta t_{02}^2$$

$$2.44\,m = V_{0y}(5\,s) + \tfrac{1}{2}(-9.8\,m/s^2)(5\,s)^2$$

$$2.44 = 5V_{0y} - 122.5$$

$$124.94 = 5V_{0y}$$

$$V_{0y} = 24.988\,m/s$$

$$\boxed{|\vec{V}_0| = 25.0\,m/s}$$

(b)

STAGE 0 → 3

| y: k | uk |
|---|---|
| $\Delta y_{03} = 0\,m$ | |
| $V_{0y} = 24.988\,m/s$ | $V_{3y}$ |
| $a_{03y} = -g$ | $\Delta t_{03}$ |

$$\Delta X = V_{ix}\Delta t + \tfrac{1}{2}a_x\Delta t^2$$

$$0 = 24.989\,\Delta t_{03} + \tfrac{1}{2}(9.8)\Delta t_{03}^2$$

$$0 = 24.988 - 4.9\,\Delta t_{03}$$

$$\Delta t_{03} = 5.09959\ \text{SEC}$$

$$\Delta t_{23} = \Delta t_{03} - \Delta t_{02}$$

$$= 5.009959 - 5$$

$$= 0.09939\ \text{SEC}$$

$$\boxed{\Delta t_{23} = 0.0996\ \text{SEC}}$$

c)

$|\vec{V}_0| \simeq 2.5 \times 10^1\,m/s$ OR $\simeq 5.6 \times 10^1\,mi/hr$

HUMANS CAN PROBABLY KICK SOCCER BALLS ON THE ORDER OF $10^1$ mi/hr :)

$\Delta t_{23} = 0.10\,s \approx 1 \times 10^{-1}\,s$

EVEN FROM REST, A BALL DROPPED ~ 2 m ABOVE THE GROUND TAKES ~ $10^{-1}$ s TO REACH GROUND :)

**Instructor Guide:**

1. Drawing a physical representation is a very important first step.
2. Many students will forget that since the acceleration is constant throughout the entire motion, you can use snapshots 0 and 2 as initial and final. I included the knowns and unknowns for the intermediate stages, and as you can see there is not enough known quantities to solve for anything. Don't let student's get too caught up trying to use two stages.