Name:________________________________ ID:________________________________

Physics 201

Final Exam

12/6/2018

Collaboration is not allowed. Allowed on your desk are: up to ten 8.5 x 11 inch doubled sided sheets of notes that are bound together, non-communicating graphing scientific calculator, 1 page of scratch paper, writing utensils, and the exam. You will have 110 minutes to complete this exam.

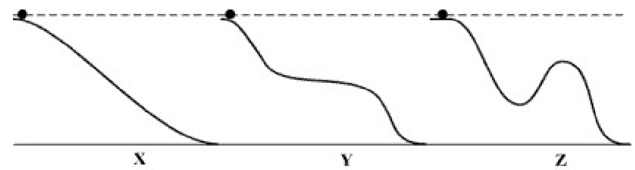
1. (6 points) Use sense-making throughout this entire exam. When you do, write it down in your solution and denote clearly which kind from the list below. Each instance is worth 1 point and you can receive up to 2 points from each type of sense-making technique. To enable the grader to find your application of sense-making, put the problem number in the boxes below of where you've used each one.

Problem Number

☐	☐	• *Sign:* Check the **sign** of their quantities makes sense
☐	☐	• *Dimensionality:* Check the **dimensionality** and units of their quantities makes sense
☐	☐	• *Order of Magnitude:* Check the **order of magnitude** of their quantities makes sense
☐	☐	• *Graphical Analysis:* Use a **graph** to see if the behavior of your solution makes sense
☐	☐	• *Proportionality:* Check the behavior of a derived equation makes sense, e.g. **proportional reasoning**
☐	☐	• *Special Cases:* Check the behavior of a derived equation in limiting (**special**) cases makes sense, e.g. as x goes to 90 degrees in sin(x)
☐	☐	• *Self-consistency:* Check derived equations, functions, or values, are **self-consistent**, e.g. check that the slope of a derived position plot matches the values of the given velocity plot
☐	☐	• *Known Values:* Compare given or derived quantities with common well **known values**
☐	☐	• *Related Quantities:* Compare the relative magnitude of two **related quantities**

For questions 2 through 6 shade in all correct answers ***like a bubble sheet***. A given problem may have more than one correct answer. Each correctly bubbled answer will receive two points. There are **8** correct answers in this section and only the first **8** bubbled answers will be graded. There is no partial credit.

2. A ball can travel down one of three different ramps, as shown below. The final elevation loss of each of the ramps is the same. Neglecting friction, for which ramp will the speed of the ball be the highest at the bottom?



[F] (a) Ramp Z
[F] (b) Ramp X
[F] (c) Ramp Y
[T] (d) The speed of the ball will be the same for all ramps.

3. Two large rockets are fired from a very high tower. Rocket A has its thruster always applying a force upward, away from Earth. Rocket B has its thrust applied horizontal, or tangent to the surface of Earth. If the thrusters fire for the same amount of time, which of the following statements are necessarily true? Ignore air resistance and assume neither rocket hit the ground.

[F] (a) The final momentum of rocket A is greater than B.
[T] (b) The final momentum of rocket A is less than B.
[F] (c) The final momentum of rocket A and B are the same.
[F] (d) The momentum of each rocket is conserved.
[T] (e) The momentum of each rocket is not conserved.
[T] (e) Both rockets received a non-zero impulse.

4. Which of the following statements are always true?

[F] (a) Friction does negative work on an object.
[F] (b) Normal forces acting on an object are perpendicular to the direction of motion.
[F] (c) Normal forces do no work on an object.
[T] (d) Friction is parallel to the surface between two objects.
[F] (e) Friction can only decrease an objects magnitude of momentum.

5. Which of the following are a vector quantities?

[S] (a) the age of the earth
[S] (b) the mass of a football
[V] (c) the earth's pull on your body
[S] (d) the temperature of an iron bar
[S] (e) the number of people attending a baseball game
[S] (f) the work done on a tennis ball hitting a racquet
[V] (g) the impulse given to a baseball by a bat

6. An object is dropped from rest into a pit, and accelerates due to gravity at roughly 10 m/s^2. It hits the ground in 5 seconds. A rock is then dropped from rest into a second pit, and hits the ground in 10 seconds. Roughly how much deeper is the second pit, compared to the first pit? Neglect air resistance.

[T] (a) four times deeper
[F] (b) two times deeper
[F] (c) square root of 2 times deeper
[F] (d) one half times deeper

7. (9 points) Skiers ride a lift to the top of the mountain and then ski down due to the reduced friction between the snow and their skies. They ski in an S pattern, which helps them avoid acquiring too much speed. They will do many rounds of this in a day. (a) Starting at the bottom of the lift, describe all the energy transfers and transformations for one entire cycle of going up and skiing down. Consider the system the Earth, the skier, and all their gear.

1. The lift does positive work on the system to increase the gravitational potential energy as it moves the skier up the mountain.
2. Gravitational potential energy is converted into kinetic energy as the skier slides down the mountain.
3. Not all of the gravitational potential energy is converted into kinetic energy, some is converted into thermal energy via the friction between the skies and the snow. This energy warms up both the skies (in the system) and the snow (out of the system). There are both internal conversion and external work occurring. This effect is larger when the frictional force is larger, which happens during the S turns.
4. Eventually all of the energy the lift put into the gravitational potential energy gets converted into thermal energy as the skier is back at the base of the mountain with zero kinetic energy but a big smile on their freezing face.

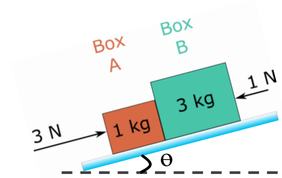
(b) Explain what features of the S pattern, as opposed to just going straight down the mountain, enable them to maintain a survivable speed. Use forces in your explanation.

While sliding straight down the mountain the skier would have a normal force and thus kinetic friction force that is taking energy and converting it to thermal energy. In many cases this is not enough to prevent the kinetic energy from increasing too much. When the skier makes a turn they undergo a form of circular motion that requires an increase in forces towards the center of the circle. This comes from a larger normal force, which in turn (pun intended) creates a larger frictional force. It also increases the distance over which the friction is applied. Larger force and a larger distance results in more work from friction that converts the collective macroscopic linear kinetic energy of the center of mass into microscopic random kinetic energy, which manifests as an increase in temperature. If only it was enough increase to keep us warm while skiing.

(c) Explain how this would differ if we lived in a world without friction.

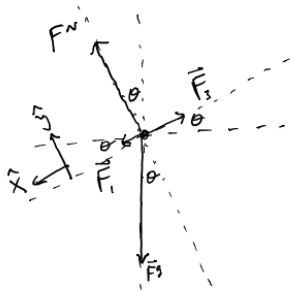
Without friction there would be no way to slow your speed and all of the gravitational potential energy would be converted into kinetic energy. Even a very small hill that's 100 meters (330 ft) high would have you speeding at nearly 45 m/s (~100 mph) at the bottom. You would also have to use some other force to slow you down which unfortunately would probably be from the normal force a tree puts on you as you crash into it.

8. (12 points) Two boxes are pushed on by different forces, on a long flat frictionless inclined surface, as shown in the figure. (a) If the angle with respect the horizontal is 25°, what is the acceleration of the boxes? (b) If the boxes start from rest, how long will it take them to travel 10 m? (c) What angle would put the boxes in equilibrium?



System AB

FBD



(a) $\Sigma F_x \Rightarrow F_1 + (M_A+M_B)g\sin\theta - F_3 = (M_A+M_B)a_x$ eq(i)

$$a_x = \frac{F_1 + (M_A+M_B)g\sin\theta - F_3}{(M_A+M_B)} = 3.6\ \text{m/s}^2$$

w/ $\Sigma F_y = 0$, $a_y = 0$, so $\vec{a} = \langle 3.6, 0 \rangle\ \text{m/s}^2$

(b) $\Delta x = \cancel{V_i}^{\,0}\Delta t + \frac{1}{2}a_x\Delta t^2$

$$\Delta t = \sqrt{\frac{2\Delta x}{a_x}} = 2.34\ \text{s}$$

(c) Starting from eq (i), w/ $a_x = 0$

$$\sin\theta_c = \frac{F_3 - F_1}{(M_A+M_B)g} \Rightarrow \theta_c = 2.92°$$

Rubric

(a)

+ 2pts - FBD
+ 0.5pts - Rotated cordinate system
+ 3pts - ΣF_x
+ 1pt - Algebra
+ 0.5pts - Answer + units

(b)

+ 2pts - Kinematic eq.
+ 0.5pts - Answer + units

(c)

+ 1pt - equilibrium $\Rightarrow (a_x = 0)$
+ 1pt - Algebra
+ 0.5pt - Answer

Rubric

+2 pts - Ranking

+2 pts - Reasoning

9. (4 points) Two boxes are initially pushed on by different forces, on a long flat frictionless inclined surface, as shown in the figure. If the incline were to be rotated clockwise such that box B is now on the downhill side, but the angle from the horizontal is the same, would the normal force between the boxes be greater than, less than, or equal to the previous orientation. Explain your reasoning.

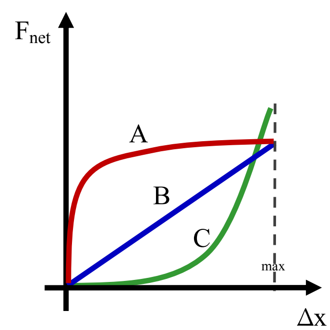
$$|F^N_{AB}|_{initial} > |\vec{F}^N_{AB}|_{final}$$

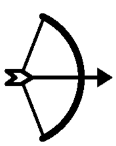
The normal force between the boxes is greater in the initial state because the net applied force is up the ramp and the heavier mass is on the uphill side. In the final state the net applied force is down the hill and the heavier mass is on the downhill side.

In the initial state box A would be pushing the larger box B up the incline, with the normal force from A on B opposing the component of the gravity of B that's down the incline. In the final state the normal force from box B on A would be opposing the portion of gravity from A up the incline, which is a smaller value than in the initial case because the force of gravity on A is smaller than on B.

Special Cases: Another way to see this is to take the angle to an extreme. Let theta go to 90 (or even 0) degrees. Then you have the two boxes on an elevator type problem. In the initial state the acceleration of the system would be smaller than in the final case. That is because the net applied force is against gravity as opposed to with it.

10. (4 points) You are building a bow and arrow. The figure shows the net force from three different bow designs as a function of the distance the bow has been pulled back from its equilibrium state. Rank the bows by the speed an arrow would be fired from each if the bow is pulled back to its maximum distance. Explain your reasoning behind your ranking.





$\Sigma W =$ Area under $\Sigma F(x)$ curve $= \Delta K$

largest Area = greatest final K.E. + Speed

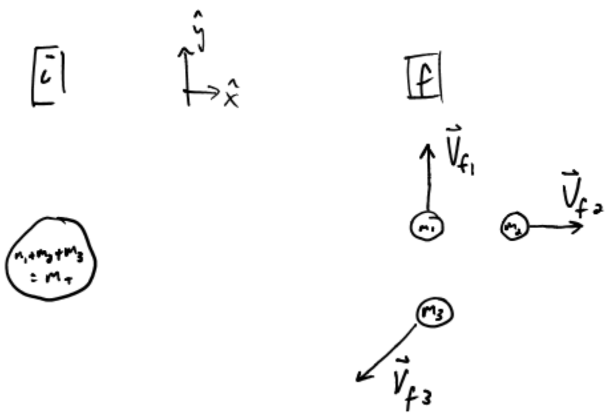
$$V_{f_A} > V_{f_B} > V_{f_C}$$

Rubric

+2 pts - Ranking

+2 pts - Reasoning

11. (10 points) An unstable nucleus of mass 17 x 10^{-27} kg, initially at rest, decays into three particles. One of the particles, of mass 5.0 x 10^{-27} kg, moves along the y axis with a speed of 6.0 x 10^6 m/s. Another particle, of mass 8.4 x 10^{-27} kg, moves along the x axis with a speed of 4.0 x 10^6 m/s. Find (a) the velocity of the third particle and (b) the total kinetic energy generated in the process.



$$m_3 = m_T - m_1 - m_2 = 3.6 \times 10^{-27}\ \text{kg}$$

a.) w/ $\Sigma \vec{F}_{ext} = 0$, $\Sigma \vec{P}_i = \Sigma \vec{P}_f$

$\boxed{x}$ $m_T \cancel{V_{ix}}^{0} = m_1 \cancel{V_{f1x}}^{0} + m_2 V_{f2x} + m_3 V_{f3x}$

$$V_{f3x} = -\frac{m_2 V_{f2x}}{m_3} = -9.33 \times 10^6\ \text{m/s}$$

$\boxed{y}$ $m_T \cancel{V_{iy}}^{0} = m_1 V_{f1y} + m_2 \cancel{V_{f2y}}^{0} + m_3 V_{f3y}$

$$V_{f3y} = -\frac{m_1 V_{f1y}}{m_3} = -8.33 \times 10^6\ \text{m/s}$$

So, $\boxed{\vec{V}_{f3} = -\langle 9.33, 8.33 \rangle \times 10^6\ \text{m/s}}$

b.) $W = K_f - \cancel{K_i}^{0}$ = Energy Released

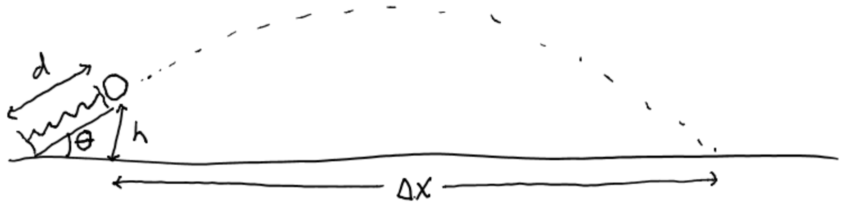
$$= \frac{1}{2}\left(m_1 V_{1f}^2 + m_2 V_{2f}^2 + m_3 V_{3f}^2\right) = \boxed{4.39 \times 10^{-13}\ \text{J}}$$

Rubric
(a)
+ 1 pts - physical representation
+ 1 pt - finding m_3
+ 2 pts - conservation of momentum analysis
+ 1.5 pts - application of conservation of momentum in x
+ 1.5 pts - application of conservation of momentum in y
+ 1 pt - algebra
+ 0.5 pts - answer + units

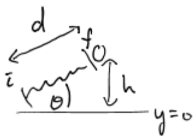
(b)
+ 1 pt - change in kinetic energy equation
+ 0.5 pts - answer + units

12. (12 points) In a freak hot tub accident you find yourself sent back in time to medieval days. To avoid death you proclaim you possess magical powers and can correctly choose the necessary ideal spring for their catapult to fire upon any distance. They setup a demonstration and ask you to hit a target 200 m away with a 100 kg boulder. What is the spring constant, in N/m, of the spring you would use? Assume the spring compresses the full distance ***d*** before the launch and that it leaves the catapult at a height ***h*** above the ground. Neglect air resistance.

d = 4 m	Θ = 30°	Δx = 200 m	m = 100 kg



Stage 1: launch - Conservation of Energy



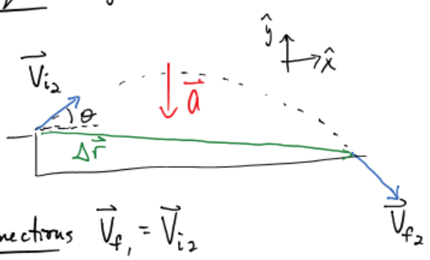
System: Boulder, Spring, Earth

$$\Sigma E_i + \Sigma W_{nc} = \Sigma E_f \Rightarrow \cancel{K_i}^0 + \cancel{U_i^g}^0 + U_i^s + \cancel{\Sigma W_{nc}}^0 = K_f + U_f^g + \cancel{U_f^s}^0$$

$$\frac{1}{2}kd^2 = \frac{1}{2}mV_{f_1}^2 + mgh \quad , \quad h = d\sin\theta$$

$$\text{so, } V_{f_1}^2 = \frac{kd^2 - 2mgd\sin\theta}{m} \quad (\text{eq I})$$

Stage 2: Projectile - Kinematics



Connections $\vec{V}_{f_1} = \vec{V}_{i_2}$

	k	uk	eq's
$\boxed{x}$	$\Delta X = 200m$, $a_x = 0$	$V_{ix} = V_{fx}$, Δt	(i) $\Delta X = V_i\Delta t + \frac{1}{2}a\Delta t^2$ (ii) $V_f = V_i + a\Delta t$ (iii) $V_f^2 = V_i^2 + 2a\Delta X$
$\boxed{y}$	$\Delta y = -d\sin\theta = -2m$, $a_y = -9.8\,m/s^2$	V_{iy}, V_{fy}, Δt	

$$\left.\begin{aligned} \boxed{x}\quad & \Delta X = V_i\cos\theta\,\Delta t \\ \boxed{y}\quad & \Delta y = V_i\sin\theta\,\Delta t - 4.9\Delta t^2 \end{aligned}\right\} \Delta y = \cancel{V_{i_2}}\sin\theta\,\frac{\Delta X}{\cancel{V_{i_2}}\cos\theta} - 4.9\,\frac{\Delta X^2}{V_{i_2}^2\cos^2\theta} \Rightarrow V_{i_2}^2 = \frac{4.9\Delta X^2}{\cos^2\theta(\Delta X\tan\theta - \Delta y)} \quad (\text{eq II})$$

$$|\vec{V}_{i_2}| = 46.77\,m/s$$

Combine eq (I) & (II) & Solve for k.

$$\frac{kd^2 - 2mgd\sin\theta}{m} = \frac{4.9\Delta X^2}{\cos^2\theta(\Delta X\tan\theta - \Delta y)}$$

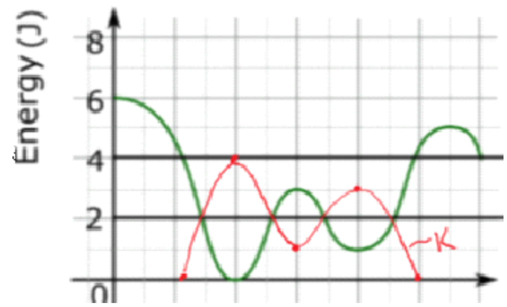
$$k = \frac{1}{d^2}\left[\frac{4.9\,m\,\Delta X^2}{\cos^2\theta(\Delta X\tan\theta - \Delta y)} + 2mgd\sin\theta\right] = \boxed{13{,}916\,\frac{N}{m}}$$

Rubric on next page

13. (4 points) Below is a plot of the potential energy as a function of position for a system. (a) If the system initially has a total energy E_1, sketch a plot of the kinetic energy as a function of position on the graph. (b) If the system is slowly losing energy and at a later time is in a state E_2, can we determine which potential well the system resides in—does it oscillate around position 2 m or 4 m? Explain your reasoning.

(b) As the energy decreases below 3 J, the position at x = 3 m becomes a forbidden region. That means if the particle found itself in the well around x = 2 m it would be stuck in that well. Likewise if it found itself in the well around x = 4 m when that happens, it would be stuck in that well. So it could end up in either well.

To go a bit deeper you can look at the average kinetic energy in each well and determine that is smaller in the well on the right. Because of this the object would be traveling slower in that region than in the region of the well on the left. If it's going slower, it spends more time and has a higher probability of being in that well when the energy falls below the 3 J threshold. So there is a higher probability it will end up in the well on the right due to the slower speeds in that region.



Rubric
(a) + 2 pts
(b) + 2 pts

Extra space:

Problem 12 rubric:

Rubric

Problem Orientation

+1pt - multiple Stages

+1pt - Stage 1: Energy

+1pt - Stage 2: Projectile Motion

Stage 1

+1pt - Conservation of Energy eq.

+2pt - Application of Energy

+1pt - $h = d\sin\theta$

Stage 2

+1pt - physical representation

+1pt - Knowns & unknowns

+1pt - eq (i)

+1pt - solving ΔX & Δy simult to get $|\vec{V}_i|$

+1pt - Combining stage 1 + 2 together