Lab 6:

Using Free-Body Diagrams to Apply Newton's Laws

Each exercise worth 1 point except for 10, 12, 17 which are worth 2 points

Purpose of the lab: To learn to analyze an object's motion by isolating it and showing all forces acting on it.

Note: This is a take-home exercise—due at the start of your Lab 5 session.

Materials needed: A ruler or other straight edge may be handy. No extra paper is needed—write on the pages provided.

Directions: For each situation,

1. **Using the given sketch (or create your own, as necessary), identify the nature and direction of all forces acting on the mass.** If you don't know a direction, make your best guess.

2. **Choose (and indicate) your coordinate system.** If the mass is known to be accelerating in a certain direction, choose a coordinate system so that one of its axes aligns with the direction of that acceleration.

3. **Draw a free-body diagram (FBD) showing all forces acting on the object.** Label all forces with variables, not numbers. Draw all force vectors with their tails beginning at the same place (use just a tiny box to represent the object in question—regardless of the actual shape of the object). Indicate any reference angles that will be used to resolve a vector into its x- and y- components. (Do not draw any acceleration vector on the free-body diagram. This is a force diagram.)

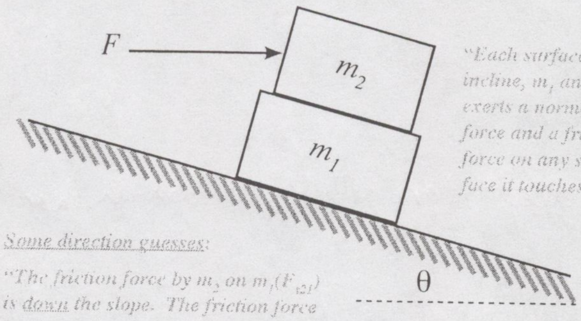
4. **Use Newton's 2nd Law to write equations for x- and y- directions.** You do not need to solve these equations. In writing each equation, don't plug in any numbers except zero. Just sum the forces on the left-hand side. Write the x- and y- components of a vector in terms of its magnitude and an angle. Also, use the expressions you know for weight ($W = mg$ or $F_G = Gm_1m_2/r^2$) and friction ($F_s^{max} = \mu_s F_N$ or $F_k = \mu_k F_N$) whenever applicable.

5. **Some situations involve more than one object. Do the above four steps separately for each object.**

The next page shows a complete example. ----->

EXAMPLE: Masses m_1 and m_2 are stacked and accelerating together down the inclined surface. Force F is horizontal. The coefficient of kinetic friction between the surface and m_1 is μ_k. The coefficient of static friction between m_1 and m_2 is μ_s. Analyze all forces on each mass.

"Gravity exerts a vertical weight force on each mass."

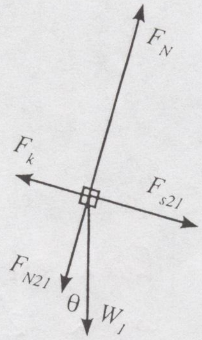


"Each surface (the incline, m_1 and m_2) exerts a normal force and a friction force on any surface it touches."

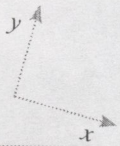
Some direction guesses:

"The friction force by m_2 on m_1 (F_{s21}) is down the slope. The friction force by the surface on m_1 is up the slope."

FBD of m_1



coord. axes



x-analysis

$$\Sigma F_x = m_1 a_x$$

$$F_{s21} + W_{1x} - F_k = m_1 a_x$$

$$\mu_s F_{N21} + m_1 g\sin\theta - \mu_k F_N = m_1 a$$

y-analysis

$$\Sigma F_y = m_1 a_y$$

$$F_N - F_{N21} - W_{1y} = m_1 a_y$$

$$F_N - F_{N21} - m_1 g\cos\theta = 0$$

The three equation forms shown are required for each analysis. Notice how you add more detail to the equation in each form. And notice how your choice of coordinate system here allows you to write a_x as just a (the total acceleration), because a_y is 0.

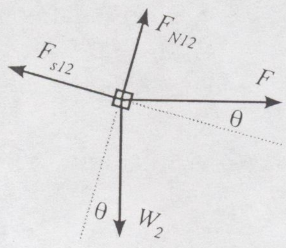
Notation examples:

*F_{N12} is "the normal force exerted **by** m_1 **on** m_2.*

*F_{s21} is "the static friction force exerted **by** m_2 **on** m_1.*

*F_k is "the kinetic friction force exerted **by** the surface (on the object it's touching)."*

FBD of m_2



coord. axes

x-analysis

$$\Sigma F_x = m_2 a_x$$

$$F_x + W_{2x} - F_{S12} = m_2 a_x$$

$$F\cos\theta + m_2 g\sin\theta - \mu_S F_{N12} = m_2 a$$

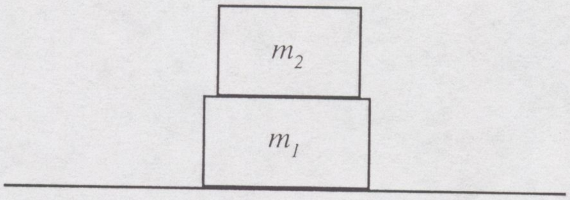
y-analysis

$$\Sigma F_y = m_2 a_y$$

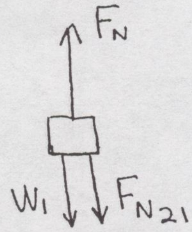
$$F_{N12} + F_y - W_{2y} = m_2 a_y$$

$$F_{N12} + F\sin\theta - m_2 g\cos\theta = 0$$

EXERCISE 1: Masses m_1 and m_2 are stacked together, at rest, on a level surface. Analyze all forces on each mass.

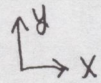


FBD of m_1



1/2

coord. axes



x-analysis

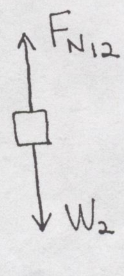
y-analysis

$$\Sigma F_y = m_1 a_y$$

$$F_N - W_1 - F_{N21} = 0$$

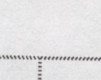
$$F_N - m_1 g - F_{N21} = 0$$

FBD of m_2



1/2

coord. axes



x-analysis

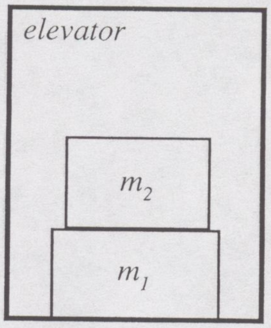
y-analysis

$$\Sigma F_y = m_2 a_y$$

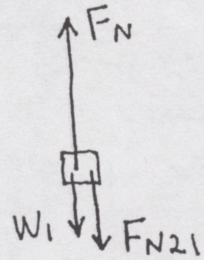
$$F_{N12} - W_2 = 0$$

$$F_{N12} - m_2 g = 0$$

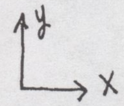
EXERCISE 2: Masses m_1 and m_2 are stacked together on the level floor of an elevator. The masses are at rest with respect to each other and to the elevator. The elevator is accelerating upward. Analyze all forces on each mass.



FBD of m_1



coord. axes



x-analysis

y-analysis

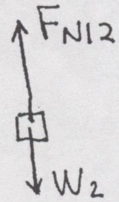
$$\Sigma F_y = m_1 a_y$$

$$F_N - W_1 - F_{N21} = m_1 a$$

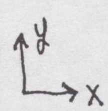
$$F_N - m_1 g - F_{N21} = m_1 a$$

1/2

FBD of m_2



coord. axes



x-analysis

y-analysis

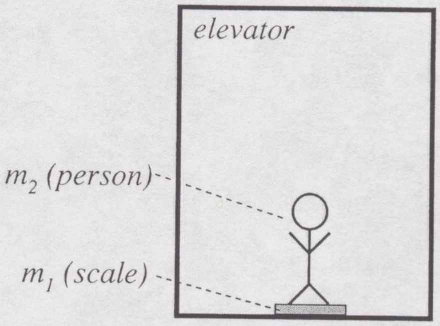
$$\Sigma F_y = m_2 a_y$$

$$F_{N12} - W_2 = m_2 a$$

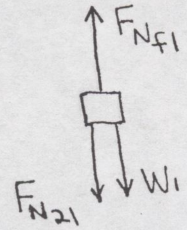
$$F_{N12} - m_2 g = m_2 a$$

1/2

EXERCISE 3: A person (m_2) is standing on a bathroom scale (m_1) on the level floor of an elevator. The person and scale are at rest with respect to each other and to the elevator. The elevator is accelerating downward. Analyze all forces on the person and scale.

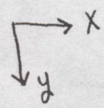


FBD of m_1



1/2

coord. axes



x-analysis

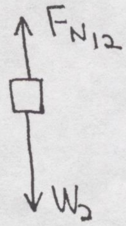
y-analysis

$$\Sigma F_y = m_1 a_y$$

$$F_{N21} + W_1 - F_{Nf1} = m_1 a$$

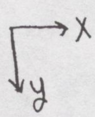
$$F_{N21} + m_1 g - F_{Nf1} = m_1 a$$

FBD of m_2



1/2

coord. axes



x-analysis

y-analysis

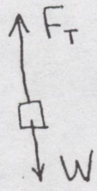
$$\Sigma F_y = m_1 a_y$$

$$W_2 - F_{N12} = m_1 a$$

$$m_2 g - F_{N12} = m_1 a$$

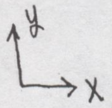
EXERCISE 4 ***(make your own sketch)*****:** A skydiver (m) is falling with his parachute opened. His speed is decreasing. The total tension force exerted on him by the parachute is F_T. Analyze all forces on the skydiver only.

FBD of m



1

coord. axes



x-analysis

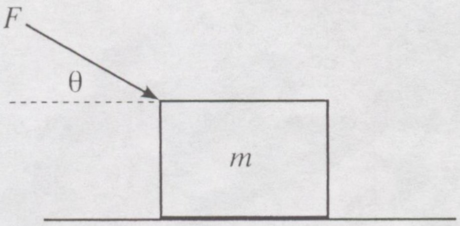
y-analysis

$\sum F_y = ma_y$

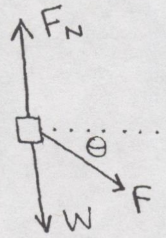
$F_T - W = ma$

$F_T - mg = ma$

EXERCISE 5: The block (m) is accelerating to the right on a level, frictionless surface. Analyze all forces on the block.

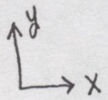


FBD of m



1

coord. axes



x-analysis

$\sum F_x = ma_x$

$F_x = ma$

$F\cos\theta = ma$

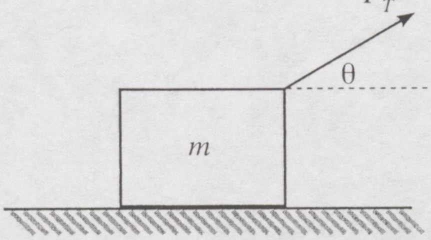
y-analysis

$\sum F_y = ma_y$

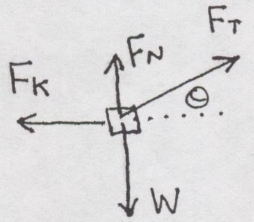
$F_N - W - F_y = 0$

$F_N - mg - F\sin\theta = 0$

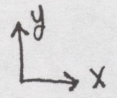
EXERCISE 6: The block (m) is being pulled to the right with constant velocity, across a level surface with friction (μ_k). Analyze all forces on the block.



FBD of m



coord. axes



1

x-analysis

$$\Sigma F_x = ma_x$$

$$F_{Tx} - F_K = 0$$

$$F_T \cos\theta - \mu_K F_N = 0$$

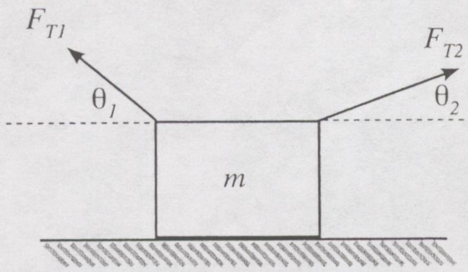
y-analysis

$$\Sigma F_y = ma_y$$

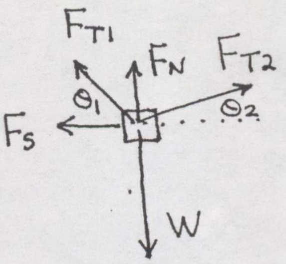
$$F_N + F_{Ty} - W = 0$$

$$F_N + F_T \sin\theta - mg = 0$$

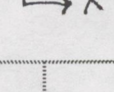
EXERCISE 7: The block (m) is at rest on a level surface with friction (μ_s). Analyze all forces on the block.



FBD of m



coord. axes



1

x-analysis

$$\Sigma F_x = ma_x$$

$$F_{T2x} - F_{T1x} - F_S = 0$$

$$F_{T2}\cos\theta_2 - F_{T1}\cos\theta_1 - F_S = 0$$

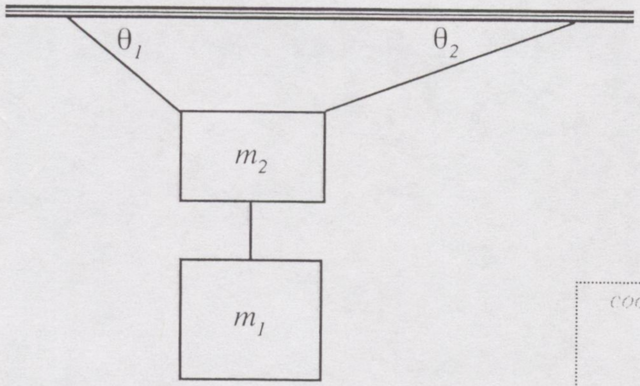
y-analysis

$$\Sigma F_y = ma_y$$

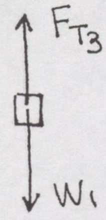
$$F_{T1y} + F_N + F_{T2y} - W = 0$$

$$F_{T1}\sin\theta_1 + F_N + F_{T2}\sin\theta_2 - mg = 0$$

EXERCISE 8: Masses m_1 and m_2 are suspended by three different wires of negligible mass. Analyze all forces on each mass.

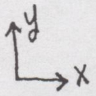


FBD of m_1



1/2

coord. axes



x-analysis

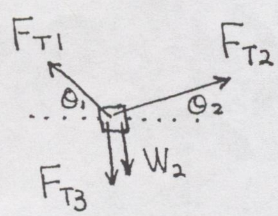
y-analysis

$$\Sigma F_y = ma_y$$

$$F_{T3} - W_1 = 0$$

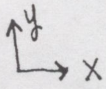
$$F_{T3} - m_1 g = 0$$

FBD of m_2



1/2

coord. axes



x-analysis

$$\Sigma F_x = ma_x$$

$$F_{T2x} - F_{T1x} = 0$$

$$F_{T_2}\cos\theta_2 - F_{T_1}\cos\theta_1 = 0$$

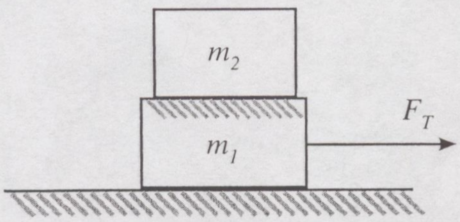
y-analysis

$$\Sigma F_y = ma_y$$

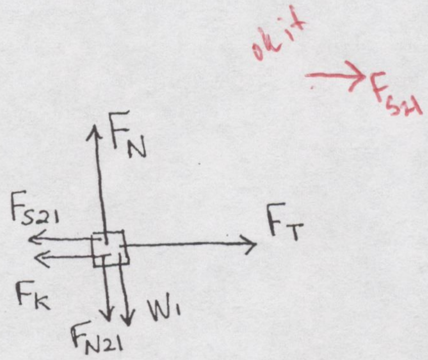
$$F_{T1y} + F_{T2y} - F_{T3} - W_2 = 0$$

$$F_{T_1}\sin\theta_1 + F_{T_2}\sin\theta_2 - F_{T3} - m_2 g = 0$$

EXERCISE 9: Masses m_1 and m_2 are stacked together on a level surface that has friction (μ_k). The blocks are accelerating together to the right. There is static friction between the two blocks (μ_s). Analyze all forces on each block.

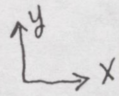


FBD of m_1



1/2

coord. axes



x-analysis

$$\sum F_x = m_1 a_x$$

$$F_T - F_{S21} - F_K = m_1 a$$

$$F_T - F_{S21} - \mu_K F_N = m_1 a$$

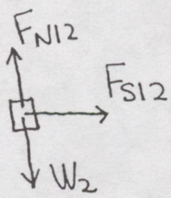
y-analysis

$$\sum F_y = m_1 a_y$$

$$F_N - F_{N21} - W_1 = 0$$

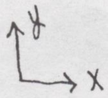
$$F_N - F_{N21} - m_1 g = 0$$

FBD of m_2



1/2

coord. axes



x-analysis

$$\sum F_x = m_2 a_x$$

$$F_{S12} = m_2 a$$

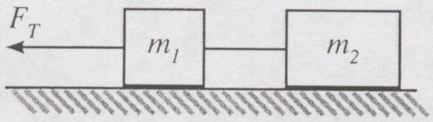
y-analysis

$$\sum F_y = m_2 a_y$$

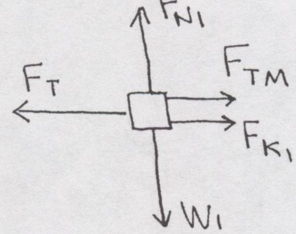
$$F_{N12} - W_2 = 0$$

$$F_{N12} - m_2 g = 0$$

EXERCISE 10: Blocks m_1 and m_2 are accelerating together to the left on a level surface that has friction (μ_k). The blocks are connected by a string of negligible mass that remains taut at all times. Analyze all forces on each block.

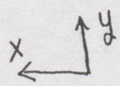


FBD of m_1



1

coord. axes



x-analysis

$$\Sigma F_x = m_1 a_x$$

$$F_T - F_{K_1} - F_{TM} = m_1 a$$

$$F_T - \mu_K F_{N_1} - F_{TM} = m_1 a$$

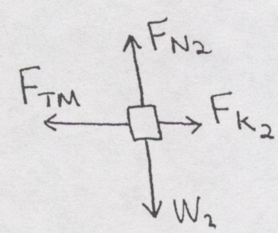
y-analysis

$$\Sigma F_y = m_1 a_y$$

$$F_{N_1} - W_1 = 0$$

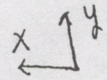
$$F_{N_1} - m_1 g = 0$$

FBD of m_2



1

coord. axes



x-analysis

$$\Sigma F_x = m_2 a_x$$

$$F_{TM} - F_{K2} = m_2 a$$

$$F_{TM} - \mu_K F_{N_2} = m_2 a$$

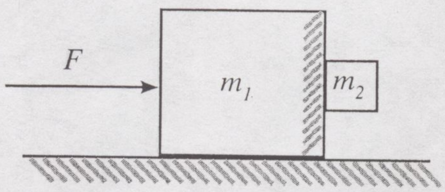
y-analysis

$$\Sigma F_y = m_2 a_y$$

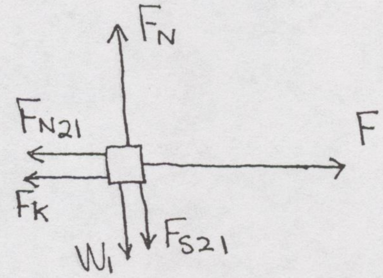
$$F_{N_2} - W_2 = 0$$

$$F_{N_2} - m_2 g = 0$$

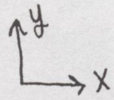
EXERCISE 11: Blocks m_1 and m_2 are not attached to one another, but they are accelerating together to the right on a level surface that has friction (μ_k). There is static friction between the two blocks (μ_s). Analyze all forces on each block.



FBD of m_1



coord. axes



1/2

x-analysis

$$\Sigma F_x = m_1 a_x$$

$$F - F_{N21} - F_K = m_1 a$$

$$F - F_{N21} - \mu_K F_N = m_1 a$$

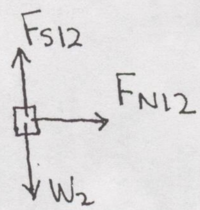
y-analysis

$$\Sigma F_y = m_1 a_y$$

$$F_N - W_1 - F_{S21} = 0$$

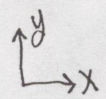
$$F_N - m_1 g - F_{S21} = 0$$

FBD of m_2



1/2

coord. axes



x-analysis

$$\Sigma F_x = m_2 a_x$$

$$F_{N12} = m_2 a$$

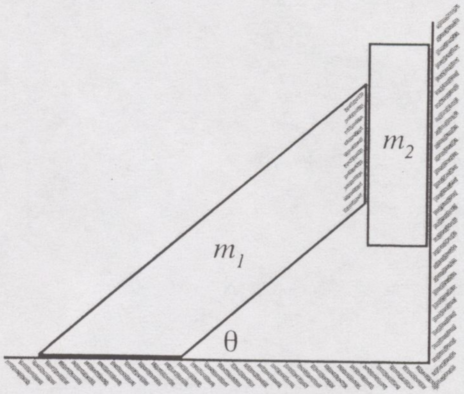
y-analysis

$$\Sigma F_y = m a_y$$

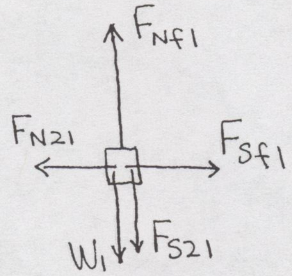
$$F_{S12} - W_2 = 0$$

$$F_{S12} - m_2 g = 0$$

EXERCISE 12: Masses m_1 and m_2 are not attached to one another, but they are at rest in the positions shown. The level floor and the vertical wall offer static friction μ_{s1} and μ_{s2}, respectively. There is also static friction (μ_{s3}) between the two masses. Analyze all forces on each mass.

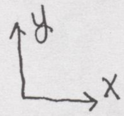


FBD of m_1



(1)

coord. axes



x-analysis

$$\sum F_x = m_1 a_x$$

$$F_{Sf1} - F_{N21} = 0$$

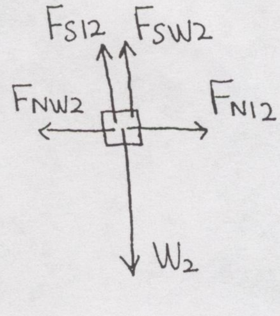
y-analysis

$$\sum F_y = m_1 a_y$$

$$F_{Nf1} - W_1 - F_{S21} = 0$$

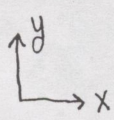
$$F_{Nf1} - m_1 g - F_{S21} = 0$$

FBD of m_2



(1)

coord. axes



x-analysis

$$\sum F_x = m_2 a_x$$

$$F_{N12} - F_{NW2} = 0$$

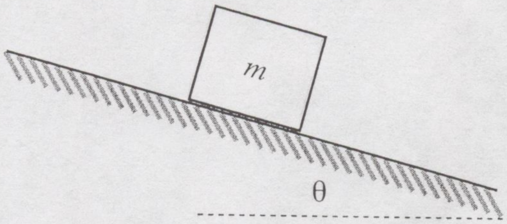
y-analysis

$$\sum F_y = m_2 a_y$$

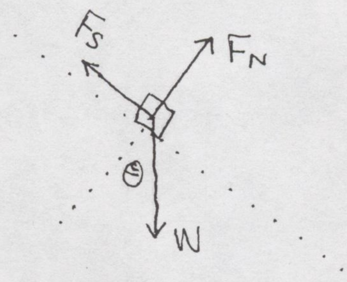
$$F_{S12} + F_{SW2} - W_2 = 0$$

$$F_{S12} + F_{SW2} - m_2 g = 0$$

EXERCISE 13: The block with mass m is at rest on the incline, which offers static friction (μ_s). Analyze all forces on the block.

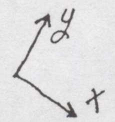


FBD of m



1

coord. axes



x-analysis

$\Sigma F_x = ma_x$

$W_x - F_s = 0$

$mg\sin\theta - F_s = 0$

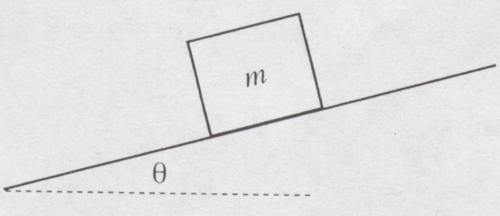
y-analysis

$\Sigma F_y = ma_y$

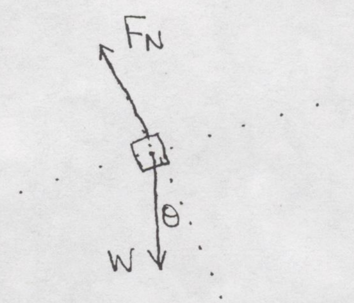
$F_N - W_y = 0$

$F_N - mg\cos\theta = 0$

EXERCISE 14: The block (m) is accelerating down the frictionless inclined surface. Analyze all forces on the block.

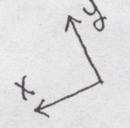


FBD of m



1

coord. axes



x-analysis

$\Sigma F_x = ma_x$

$W_x = ma$

$mg\sin\theta = ma$

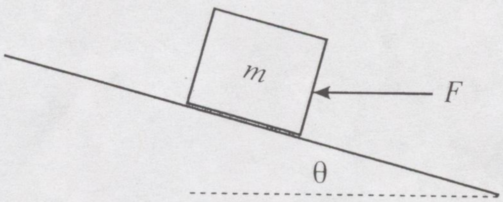
y-analysis

$\Sigma F_y = ma_y$

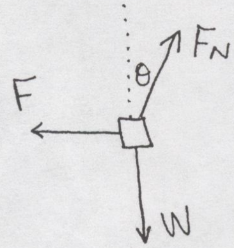
$F_N - W_y = 0$

$F_N - mg\cos\theta = 0$

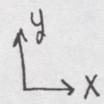
EXERCISE 15: The block with mass m is moving up the frictionless incline at a constant velocity. Analyze all forces on the block.



FBD of m



coord. axes



x-analysis

$\Sigma F_x = ma_x$

$F_{Nx} - F = 0$

$F_N \sin\theta - F = 0$

y-analysis

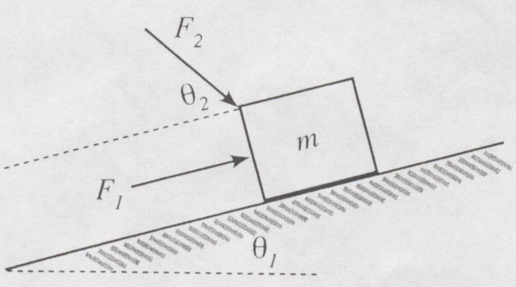
$\Sigma F_y = ma_y$

$F_{Ny} - W = 0$

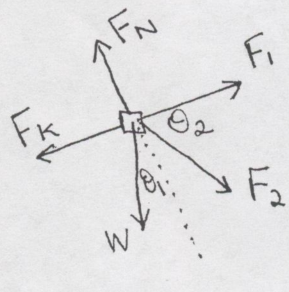
$F_N \cos\theta - mg = 0$

(1)

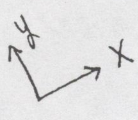
EXERCISE 16: The block (m) is accelerating up the inclined surface. The coefficient of kinetic friction between the block and the incline is μ_k. Analyze all forces on the block.



FBD of m



coord. axes



x-analysis

$\Sigma F_x = ma_x$

$F_1 + F_{2x} - F_K - W_x = ma$

$F_1 + F_2 \cos\theta_2 - \mu_k F_N - mg \sin\theta_1 = ma$

y-analysis

$\Sigma F_y = ma_y$

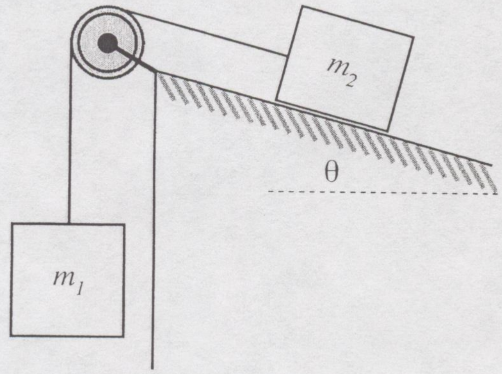
$F_N - W_y - F_{2y} = 0$

$F_N - mg\cos\theta_1 - F_2 \sin\theta_2 = 0$

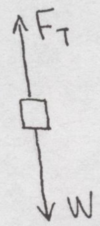
(1)

18

EXERCISE ~~18~~ 17: Blocks with masses m_1 and m_2 are connected by a string (with negligible mass) that passes over a frictionless pulley. The coefficient of kinetic friction between m_2 and the incline is μ_k. The masses are accelerating as one unit—the string is taut at all times. Analyze all forces on each mass.

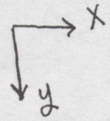


FBD of m_1



(1)

coord. axes

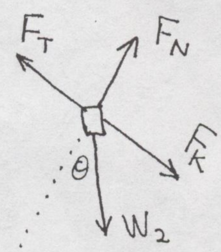


x-analysis

y-analysis

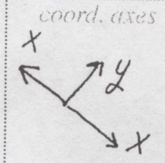
$$\Sigma F_y = m_1 a_y$$
$$W_1 - F_T = m_1 a$$
$$m_1 g - F_T = m_1 a$$

FBD of m_2



(1)

coord. axes



x-analysis

$$\Sigma F_x = m_2 a$$
$$F_T - F_K - W_{2x} = m_2 a$$
$$F_T - \mu_K F_N - m_2 g \sin\theta = m_2 a$$

y-analysis

$$\Sigma F_y = m_2 a_y$$
$$F_N - W_{2y} = 0$$
$$F_N - m_2 g \cos\theta = 0$$