PH201 U2022 Final Exam Solutions

Thursday, July 14, 2022 12:02 PM

Name:______________________________ ID:______________________________

Physics 201

Final Exam

7/14/2022

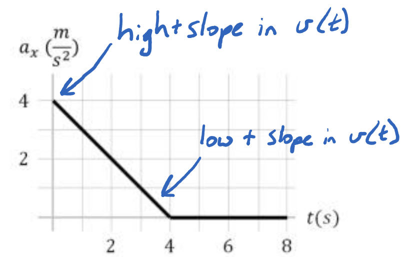
Collaboration is not allowed. Allowed on your desk are: four 8.5 x 11 inch doubled sided sheets of notes, a non-communicating graphing scientific calculator, scratch paper, writing utensils, and the exam. You will have 110 minutes to complete this exam.

For questions 1 and 2, **fill in the square** next to all correct answers. A given problem may have more than one correct answer. Each correctly bubbled answer will receive two points. There are **6** correct answers in this section and only the first **6** filled in answers will be graded. There is no partial credit.

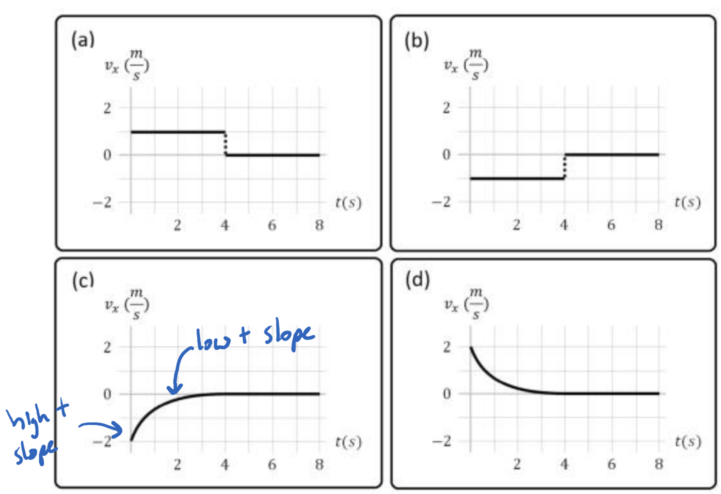
1. Which of the following statements MUST be true?

   ☐ (a) An object in motion has a net force pointed in the direction of motion.
   ☑ (b) Energy is not always conserved in all systems.
   ☐ (c) The weight of an object is equal to the normal force acting on it .
   ☐ (d) Velocity is constant for an object in uniform circular motion.
   ☐ (e) Two identical caterpillars are moving in opposite directions with the same speed. Their total kinetic energy is zero.
   ☑ (f) An object can have zero velocity and a non-zero net force acting on it at the same time.
   ☑ (g) An object can have a non-zero velocity and also not be accelerating.
   ☐ (h) If an object has only the force of gravity acting on it, it must have a downward velocity.
   ☐ (i) The friction force always points opposite the direction of motion.
   ☐ (j) If the potential energy of an object decreases, its kinetic energy will increase.
   ☑ (k) The average acceleration of an object points the same direction as its change in velocity.
   ☐ (l) If an object changes direction, but not speed, its momentum stays the same.
   ☑ (m) If an object changes direction, but not speed, its kinetic energy stays the same.

2. A non-horse object undergoes motion described by the graph to the right. Which of the following graphs could represent the same object during the same motion? (make sure to fill in the square(s) below!)

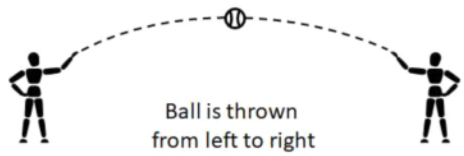


☐ (a) ☐ (b) ☑ (c) ☐ (d)

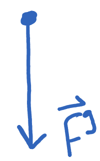


3. (9 points) Carefully draw a free body diagram for each of the following scenarios. Make sure that the forces on the free body diagrams are appropriately scaled relative to each other (the vectors should be appropriate lengths compared to each other).

(a)

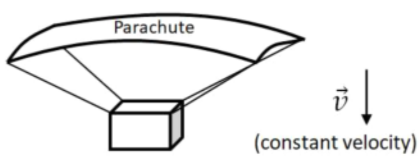


(draw free body diagram for the ball at the moment shown)

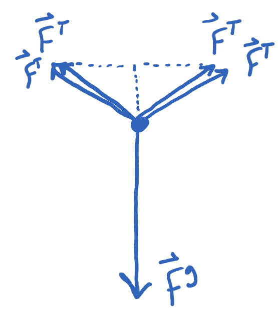


(could also include small drag force to the left)

(b)



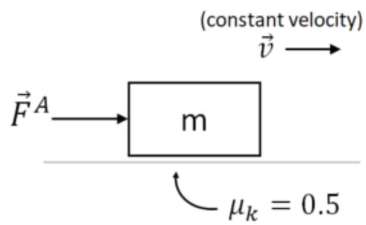
(draw free body diagram for the box)

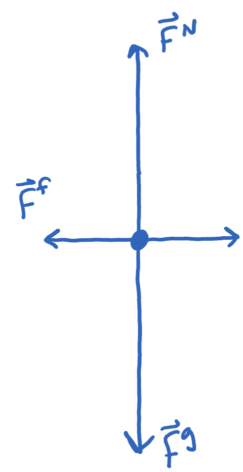


four forces of tension.

Horizontal components cancel, vertical comp. add to equal $-\vec{F}^g$

(c)





$|\vec{F}^f| = \frac{1}{2} |\vec{F}^N|$

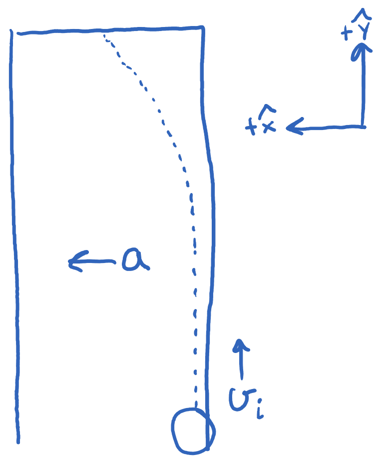
$\uparrow$

$\mu = 0.5$

$|\vec{F}^A| = |\vec{F}^f|$

4. (10 points) While bowling, you throw the bowling ball forward down the right side of the lane. The ball has a constant forward velocity of 8.9 m/s. A bowling lane is 18.3 meters long and 1.05 meters wide. Since you are a professional bowler, you release the ball with spin that causes the ball to accelerate to the left at a constant rate, a. To get a strike, you need your bowling ball to hit the head pin which stands at the end of the lane in the center.

(a) What is the magnitude of acceleration, |a|, needed to hit the head pin? (ignore the width of the pin and ball)



y

$$\Delta y = v_{iy}\Delta t + \tfrac{1}{2}\cancel{a_y}^{\,0}\Delta t^2$$

$$\Delta t = \frac{\Delta y}{v_{iy}}$$

x

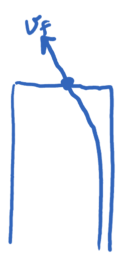
$$\Delta x = \cancel{v_{ix}}^{\,0}\Delta t + \tfrac{1}{2}a_x\Delta t^2$$

only half the lane

$$a_x = \frac{2\Delta x}{\Delta t^2} = \frac{2\Delta x}{\left(\frac{\Delta y}{v_{iy}}\right)^2} = \frac{2\left(\frac{1}{2}\times 1.05\,m\right)}{\left(18.3\,m/8.9\,m/s\right)^2}$$

$$a_x = 0.248\ m/s$$

(b) What is the final speed of the ball?



$$\vec{v}_f = \langle v_{fx}, v_{fy} \rangle\ m/s$$

$$v_{fy} = 8.9\ m/s$$

$$v_{fx} = \cancel{v_{ix}}^{\,0} + a_x\Delta t = \frac{2\Delta x}{\Delta t^2}\Delta t = \frac{2\Delta x}{\Delta t} = \frac{2\Delta x\, v_{iy}}{\Delta y} = 0.51\ m/s$$

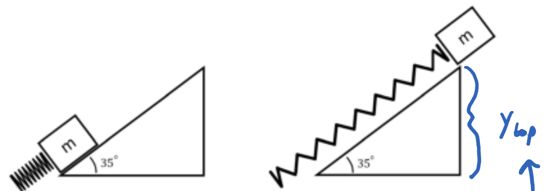
$$|\vec{v}_f| = \sqrt{v_{fx}^2 + v_{fy}^2} = 8.91\ m/s$$

(c) Use the Order of Magnitude and Known Values or Related Quantities sense-making techniques to check the validity of your answer to part (b).

- the initial velocity is ~10 m/s
- the final velocity is also on the same order of magnitude (~10 m/s)
- also, baseballs can be thrown @ ~100 mph ⇒ ~50 m/s which is also on the same order of magnitude as the velocities we found

→ both of these lead me to think my answer is reasonable

5. (12 points) A compressed spring pushes a box of mass 3 kg up a frictionless 35 degree incline of length 5 meters (the hypotenuse is 5 meters). The spring's equilibrium position is when it reaches the top of the incline (second picture). The spring constant is 30 N/m. Explain all answers using any combination of words, diagrams, figures, etc.



$Y_{top} = (5m)\sin 35° = 2.87m$

(a) find the speed of the box at the top of the ramp (second picture above).

$W_{ext} = 0 \Rightarrow E_i = E_f$

$U_i^s = U_{top}^g + KE_{top}$

$\frac{1}{2}kx_i^2 = mgY_{top} + \frac{1}{2}mv_{top}^2$

$\frac{1}{2}mv_{top}^2 = \frac{1}{2}kx_i^2 - mgY_{top}$

$v_{top}^2 = \frac{k}{m}x_i^2 - 2gY_{top} = \frac{30\ N/m}{3\ kg}(5m)^2 - 2(9.8\ m/s^2)(2.87m)$

$|v_{top}| = 13.9\ m/s$

(b) find the maximum height reached by the box during its ensuing flight.

$v_{iy} = |\vec{v}_{top}| \sin 35°$

can use kinematics or conservation of energy!

@ top of ramp

above the ramp

$\frac{1}{2}\cancel{m}v_{iy}^2 = \cancel{m}g\Delta y$

$\Rightarrow \Delta y = 3.25m$

$\Rightarrow$ max height $= 3.25m + 2.87m = 6.12m$

ramp height

alternative method

$v_{iy} = v_{top}\sin 35°$

$v_{fy}^2 = v_{iy}^2 + 2a_y\Delta y$

$\Rightarrow 0 = v_{top}^2(\sin 35°)^2 - 2g\Delta y$

$\Rightarrow \Delta y = \frac{v_{top}^2(\sin 35°)^2}{2g}$

$\Rightarrow \Delta y = 3.25m$

$\Rightarrow$ max height $= 6.12m$

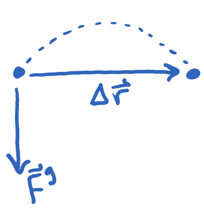
(c) find the speed of the box just before it contacts the ground.

$W^{ext} = 0 \Rightarrow E_i = E_f$

$\Rightarrow \frac{1}{2}kx^2 = \frac{1}{2}mv_f^2$

$\Rightarrow v_f = \sqrt{\frac{kx^2}{m}} = 15.8\ m/s$

(d) how much total work does gravity do on the box from the start of the motion until the box hits the ground?

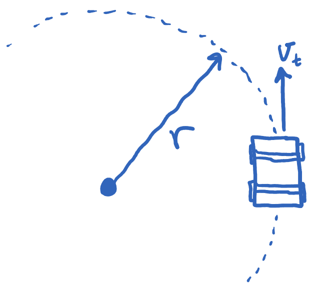


$W^g = \vec{F}^g \cdot \Delta\vec{r} = 0 \Rightarrow$ gravity does zero total work

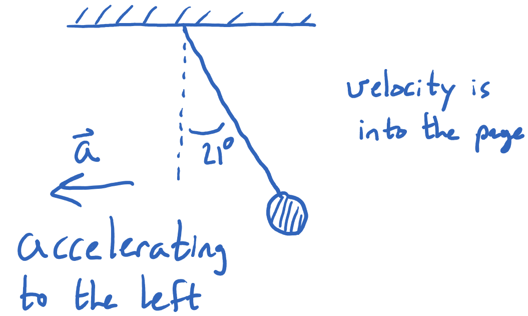
gravity does negative work as the box moves upwards, but does an equal amount of positive work as the box falls.

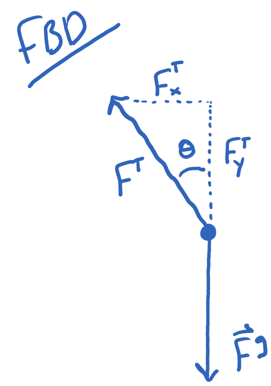
6. (9 points) While riding in a car travelling at a steady 55 miles per hour, you hold a weight on a string in front of you. As the car goes around a turn in the highway, you measure the angle the string makes with the vertical to be 21 degrees. What is the radius of the turn your car is making?

$$\left(55\ \frac{mi}{hr}\right)\left(\frac{5280}{1}\ \frac{ft}{mi}\right)\left(\frac{12}{1}\ \frac{in}{ft}\right)\left(\frac{2.54}{1}\ \frac{cm}{in}\right)\left(\frac{1}{100}\ \frac{m}{cm}\right)\left(\frac{1}{60}\ \frac{hr}{min}\right)\left(\frac{1}{60}\ \frac{min}{s}\right) = 24.59\ m/s$$



$\vec{F}_{net} = \frac{mv^2}{r}$ towards center





y/ $F^T_y - mg = 0$

$F^T_y = mg$

$\Rightarrow F^T_y = |F^T| \cos\theta = mg$

$\Rightarrow |F^T| = \frac{mg}{\cos\theta}$ (1)

x/ $F^T_x = ma_x = m\frac{v_t^2}{r}$

$\underbrace{F^T_x = |F^T| \sin\theta = m\frac{v_t^2}{r}}_{(2)}$

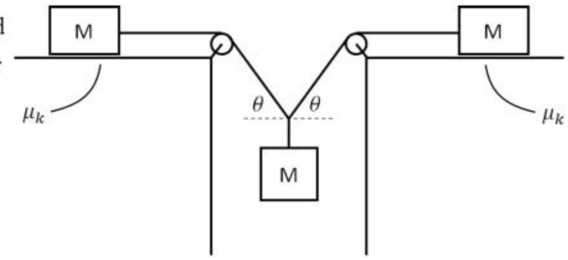
(1) + (2) $\Rightarrow \left(\frac{\cancel{m}g}{\cos\theta}\right)\sin\theta = \cancel{m}\frac{v_t^2}{r}$

$\Rightarrow g\tan\theta = \frac{v_t^2}{r}$

$\Rightarrow r = \frac{v_t^2}{g\tan\theta}$

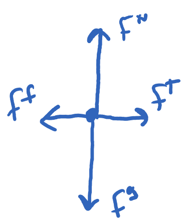
$r = 161m$

7. (10 points) Three boxes of the same mass are arranged as shown using frictionless pulleys and massless string. The coefficient of kinetic friction between the tables and the boxes is $\mu_k = 0.6$ and the pictured angles, θ, are identical.



(a) For an instant, the masses are moving with constant velocity. At this instant, what is the angle of the string, θ? (hint: part b will be slightly easier if you solve part a symbolically, but it's not necessary)

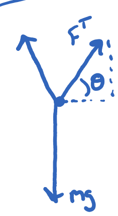
FBD for blocks on table



x: $-\mu m g + F^T = m \cancel{a_x}^{0}$

$\Rightarrow F^T = \mu m g$

FBD for hanging block



y: $-mg + 2F^T \sin\theta = m\cancel{a_y}^{0}$

$\Rightarrow F^T = \frac{mg}{2\sin\theta}$

Set $F^{T\uparrow} = F^{T\rightarrow}$

$\Rightarrow \mu \cancel{m}\cancel{g} = \frac{\cancel{m}\cancel{g}}{2\sin\theta}$

$\sin\theta = \frac{1}{2\mu}$

$\theta = \sin^{-1}\left(\frac{1}{2(0.6)}\right) = \theta = 56.4^\circ$

(b) Examine what happens to your answer to part (a) when the coefficient of friction is changed. Are there minimum or maximum values of μ? What physical significance do these values of μ have? (in other words, what happens to the system at these values of μ?)

We found that $\sin\theta = \frac{1}{2\mu}$

b/c $\sin\theta \leq 1$, this means that μ can't be less than 0.5!

(if μ were to be less than 0.5, something about the situation would change. If $\mu < 0.5$, then we can no longer have a constant velocity at any angle. F^f is not strong enough to ever equal the forces of tension.)