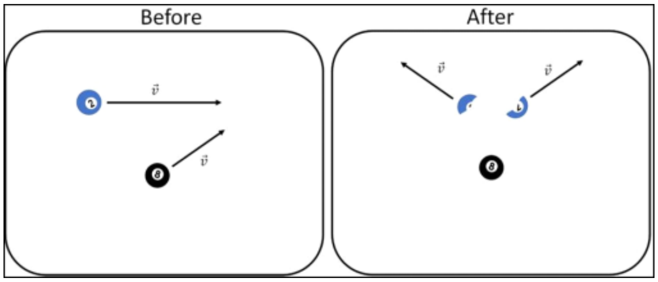
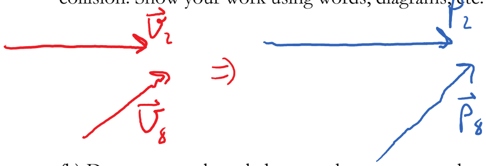
Final Exam Solutions (p1q1)

Tuesday, December 8, 2020 5:42 PM

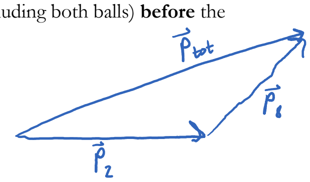
(8 points) Bill Yards, the astronaut, is playing a game of space billiards in outer space, far from any other objects. Two identical billiard balls collide. The blue ball breaks into two even pieces. The velocities of both balls before the collision, and the two pieces of the blue ball after the collision are given in the diagrams below. The velocities are drawn to scale. The velocity of the black ball (8 ball) after the collision is unknown. The velocity of one of the broken pieces after the collision is identical to the velocity of the black ball before the collision.



(a) Draw a properly scaled vector that represents the total **momentum** of the system (including both balls) **before** the collision. Show your work using words, diagrams, etc.

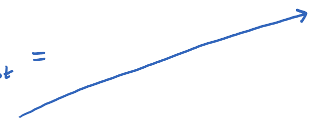


$\vec{P}_{tot} = \vec{P}_2 + \vec{P}_8 \Rightarrow$



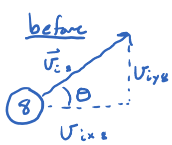
(b) Draw a properly scaled vector that represents the total **momentum** of the system (including the two blue pieces and the black ball) **after** the collision. Show your work using words, diagrams, etc.

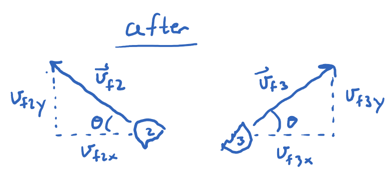
$\vec{J}_{ext} = 0 = \Delta\vec{P}_{sys} \Rightarrow \vec{P}_{f,tot} = \vec{P}_{i,tot} \Rightarrow \vec{P}_{f,tot} =$



(c) Draw a properly scaled vector that represents the **velocity** of the black ball **after** the collision. Show your work using words, diagrams, etc.

Consider y-component of $\vec{p}$





I called this one "3" for simplicity

$P_{iy8} = P_{i,tot} = m|v_{i8}|\sin\theta$

(8-ball is only ball with $p_y > 0$)

$P_{fy} = P_{f2y} + P_{f3y}$

$= \frac{m}{2}|v_{f2}|\sin\theta + \frac{m}{2}|v_{f3}|\sin\theta$

$= m|v_{f2}|\sin\theta$

Since $|v_{i8}| = |v_{f2}| = |v_{f3}|$,

the broken two ball after the collision has all the y-momentum that the system had before the collision. Thus the 8-ball has zero y-momentum, and $v_{f8y} = 0$.

$P_{f2x} + P_{f3x} = 0 \Rightarrow P_{f8x} = P_{i,tot,x}$

Rubric

~~ Part (a) ~~
0.5 pt - Answer
1.5 pt - Reasoning

~~ Part (b) ~~
0.5 pt - Answer
1.5 pt - Reasoning

~~ Part (c) ~~
1.0 pt - Accounting for mass cut in half
2.5 pts - Vertical momentum analysis
0.5 pts - Final velocity is horizontal

Part (c) - Instructor notes

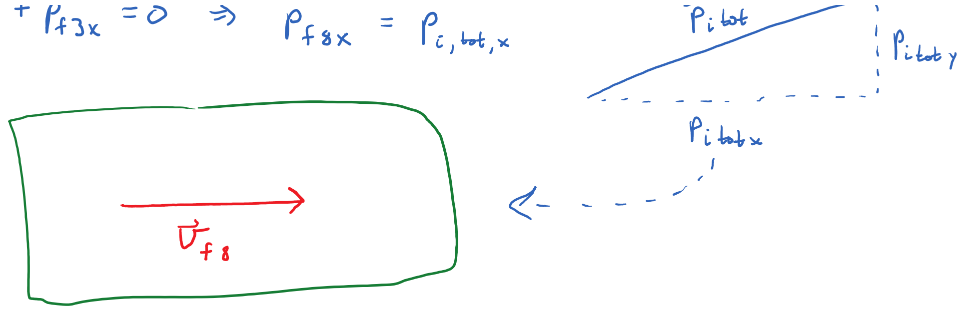
("D/F level") some sort of vector addition or subtraction operation or knowledge of momentum shown

("C level") vector algebra with use of total momentum vectors and broken ball momentum

("B level") vector algebra + accounting for change in mass of ball pieces (momentum is not just velocity)

("A level") final velocity is horizontal

$$p_{f2x} + p_{f3x} = 0 \Rightarrow p_{f8x} = p_{i,tot,x}$$



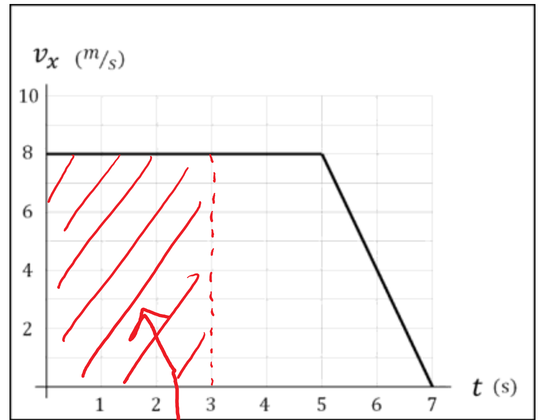
Final Exam Solutions (p1q2)

Tuesday, December 8, 2020 8:33 PM

(6 points) Bernice and Daisy are racing in the Annual Mascot Race, which is 100 m long. Benny is supposed to be recording the race, but makes a mistake and only records the last few seconds of Bernice winning the race. He manages to record Bernice crossing the finish line and the few moments of motion afterwards. A plot of Bernice's motion is made from the recording that Benny made, and is displayed on the right.

(a) Describe Bernice's recorded motion in words.

(b) At **t** = 0 seconds on the graph Bernice is 24 meters from the finish line. How many more seconds until she cross the finish line?



$\Delta x = (8\,m/s)(3\,s) = 24\,m$

(a)

Bernice moves @ a constant speed of 8 m/s for 5 seconds, then slows down to a stop with a constant acceleration over the next 2 seconds.

(b) Δx = area under $v_x(t)$ graph

$24 \text{ meters} = (8\ m/s)(3\ sec)$

Bernice crosses finish line 3 seconds after Benny starts recording.

Rubric

~~ Part (a) ~~
2 pts - Description

~~ Part (b) ~~
3 pts - Reasoning
1 pt - Answer

Final Exam Solutions (p2q1)

Monday, December 7, 2020 4:08 PM

(6 points) Provide one example for each of Newton's 3 laws of motion that you have (or can) observe in your everyday life. Using words, phrases, diagrams, etc… briefly explain how each situation exemplifies each specific law.

(a) Example of Newton's 1st Law

Example and explanation should include something "wanting" to maintain a constant velocity in a scenario where there are little to no external net force. E.g. Hockey puck on very icy surface slides at almost a constant velocity, it would if there was no friction; astronaut on a video released an object at rest and it just floated there at rest; etc…

Rubric (for each a, b, c)

1 pt - Example demonstrates the force law
1 pt - Explanation

(b) Example of Newton's 2nd Law

Example and explanation should include a net external force causing something to accelerate/change-velocity AND how the mass of that something affects the magnitude of the resulting acceleration. E.g. Pushing a broken down car from rest to a speed of 2 miles per hour changed the velocity of the car, but not by much because a car's mass is very large and a person can only push with a small amount of force relative to the weight of the car; etc…

(c) Example of Newton's 3rd Law

Example and explanation should include a force and the two objects/systems (1 and 2) interacting involved highlighting how the force from (1 on 2) is equal in magnitude and opposite in direction from the force of (2 on 1). E.g. Sitting on a chair at rest a person exerts a normal force of say 700 N on the chair downwards, and the chair exerts a normal force on the person with exactly 700 N upwards.

Final Exam Solutions (p2,q2)

Wednesday, December 9, 2020 6:12 PM

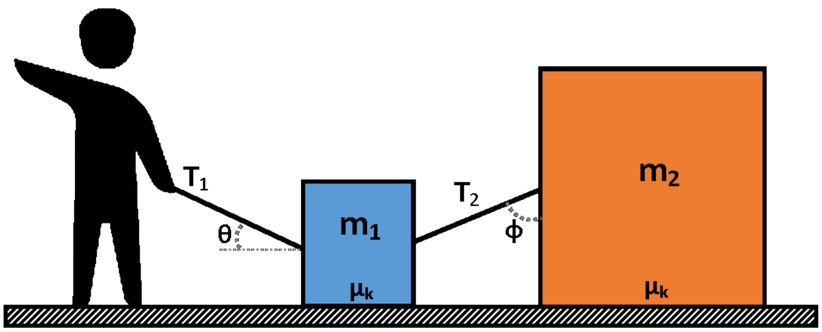
(14 point) A child ties a rope (T_2) between two toy boxes. They then tie another rope (T_1) to the other side of the first box (m_1) and begin dragging both of them across the floor at a constant speed. They call it their toy train! The mass of both boxes are equivalent as is the coefficient of kinetic friction (mu) between them and the floor.

(a) Draw two Free-body Diagrams, one for each toy box. Pay careful attention to scale of each vector, making sure they are all scaled relative to each other. Use what you know about the net force to aid in scaling the vectors.

(b) Compare the velocity of the two boxes. Compare the acceleration of the two boxes.

(c) Choose a coordinate system and write out Newton's 2nd Law for box 1 (m_1) in terms of the variables: T_1, m_1, g, T_2, θ, ϕ, F^N, and μ_k. You do not have to solve for anything.

(d) Is the friction force from the floor acting on m_1 greater than, less than, or equal to $\mu_k(m_1g)$? Explain your reasoning in words, phrases, diagrams, etc...



Rubric

~~ Part (a) ~~

2.5 pts - FBD forces for m1

2 pts - FBD forces for m2

1.5 pts - Relative scaling of vectors

~~ Part (b) ~~

1 pt - compare velocity and acceleration

~~ Part (c) ~~

2.5 pts - sum of forces in the x

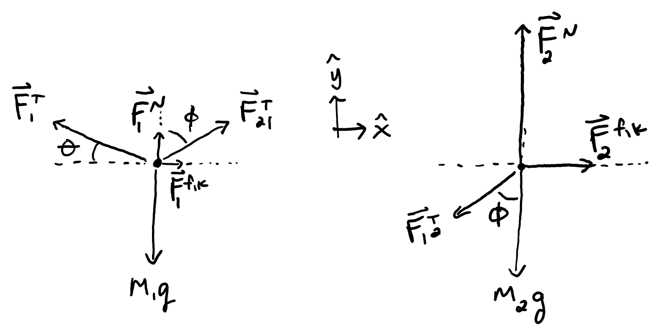
2.5 pts - sum of forces in the y

~~ Part (d) ~~

1.5 pts - Reasoning

0.5 pts - Answer

(a) FBD(m_1) FBD(m_2)



(b) $\vec{V}_1 = \vec{V}_2$, $\vec{a}_1 = \vec{a}_2 = 0$, $M_1 = M_2 = M$

(c) $\Sigma F_y = Ma_y$

$$T_2 \cos\phi + F^N + T_1 \sin\theta - Mg = M\cancel{a_y}^{\,0}$$

$\Sigma F_x = Ma_x$

$$T_2 \sin\phi + \mu_k F_1^N - T_1 \cos\theta = M\cancel{a_x}^{\,0}$$

(d) $|\vec{F}^{f,k}| = \mu_k |\vec{F}^N|$

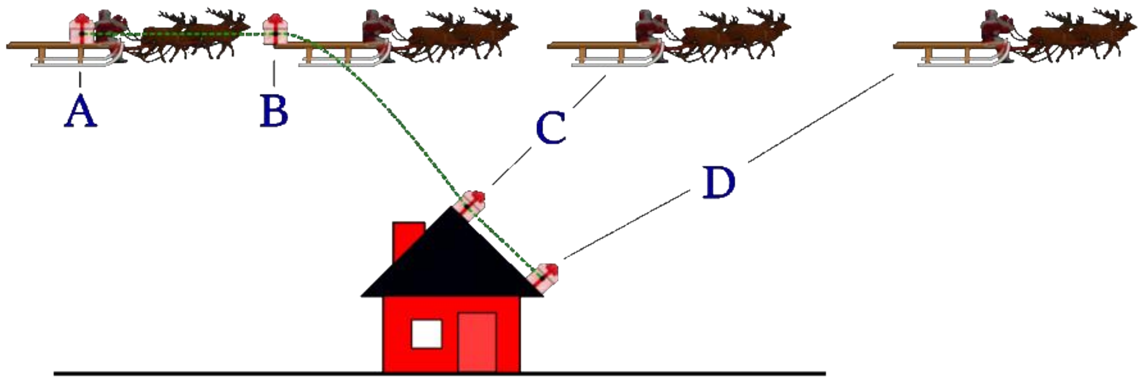
$F^N < Mg$ b/c $\vec{F}_1^T + \vec{F}_2^T$ both have vertical components

so, $|\vec{F}^{f,k}| < \mu_k Mg$

Final Exam Solutions (p3q1)

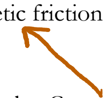
Monday, December 7, 2020 4:10 PM

(6 points) Santa is using a new sled this year. Santa is flying horizontally to the right and accelerating to the right when a gift box slides off the back of the sled and then proceeds to land on a roof as seen in the image below. Snapshots in time were taken (**A** through **D**). Between snapshot **A** and **B**, the gift box slides off the horizontal sled. Between **B** and **C** the gift box is a projectile falling towards a house. Between **C** and **D**, the gift box slides down the inclined roof and comes to a rest at the bottom of the roof. The dashed green line represents the trajectory of the gift box.



(a) Draw a vector to represent the direction of the kinetic friction force on the gift box from the sled between **A** and **B**?

(b) Draw a vector to represent the direction of the kinetic friction force on the gift box from the inclined roof between **C** and **D**?



(c) Consider the motion from snapshot **B** through **D** only. Complete the Energy Flow Diagram for this scenario when the following objects are included in the system: Gift box, sled, roof, Earth.

Key features are highlighted and explained below:

A: There is KE and Gravitational Potential energy at B

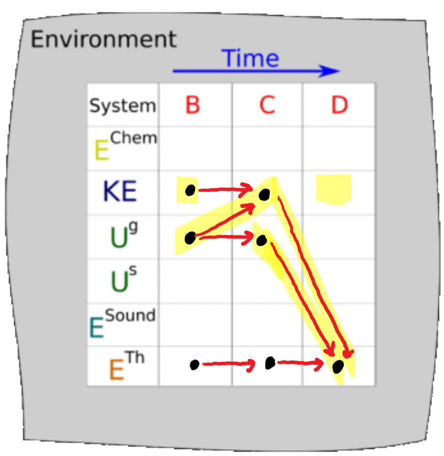
B->C : Gravitational Potential energy transforms to KE

C: There is KE and Gravitational Potential energy at C.

C->D: KE and Gravitational Potential energy transforms to Thermal Energy. It's ok to have KE and U^g transform into sound energy in addition to thermal energy as well.

D: There is no KE.

There is no spring or chemical energy relevant to this situation.



Rubric

~~ Part (a) ~~
1.5 pts - answer to direction

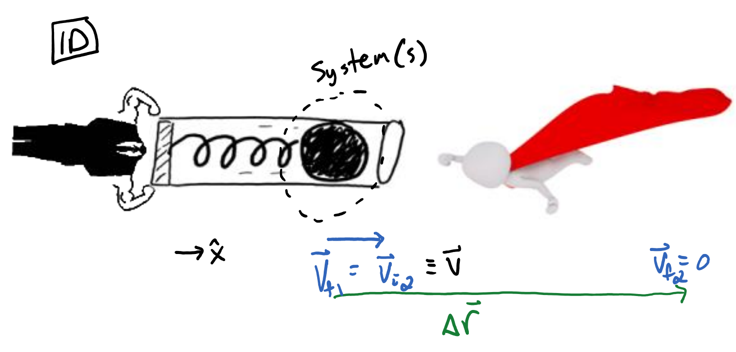
~~ Part (b) ~~
1.5 pts - answer to direction

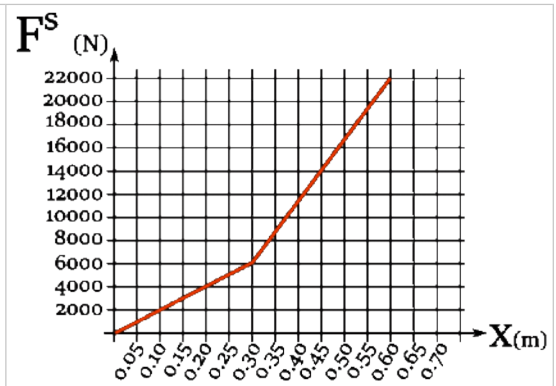
~~ Part (c) ~~
1.0 pts - correct types of energies (K, U, E_thermal)
1.0 pts - U => KE
1.0 pts - correct diagram

Final Exam Solutions (p3,q2)

Wednesday, December 9, 2020 6:12 PM

(10 points) The evil Headless Businessman is a really bad guy and has stolen Benny's spring loaded weather probe launcher. Benny recently unveiled his invention, providing a plot of the spring's force as a function of distance. While Headless threatens a crowd of people, Superman comes to the rescue. During the scuffle Headless compresses the spring all the way to its maximum value of 0.60 m and then fires a mysterious dark sphere directly horizontal towards the crowd. Assume all of the work from the spring goes into the sphere's kinetic energy. Superman, flying in midair, immediately applies a constant 250 N force for 3.00 seconds stopping the sphere. What is the mass of the mysterious dark sphere?





2-Stages ① Work → Kinetic Energy ② Impulse Applied to Stop

① Work-KE Theorem: $W = \Delta KE$

$= \frac{1}{2} m V_f^2 - \frac{1}{2} m V_i^2$ ($V_i \to 0$) $\Rightarrow$ So, Work $= \frac{1}{2} m V^2$, W is also area under F(x) curve.

w/ Area = 5100 J (from previous quiz), $\frac{1}{2} m V^2 = 5100\ J$ (i)

② Impulse-Momentum Theorem: $\Delta \vec{P} = \Sigma \vec{F}_{ext} \Delta t$, w/out gravity, $\Sigma \vec{F}_{ext} = \Sigma \vec{F}_s$, FBD(s): $F^s \leftarrow$

x̂ So, $m V_{f_2} - m V_{i_2} = -F^s \Delta t$ ($V_{f_2} \to 0$; $V_{i_2} = V_{f_1}$; b/c +x̂ definition) $\Rightarrow$ $m V = F^s \Delta t$ (ii)

Combine (i) + (ii) $\frac{1}{2} m \left[\frac{F^{s\,2} \Delta t^2}{m^2} \right] = 5100\ J$

m = 55.1 Kg

Rubric

~~ Problem orientation ~~	~~ Solution exploration ~~	~~ Solution execution ~~
1 pt - Recognizing more than 1 stage 1 pt - Recognizing energy stage 1 pt - Recognizing momentum stage	1 pt - W = K.E. equation 1 pt - K.E. = 1/2*m*v^2 equation 1 pt - Impulse and momentum theorem equation 1 pt - P = m*v equation	2.5 pts - Setting equation (i) equal to (ii) and solving 0.5 pts - Answer and units

Note: You can also use forces and kinematics instead of impulse and momentum. The graders gave equivalent credit for this solution path.