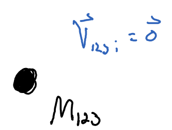
Week 8 Challenge Homework Solutions

An evil doer has stolen the unstable atomic nuclear mass (17.0 x 10^{-27} kg) from the unstable nuclear mass facility. While waiting at a stop light the unthinkable happens (actually quite expected and predictable up to a probability) and the mass disintegrates into three particles. One of the particles, of mass 5.00 x 10^{-27} kg, moves in the y-direction with a speed of 6.00 x 10^{6} m/s. Another particle, of mass 8.40 x 10^{-27} kg, moves in the x-direction with a speed of 4.00 x 10^{6} m/s.

(a) (a) Find the magnitude and direction of the velocity of the third particle.

(b) (b) If the evil doer had been driving with a speed of 30 m/s when the disintegration occurred, how would this have changed your answer to part (a)? Use an *Order of Magnitude* sense-making argument to help with this analysis.

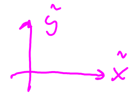
I



$\sum \vec{F}_{Ext}\Delta t \approx \vec{0}$

... so..

$\sum \vec{P}_i = \sum \vec{P}_f$



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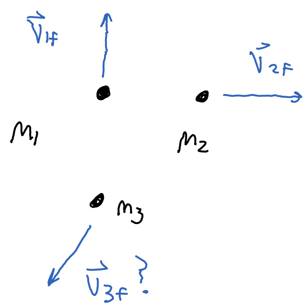


$M_{123} = m_1 + m_2 + m_3$

$\cancel{\sum P_{ix}}^{0} = \sum P_{fx}$

$0 = m_1 \cancel{V_{1fx}}^{0} + m_2 V_{2fx} + m_3 V_{3fx}$

$0 = m_2 V_{2fx} + m_3 V_{3fx}$

$\cancel{\sum P_{iy}}^{0} = \sum P_{fy}$

$0 = m_1 V_{1fy} + m_2 \cancel{V_{2fy}}^{0} + m_3 V_{3fy}$

$0 = m_1 V_{1fy} + m_3 V_{3fy}$

3 Eqns 3 Unkns

$M_{123} = m_1 + m_2 + m_3$ $\qquad 0 = m_2 V_{2fx} + m_3 V_{3fx}$ $\qquad 0 = m_1 V_{1fy} + m_3 V_{3fy}$

$m_3 = M_{123} - m_1 - m_2$

$= (17 \times 10^{-27}\text{ kg}) - (5 \times 10^{-27}\text{ kg}) - (8.4 \times 10^{-27}\text{ kg})$

$m_3 = 3.6 \times 10^{-27}\text{ kg}$

$0 = (8.4\times10^{-27})(4\times10^{6}) + (3.6\times10^{-27})V_{3fx}$

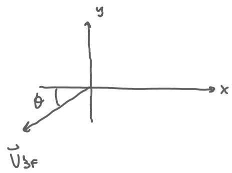
$0 = (5\times10^{-27})(6\times10^{6}) + (3.6\times10^{-27})V_{3fy}$

$V_{3fx} \approx -9.33\times10^{6}\ m/s$

$V_{3fy} = -8.33\times10^{6}\ m/s$

$|\vec{V}_{3f}| = \sqrt{V_{3fx}^2 + V_{3fy}^2}$

$|\vec{V}_{3f}| \approx 1.25\times10^{7}\ m/s$



$\tan\theta = \frac{V_{3fy}}{V_{3fx}}$

$\theta = 41.8°$

(a) $\boxed{\vec{V}_{3f} = 1.25\times10^{7}\ m/s \ @ \ \theta = 41.8°}$

(b)

IF $|\vec{V}_{123i}| = 30\ m/s$

$\Sigma P_{ix} = (1.7\times10^{-26}\ kg)(3\times10^{1}\ m/s)$

$\Sigma P_{ix} \approx 10^{-25}\ Ns$

AND

$\Sigma P_{fx} \approx 10^{-27}\ kg\ 10^{6}\ m/s$

$\Sigma P_{fx} \approx 10^{-21}\ Ns$

4 ORDERS OF MAGNITUDE DIFFERENT

SO $|\vec{V}_{3f}|$ WOULD BASICALLY BE THE SAME ....

... DIFFERENT @ ≈ 3-4 DECIMAL PLACE

Question 2

In the school cafeteria, a trouble-making child blows a 12.0 g spitball through a 25.0 cm straw. The force (**F**) in Newtons, of his breath as a function of the distance along the straw (**x**) in meters, can be modeled as a linearly decreasing function for the first half of the straw then a constant force through the rest of the straw. The force decreases by half along the first half of the straw. Assume there is negligible friction and the straw is held horizontally.

(a) (a) Sketch a plot of the force of his breath as a function of position along the straw, labeling the force at x = 0 as $\mathbf{F_0}$.

(b) (b) If the spitball begins from rest and leaves the straw with a speed of 16 m/s, how much work is done on the spitball?

(c) (c) What is the maximum force $\mathbf{F_0}$, that acts on the spitball?

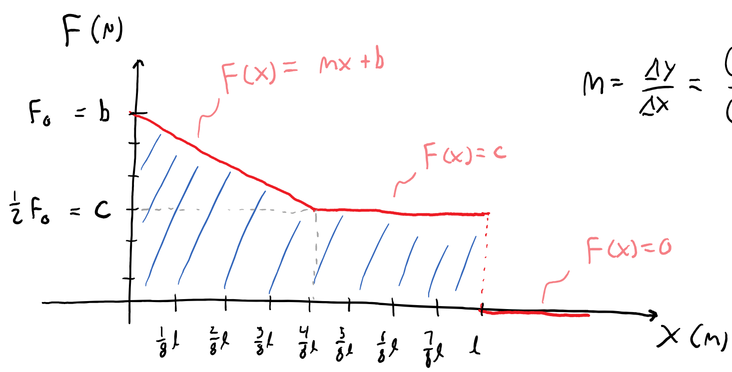
(d) (d) Use Proportionality sense-making to analyze your answer to part (c). Would you expect the maxi-mum force to increase or decrease if the same amount of work is done on the spitball while the length of the straw decreased? Does your expression for the maximum force portray your expectations?

a) GENERIC PIECEWISE FUNCTION

$$F(x) = \begin{cases} mx + b & x \le \frac{1}{2}\ell \\ c & \frac{1}{2}\ell < x \le \ell \\ 0 & x > \ell \end{cases}$$

KNOWNS

$\ell = 25$ cm



$$m = \frac{\Delta y}{\Delta x} = \frac{(\frac{1}{2}F_0 - F_0)}{(\frac{1}{2}\ell - 0)} = \frac{-\frac{1}{2}F_0}{\frac{1}{2}\ell} = -\frac{F_0}{\ell}$$

UPDATED PIECEWISE FUNCTION

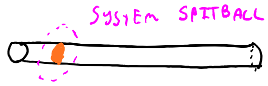
$$F(x) = \begin{cases} -\frac{F_0}{\ell}x + F_0 & x \le \frac{1}{2}\ell \\ \frac{1}{2}F_0 & \frac{1}{2}\ell < x \le \ell \\ 0 & x > \ell \end{cases}$$

OR,

$$F(x) = \begin{cases} -4F_0 x + F_0 & x \le 0.125\ \text{m} \\ \frac{1}{2}F_0 & 0.125\ \text{m} < x \le 0.25\ \text{m} \\ 0 & 0.25\ \text{m} > x \end{cases}$$

b)

SYSTEM SPITBALL (m_1)



Knowns

$V_{1i} = 0$

$V_{1f} = 16\ m/s$

$m_1 = 0.012\ kg$

$$\cancel{KE_{1i}}^{\,0} + W_{Ext} = KE_{1f}$$

$$W^{F(x)} = \frac{1}{2} m_1 V_{1f}^2$$

$$W^{F(x)} = \frac{1}{2}(0.012)(16)^2$$

$$\boxed{W^{F(x)} = 1.536\ J}$$

c)

$W^{F(x)}$ = AREA UNDER $F(x)$ VS x CURVE

$$W^{F(x)} = 1.536$$

AREA = 1.536

$\square + \triangle = 1.536$

$$\ell\left(\tfrac{1}{2}F_0\right) + \tfrac{1}{2}\left(\tfrac{1}{2}\ell\right)\left(\tfrac{1}{2}F_0\right) = 1.536$$

$$\tfrac{1}{2}\ell F_0 + \tfrac{1}{8}\ell F_0 = 1.536$$

$$\tfrac{5}{8}\ell F_0 = 1.536$$

$$\boxed{F_0 = 9.83\ N}$$

d)

$$\frac{5}{8}\ell F_0 = W^{F(x)}$$

$$F_0 = \frac{8}{5}\,\frac{W^{F(x)}}{\ell}$$

w/ $W^{F(x)}$ = CONST

$$F_0 \propto \frac{1}{\ell}$$

IF $\ell \uparrow$ THEN $F_0 \downarrow$ OR IF $\ell \downarrow$ THEN $F_0 \uparrow$

MAKES SENSE... TO GET SAME WORK (i.e. SAME AREA) w/ SMALLER LENGTH THE $\bar{F}_x$ WOULD HAVE TO BE LARGER :)