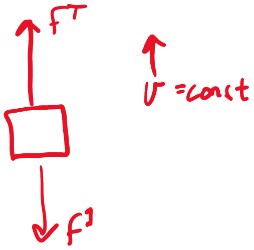
Final Exam Solutions

Thursday, July 20, 2023 9:42 AM

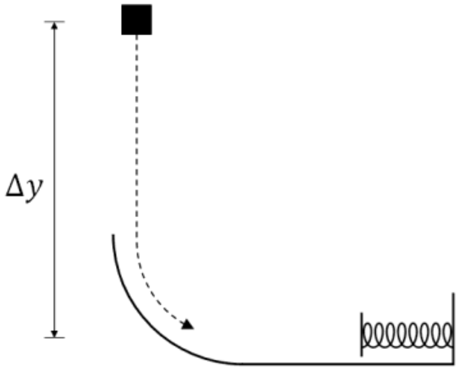
For questions 1 through 5, **fill in the square** next to all correct answers. A given problem may have more than one correct answer. Each correctly bubbled answer will receive two points. There are **14** correct answers in this section and only the first **14** filled in answers will be graded. There is no partial credit.

1. Which of the following quantities are **NOT** vectors?
   - ☐ (a) Force $\vec{F}$
   - ☐ (b) Acceleration $\vec{a}$
   - ☑ (c) Speed
   - ☐ (d) Position $\vec{r}$
   - ☑ (e) Energy
   - ☐ (f) Momentum $\vec{p}$
   - ☐ (g) Impulse $\vec{J}$
   - ☑ (h) Work

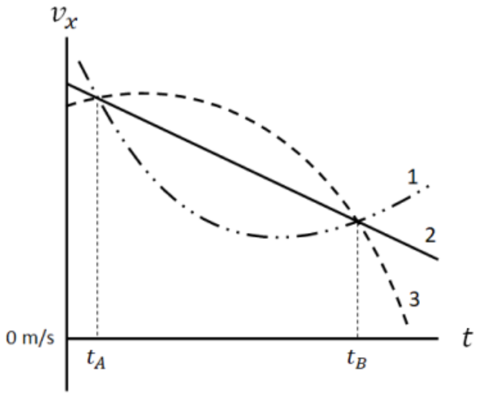
2. An cardboard box is lifted upwards by a rope at a constant speed. Which of the following statements must be true? → $\vec{a} = 0 \Rightarrow F_{net} = 0$
   - ☐ (a) The force of tension on the box is **larger** than the force of gravity on the box
   - ☑ (b) The force of tension on the box is **equal to** the force of gravity on the box
   - ☑ (c) The box is in equilibrium dynamic equilib.
   - ☐ (d) The acceleration of the box is g
   - ☑ (e) The work done by gravity is **negative** $\Delta\vec{r}\uparrow$ $F^g\downarrow$
   - ☐ (f) The work done by gravity is **positive**
   - ☐ (g) The work done by gravity is **zero**



3. A box of mass M is released from rest, falls onto a frictionless ramp, then slides into an ideal spring after falling a distance Δy. Which of the following **must** we include in our system if we are to use a conservation of energy analysis?
   - ☑ (a) The box.
   - ☑ (b) The Earth. → U^g not W^g_{ext}
   - ☐ (c) The ramp.
   - ☑ (d) The spring.



4. Three squirrels (labeled 1, 2, and 3) running along the x-axis have motion described by the following graph. Which of the following statements must be true?



☐ (a) At time t_A the squirrels have the same net force acting on them. $a = \text{diff}$

☐ (b) At time t_B, **squirrel 3** is at the largest x-position. don't know x, only Δx

☐ (c) At time t_B, **squirrel 1** has a larger velocity than the other squirrels.

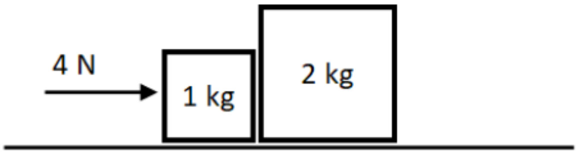
✓☑ (d) Between t_A and t_B, **squirrel 3** has the largest displacement. → Δx

✓☑ (e) Between t_A and t_B, all three squirrels have a positive Δx. $v > 0$ for

☐ (f) For all of the time between t_A and t_B, **squirrel 3** has the largest acceleration.

✓☑ (g) **Squirrel 2** has a constant net force acting on it. $a = \text{slope} = \text{const} \Rightarrow F_{net} = \text{const}$

5. Two boxes are placed next to each other on a frictionless surface as shown. Starting from rest, a constant force of **4 N** is applied to the smaller box. Which of the following statements must be true?



✓☑ (a) The net force on the **2 kg** box is twice as big as the net force on the **1 kg** box. $\underbrace{F_{net}}_{\uparrow \times 2} = \underbrace{m}_{\uparrow \times 2}\underbrace{a}_{\text{same}}$

☐ (b) The net force on both boxes is equal.

☐ (c) The **2 kg** box has a net force of **4 N** acting on it.

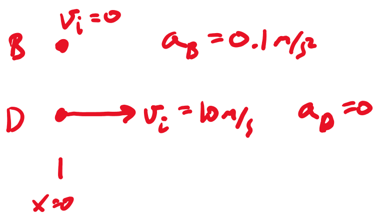
☐ (d) The **1 kg** box has a net force of **4 N** acting on it.

✓☑ (e) The **1 kg** box has a net force less than **4 N** acting on it.

☐ (f) After 2 seconds has elapsed, the change in momentum of each box is equal.

$$\underbrace{\Delta p}_{\text{diff}} = \underbrace{\Sigma F}_{\text{diff}}\,\underbrace{\Delta t}_{\text{same}}$$

6. (6 points) Benny and Donald are racing their scooters. Both start at **x = 0 m** at the same time. Donald starts with an initial velocity of **10 m/s** in the positive x-direction, and travels with a constant velocity the entire race. Benny starts with an initial velocity of **0 m/s**, but accelerates in the positive x direction at a constant rate of **0.1 m/s²**. How far do they travel before Benny passes Donald?



$\Delta x_B = \Delta x_D$ Δt = same as well

$$\Delta x_B = \cancel{v_{iB}}^{0}\Delta t + \frac{1}{2}a_B\Delta t^2$$

$$\Delta x_D = v_{iD}\Delta t + \frac{1}{2}\cancel{a_D}^{0}\Delta t^2$$

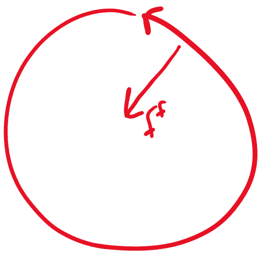
$$\Rightarrow \frac{1}{2}a_B\Delta t^2 = v_{iD}\Delta t$$

$$\Rightarrow \Delta t = 0 \quad \text{or} \quad \Delta t = \frac{2v_{iD}}{a_B} = 200\,s$$

$$\Delta x_D = v_{iD}(200\,s)$$

$$\boxed{\Delta x = 2000\,m}$$

7. (5 points) A sports car is travelling in a circle on flat pavement. The coefficient of static friction between the pavement and car tires is **0.95**. What is the smallest radius of circle the car is able to achieve if its speed is a constant **20 m/s**?



$\vec{r}$

$$\Sigma f_r = \frac{mv^2}{r}$$

$$f^f = \frac{mv^2}{r}$$

$$\mu f^N = \frac{mv^2}{r}$$

$$\mu \cancel{m}g = \frac{\cancel{m}v^2}{r}$$

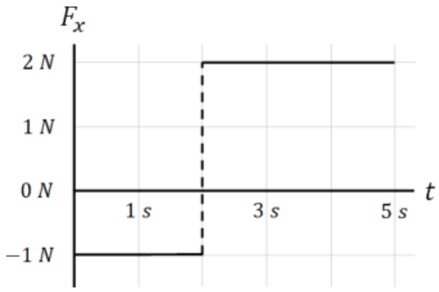
$$r = \frac{v^2}{\mu g} = \frac{v^2}{9.32}$$

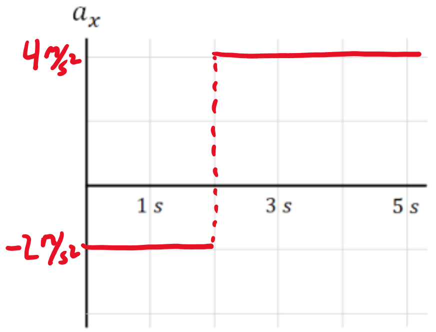
$$\boxed{r = 42.9\,m}$$

8. (7 points) A **0.5 kg** object, initially at rest, experiences a net force along the x-axis described by the given graph. Clearly and neatly draw the graphs of the object's corresponding acceleration, momentum, and velocity. Please make sure to label the vertical axis of each graph with values and units!

Hint: you may notice common relationships between these quantities!

$\Sigma F = ma \Rightarrow a = \Sigma F/m \Rightarrow 2N/0.5\,kg = 4\,m/s^2$





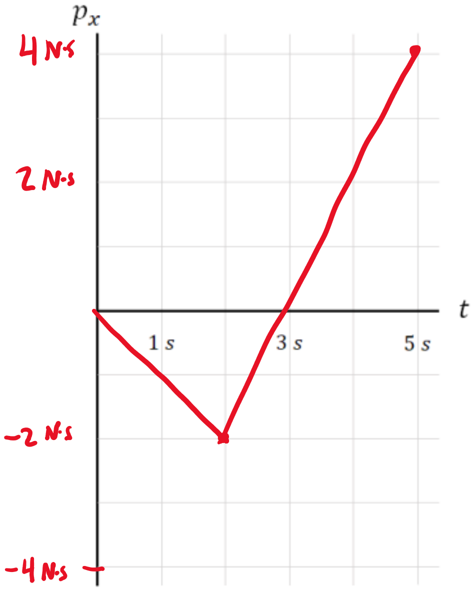
$\Delta p = \Sigma F \Delta t$ = area under curve

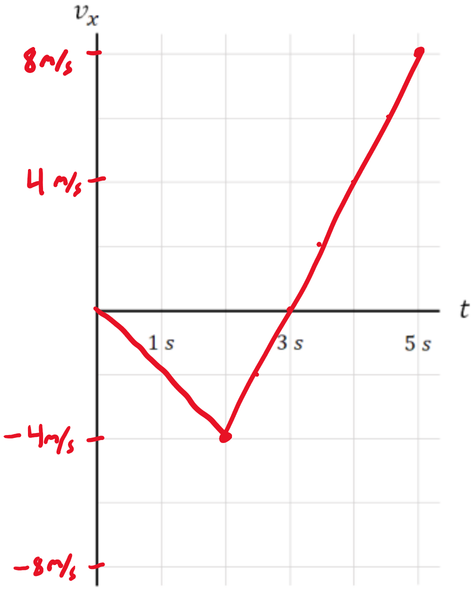
0→2

−2 N·s

2→5

+6 N·s





$p = mv$

$\Rightarrow \underbrace{-2\,N\cdot s}_{\downarrow} = \underbrace{(0.5\,kg)(-4\,m/s)}_{\downarrow}$

$kg\frac{m}{s^2}s = kg\frac{m}{s}$ ✓

$a = \frac{\Delta v}{\Delta t}$

$\Delta v = a\Delta t$

0→2

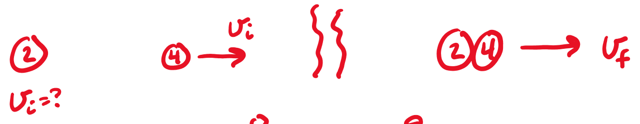
$\Rightarrow \Delta v = -4\,m/s$

2→5

$\Rightarrow \Delta v = +12\,m/s$

9. (6 points) A **2 kg** clay puck is sliding across frictionless ice along the x-axis. It collides with a **4 kg** puck travelling at **8 m/s** in the positive x-direction. The two pucks then stick together and travel together at a speed of **4 m/s** in the positive x-direction.

(a) What was the initial velocity (include direction) of the **2 kg** puck?



$$\Sigma \cancel{F_{ext}}^{0} \Delta t = \cancel{\Delta p_{sys}}^{0} \Rightarrow p_{ix} = p_{fx}$$

$$m_2 v_{i2} + m_4 v_{i4} = (m_2 + m_4) v_f$$

$$(2\,kg) v_{i2} + (4\,kg)(8\,m/s) = (6\,kg)(4\,m/s)$$

$$\boxed{v_{i2} = 4\,m/s \text{ in } -x \text{ direction}}$$

(b) How much kinetic energy was converted to thermal energy by the collision? (sound energy is negligible in this situation and can be neglected)

$$KE_i = \frac{1}{2} m_2 v_{i2}^2 + \frac{1}{2} m_4 v_{i4}^2 = 144\,J$$

$$KE_f = \frac{1}{2}(m_2 + m_4)(v_f)^2 = 48\,J$$

$$\Rightarrow -\Delta KE = \boxed{\Delta E^{th} = 96\,J}$$

10. (7 points) A box is sliding up a frictionless ramp. The box starts with a speed of **12.4 m/s**. The ramp makes an angle of **30 degrees** with the horizontal.

(a) On midterm 3, you found that a box with the same initial conditions, experiencing friction, would reach a height of $\mathbf{h_{friction} = 4.88}$ **meters**. You will be asked to solve for the height achieved by this frictionless box, $\mathbf{h_{frictionless}}$. Before you solve for this new height, use related quantities sensemaking to explain how these two heights should compare with each other **and why**. (Do not give a numerical answer yet)

I expect $h_{frictionless}$ will be larger than $h_{friction}$ b/c when friction converts KE to E^{th}, there is less total energy that goes into $U_f^g = mg\,h_{friction}$

$\Rightarrow$ $h_{friction}$ will be smaller than $h_{fric\underline{tionless}}$

(b) What maximum vertical height above the starting location does the box reach?
(hint: you have a choice of solution method!)

$$\Sigma W_{ext} = 0 = \Delta E \quad \Rightarrow \quad E_i = E_f$$

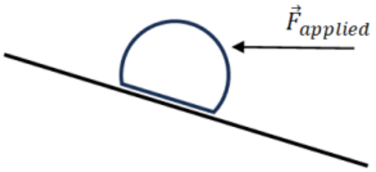
$$KE_i = U_f^g$$

$$\frac{1}{2}\cancel{m}v^2 = \cancel{m}g\,y_f$$

$$y_f = \frac{v^2}{2g} = \boxed{7.84m = h_{frictionless}}$$

⇗ $h_{frictionless} > h_{friction}$ ✓

11. (12 points) On Earth, a piece of clay of mass **m = 7 kg** is placed on a ramp as shown. The ramp makes an angle of **30 degrees** with the horizontal. A force of **20 N** is applied horizontally to the left. The coefficient of friction between the object and the ramp is unknown. The system is in static equilibrium.



(a) In which direction is the force of friction? Explain using any combination of words, diagrams, and algebra.

x

$\sum f_x = m\cancel{a_x}^{0}$

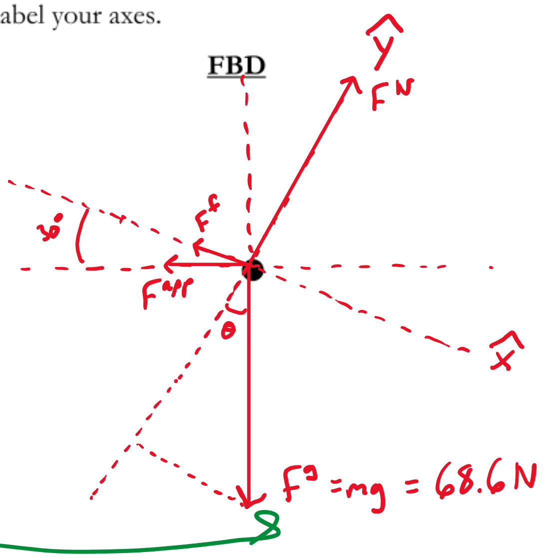
$F_x^{apr} + F_x^{g} + f^{f} = 0$

$-(20N)\cos\theta + 68.6N\sin\theta + f^{f} = 0$

$\Rightarrow f^{f} = -16.98\ N$

Up the incline b/c F_x^{g} is larger than F_x^{app}

(b) Draw a clear and precise free body diagram for this situation. Make sure to label each force vector and scale them relative to each other. Also include any relevant angles and label your axes.



(c) What is the coefficient of static friction between the ramp and object?

Y

$\sum f_y = m\cancel{a_y}^{0}$

$F^N - mg\cos\theta - F^{app}\sin\theta = 0$

$F^N = (68.6N)\cos\theta + (20N)\sin\theta$

$F^N = 69.4\ N$

$|F^{f,s}| = \mu_s|F^N|$

$|-16.98\ N| = \mu_s\,(69.4\ N)$

$\boxed{\mu_s = 0.245}$