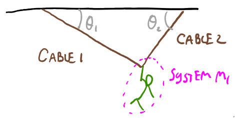
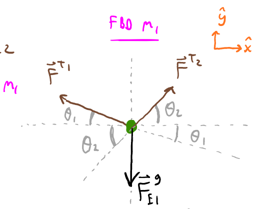
Week 5 Challenge homework Solutions

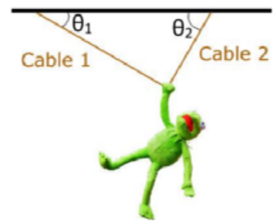
A chandelier with mass m is attached to the ceiling of a large concert hall by two cables. Cable 1 has tension T_1 and makes an angle of θ_1 with the ceiling. Cable 2 has tension T_2 and makes an angle of θ_2 with the ceiling.

(a) (a) Find an expression for T_1, the tension in cable 1, that does not depend on T_2. Express your answer in terms of some, or all of the variables m, θ_1, θ_2, and the magnitude of the acceleration due to gravity g.

(b) (b) Use the *Special Cases* sense-making technique to check your expression in part (a). Specifically how does the tension change as m, θ_1, θ_2, and g change to extreme values?







Instructor Guide:

1. This is a tricky problem mathematically. Encourage student to focus on the set-up of the FBD and x and y analysis of Newton's 2nd law.
2. To help with the mathematical stage, perhaps create to equations with simple variables like -a*b + c*d = 0 and a*e + c*f - g = 0 where b, d, e, f, and g are all "known" quantities. Then have them solve for a and b. If this is too much then try putting in random numbers for b,d,e,f, and g.

$$\sum F_x = m_1 a_x^{\;0}$$

$$-|\vec F^{T_1}|\cos\theta_1 + |\vec F^{T_2}|\cos\theta_2 = 0$$

$$\sum F_y = m_1 a_y^{\;0}$$

$$|\vec F^{T_1}|\sin\theta_1 + |\vec F^{T_2}|\sin\theta_2 - |\vec F^g_{E1}| = 0$$

$$|\vec F^{T_1}|\sin\theta_1 + |\vec F^{T_2}|\sin\theta_2 - m_1 g = 0$$

PHYSICS ↑

MATH ↓

$$|\vec F^{T_2}|\cos\theta_2 = |\vec F^{T_1}|\cos\theta_1$$

$$|\vec F^{T_2}| = \frac{\cos\theta_1}{\cos\theta_2}|\vec F^{T_1}|$$

$$|\vec F^{T_1}|\sin\theta_1 + |\vec F^{T_1}|\frac{\cos\theta_1\sin\theta_2}{\cos\theta_2} - mg = 0$$

$$|\vec F^{T_1}|\left(\sin\theta_1 + \frac{\cos\theta_1\sin\theta_2}{\cos\theta_2}\right) = mg$$

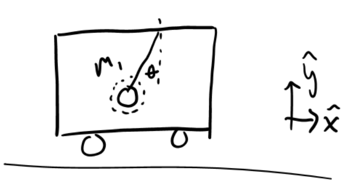
$$|\vec F^{T_1}| = \frac{mg}{\left(\sin\theta_1 + \frac{\cos\theta_1\sin\theta_2}{\cos\theta_2}\right)}$$

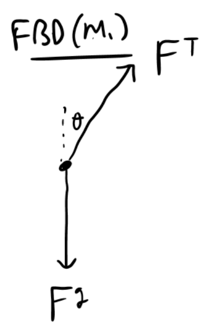
$$|\vec F^{T_1}| = \frac{mg}{\frac{1}{\cos\theta_2}\left(\sin\theta_1\cos\theta_2 + \cos\theta_1\sin\theta_2\right)}$$

$$|\vec F^{T_1}| = \frac{mg\cos\theta_2}{\sin(\theta_1+\theta_2)} \leftrightarrow \frac{mg\cos\theta_2}{\sin\theta_1\cos\theta_2 + \cos\theta_1\sin\theta_2}$$

$$|\vec F^{T_2}| = \frac{mg\cos\theta_1}{\sin(\theta_1+\theta_2)} \updownarrow \frac{mg\cos\theta_1}{\sin\theta_1\cos\theta_2 + \cos\theta_1\sin\theta_2}$$

Question 2





(a) $\sum \vec F$ to right from horizontal tension component. So, $\vec a$ to right

(b) No information about velocity. Could be speed up to right or moving left but slowing down

(c) $\sum F_y = ma_y \Rightarrow F^T\cos\theta - mg = m a_y^{\;0} \Rightarrow F^T = \frac{mg}{\cos\theta}$

$\sum F_x = ma_x \Rightarrow F^T\sin\theta = ma_x \Rightarrow mg\tan\theta = ma_x$, $a_x = g\tan\theta = 1.03\,m/s^2$

(d) look @ $a_x = g\tan\theta$, as $\theta\uparrow$, $\tan\theta\uparrow$, + $a_x\uparrow$. the greater the acceleration the larger θ