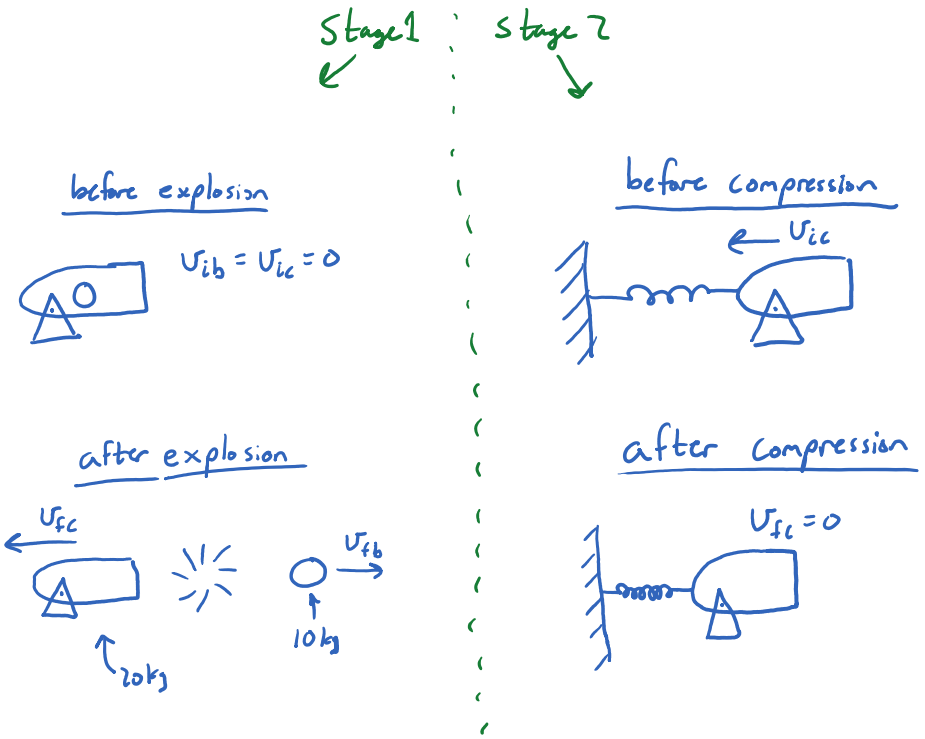
Week 9 Challenge Homework Solutions

Old naval ships fired 10 kg cannon balls from a 200 kg cannon. It was very important to stop the recoil of the cannon, since otherwise the heavy cannon would go careening across the deck of the ship. In one design, a large spring with spring constant 20,000 N/m was placed behind the cannon. The other end of the spring braced against a post that was firmly anchored to the ship's frame. When the cannon fired, the spring compressed 50 cm.

(a) (a) How fast could this cannon fire cannon balls?

(b) (b) Use *Dimensionality* sense-making to check the expression you derive in part (a).



Stage 1

explosion ⇒ conservation of momentum

(energy is conserved too, but I don't know how to quantify thermal energy in the system)

after

$$p_{tot,xf} = m_c v_{cf} + m_b v_{bf}$$

before

$$m v_{ci} = m v_{bi} = 0$$

$$p_{tot,xi} = 0 + 0 = 0$$

because $\sum \vec{J}_{ext} = 0 \Rightarrow p_{tot,xi} = p_{tot,xf}$

$$\Rightarrow m_c v_{cf} + m_b v_{bf} = 0$$

$$\Rightarrow m_c v_{cf} = -m_b v_{bf}$$

$$\Rightarrow v_{bf} = -\frac{m_c}{m_b} v_{cf}$$

final answer — known — must find from stage 2

(b)

stage 2

$$v_{ic} = \sqrt{\frac{k}{m}}\, x$$

$$= \sqrt{\frac{\text{force/dist}}{\text{mass}}}\ (\text{dist})$$

$$= \sqrt{\frac{\text{mass}\ \frac{1}{\text{time}^2}}{\text{mass}}}\ (\text{dist})$$

$$= \sqrt{\frac{1}{\text{time}^2}}\ (\text{dist})$$

$$= \frac{\text{dist}}{\text{time}} \Rightarrow \text{velocity} \checkmark$$

stage 1

$$v_{bf} = -\frac{m_c}{m_b} v_{cf}$$

$$= \frac{\text{mass}}{\text{mass}}\ \text{veloc.}$$

$$= \text{velocity} \checkmark$$

Stage 2

Conservation of energy

$$W_{ext} = 0 \Rightarrow \Delta E = 0$$

$$\Rightarrow E_i = E_f$$

$$\Rightarrow KE_i + \cancel{U_i^s}^{\,0} = \cancel{KE_f}^{\,0} + U_f^s$$

$$\Rightarrow \frac{1}{2} m_c v_{ci}^2 = \frac{1}{2} k x^2$$

(x: distance compressed from equilibrium)

$$\Rightarrow v_{ci}^2 = \frac{k}{m_c} x^2$$

$$\Rightarrow v_{ci} = \sqrt{\frac{k}{m_c}}\, x = \sqrt{\frac{20000\ \text{N/m}}{200\ \text{kg}}}\ (0.5\ \text{m})$$

$$= 5\ \text{m/s}$$

Combining stage 1 & stage 2

v_{cf} from stage 1 = v_{ci} from stage 2

$$\Rightarrow v_{bf} = -\frac{m_c}{m_b}\ (5\ \text{m/s})$$

$$= -\frac{200\ \text{kg}}{10\ \text{kg}}\ (5\ \text{m/s})$$

$$|v_{bf}| = +100\ \text{m/s}$$

(asked for "how fast" ⇒ speed)