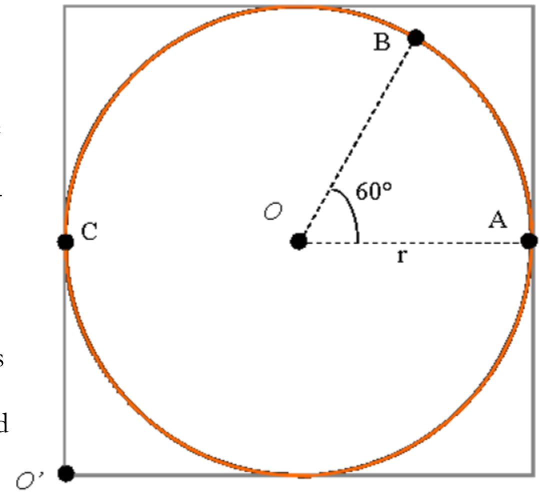
Example Challenge Homework Solution



This is a question which in the past has been a challenge homework we assigned. This time it will be an example of the type of solution we are looking for on a challenge homework. You may also consider solving this problem yourself for practice!

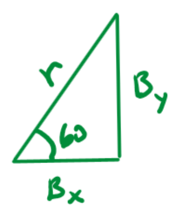
Use the diagram to answer the following questions. Assume a standard coordinate system.

(a) Find the position vector, as a function of the radius (variable **r**), for points **A**, **B**, and **C**, using origin ***O***.
(b) Find the change in position vector from **A** to **B** and **B** to **C** using origin ***O***.
(c) Repeat parts (a) and (b) using origin ***O'***.
(d) Use the *Related Quantities* sensemaking technique by answering the following prompts:
- Make a prediction about the relationship between position vectors before and after the origin changes. Similarly, make a prediction about the relationship between change in position vectors before and after the origin changes.
- Explain the above predictions using any combination of words, diagrams, algebra, etc.
- Compare your prediction with the results from part (a), (b), and (c).

(a) $\vec{A}_o = \langle r, 0 \rangle$

$\vec{B}_o = \langle \frac{1}{2} r, \frac{\sqrt{3}}{2} r \rangle$

$\vec{C}_o = \langle -r, 0 \rangle$



$\sin 60° = \frac{B_y}{r}$

$B_y = r \sin(60°)$

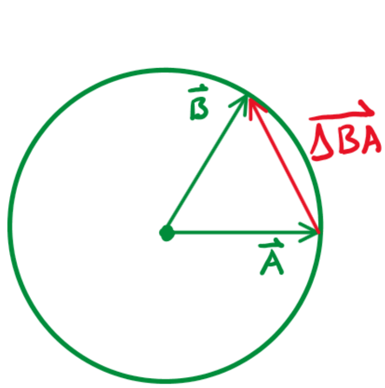
$= \frac{\sqrt{3}}{2} r$

$\cos(60°) = \frac{B_x}{r}$

$B_x = r \cos(60°)$

$= \frac{1}{2} r$

(b)

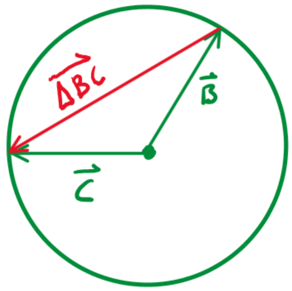


$\Delta \Rightarrow$ final − initial

$\overrightarrow{\Delta(AB)} = \vec{B}_o - \vec{A}_o$

$= \langle \frac{1}{2} r, \frac{\sqrt{3}}{2} r \rangle - \langle r, 0 \rangle$

$= \langle -\frac{1}{2} r, \frac{\sqrt{3}}{2} r \rangle$

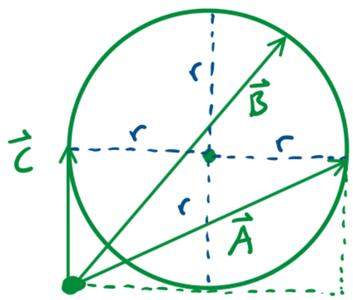


$$\overrightarrow{\Delta BC} = \vec{C}_0 - \vec{B}_0$$

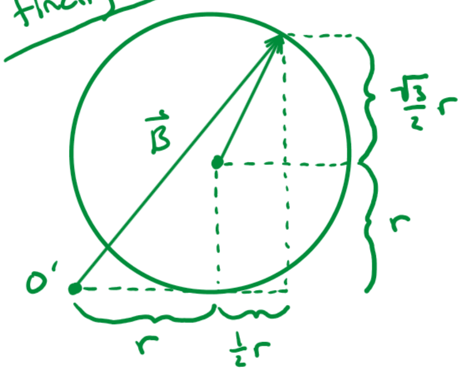
$$= \langle -r, 0 \rangle - \langle \tfrac{1}{2} r, \tfrac{\sqrt{3}}{2} r \rangle$$

$$= \langle -\tfrac{3}{2} r, -\tfrac{\sqrt{3}}{2} r \rangle$$

(c)



finding $\vec{B}$ components



$$\vec{A}_{0'} = \langle 2r, r \rangle$$

$$\vec{B}_{0'} = \langle \tfrac{3}{2} r, (1 + \tfrac{\sqrt{3}}{2}) r \rangle$$

$$\vec{C}_{0'} = \langle 0, r \rangle$$

$$B_x = r + \tfrac{1}{2} r$$

$$B_y = r + \tfrac{\sqrt{3}}{2} r$$

$$\overrightarrow{\Delta AB} = \vec{B}_{0'} - A_{0'} = \langle \tfrac{3}{2} r, r + \tfrac{\sqrt{3}}{2} r \rangle - \langle 2r, r \rangle$$

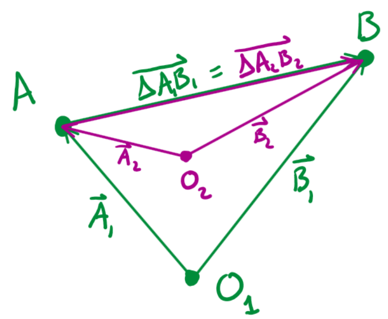
$$= \langle -\tfrac{1}{2} r, \tfrac{\sqrt{3}}{2} r \rangle$$

$$\overrightarrow{\Delta BC} = \vec{C}_{0'} - \vec{B}_{0'} = \langle 0, r \rangle - \langle \tfrac{3}{2} r, r + \tfrac{\sqrt{3}}{2} r \rangle$$

$$= \langle -\tfrac{3}{2} r, -\tfrac{\sqrt{3}}{2} r \rangle$$

(d) I predict that position vectors will change when the origin changes, but change in position vectors (displacement vectors) will not.

explanation



Points A & B are depicted with two origins O_1 & O_2. The position vectors $\vec{A}_1$ & $\vec{A}_2$ differ b/c their origins are different. However, $\overrightarrow{\Delta AB}$ is the same for both origins.

Comparison

I see in parts (a) and (c) that the position vectors are:

$$A_o = \langle r, 0 \rangle \neq A_{o'} = \langle 2r, r \rangle$$

$$B_o = \langle \tfrac{1}{2}r, \tfrac{\sqrt{3}}{2}r \rangle \neq B_{o'} = \langle \tfrac{3}{2}r, r + \tfrac{\sqrt{3}}{2}r \rangle$$

$$C_o = \langle -r, 0 \rangle \neq C_{o'} = \langle 0, r \rangle$$

But, I see from (b) and (c) that the change in position vector $\overrightarrow{\Delta AB}$ is the same for both origins O and O'. This is also true for $\overrightarrow{\Delta BC}$. This matches my prediction, therefore my answer is reasonable!