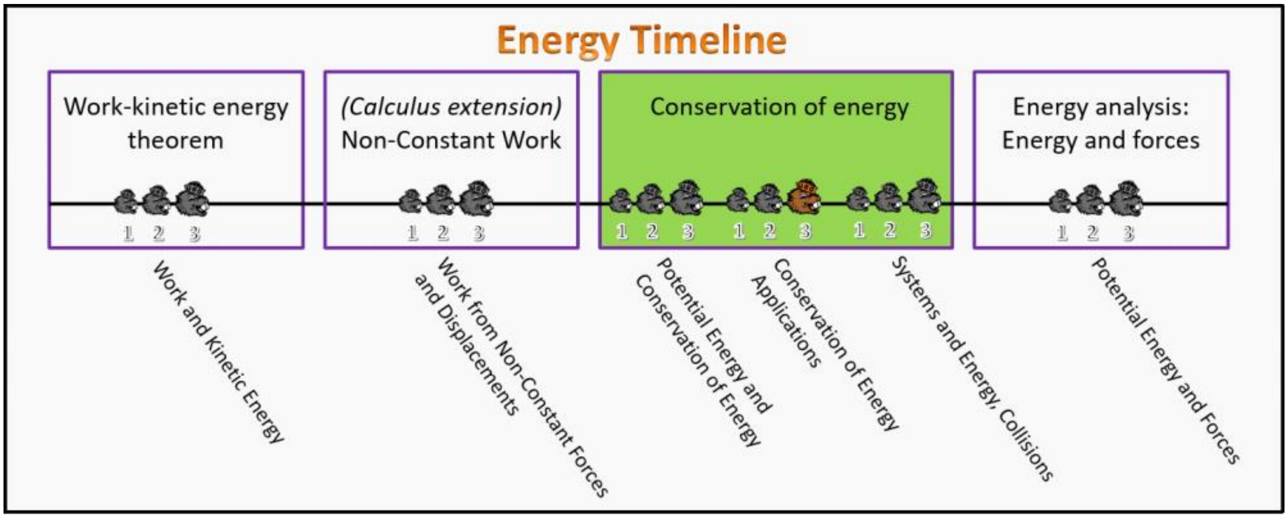
(CE.L2.3-sols) Practice Stage Solutions

Thursday, March 29, 2018 8:34 PM

Conservation of Energy
Foundation Stage (CE.L2.3)

Post-Lecture 2
Conservation of Energy Applications



Questions

CE.L2.3-01

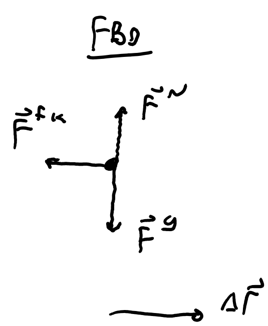
Description: Conservation of energy calculations with non-conservative forces

Learning Objectives: [x]

Problem Statement: A cardboard box of unknown mass is sliding upon a horizontal frictionless surface. The box has a velocity of 4.56 m/s when it encounters friction. After sliding 0.700 m, the box has a velocity of 3.33 m/s. What is the coefficient of friction of the surface?

(1) 0.0707
(2) 0.141
(3) 0.707
(4) 1.41

System m + surface



$$KE_i + E_i^{th} + \cancel{\sum W_{ext}}^{0} = KE_f + E_f^{th}$$

$$KE_i = \Delta E^{th,f} + KE_f$$

$$\frac{1}{2}mv_i^2 = -F^{f_k}\Delta r\,\cancel{\cos(180)}^{-1} + \frac{1}{2}mv_f^2$$

$$\frac{1}{2}mv_i^2 = \mu_k F^N \Delta r + \frac{1}{2}mv_f^2$$

$$\frac{1}{2}\cancel{m}v_i^2 = \mu_k \cancel{m}g\Delta r + \frac{1}{2}\cancel{m}v_f^2$$

$$\frac{1}{2}\cancel{m}v_i^2 = \mu_k \cancel{m}g\Delta r + \frac{1}{2}\cancel{m}v_f^2$$

$$\frac{1}{2}v_i^2 = \mu_k g\Delta r + \frac{1}{2}v_f^2$$

$$\frac{\frac{1}{2}v_i^2 - \frac{1}{2}v_f^2}{g\Delta r} = \mu_k$$

$$\frac{\frac{1}{2}(4.56)^2 - \frac{1}{2}(3.33)^2}{(9.8)(0.7)} = \mu_k$$

$$\boxed{\mu_k = 0.707}$$

CE.L2.3-02

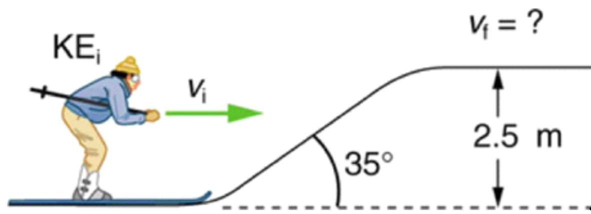
Description: Conservation of energy calculations with non-conservative forces

Learning Objectives: [x]

Problem Statement: A 60.0-kg skier with an initial speed of 12.0 m/s coasts up a 2.50-m-high rise as shown in the figure. Find her final speed at the top, given that the coefficient of friction between her skis and the snow is 0.0800. (**Hint**: *Find the distance traveled up the incline assuming a straight-line path as shown in the figure.*)

(1) 9.46 m/s
(2) 8.42 m/s
(3) 9.75 m/s
(4) 9.58 m/s

Energy System: Person + Hill



$y=0$

$$KE_i + E^{TH}_i + \cancel{U^g_i}^{0} + \cancel{W_{EXT}}^{0} = KE_f + E^{th}_f + U^g_f$$

$$KE_i = KE_f + \Delta E^{TH} + U^g_f$$

$$\frac{1}{2}mv_i^2 = \frac{1}{2}mv_f^2 - F^{fk}\Delta r \cancel{\cos\theta_f}^{-1} + mgy_f$$

$$\frac{1}{2}mv_i^2 = \frac{1}{2}mv_f^2 + \mu_k F^N\Delta r + mgy_f$$

$$\frac{1}{2}\cancel{m}v_i^2 = \frac{1}{2}\cancel{m}v_f^2 + \mu_k \cancel{m}g\cos\theta\Delta r + \cancel{m}gy_f$$

$$\frac{1}{2}v_i^2 = \frac{1}{2}v_f^2 + \mu_k g\cos\theta\Delta r + gy_f$$

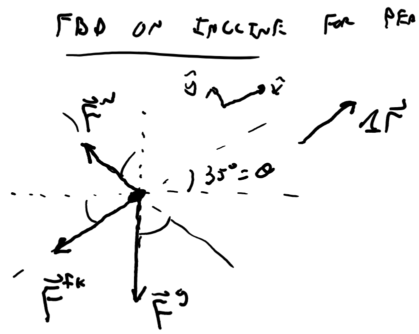
$$\frac{1}{2}v_i^2 = \frac{1}{2}v_f^2 + \mu_k g\frac{\cos\theta\, y_f}{\sin\theta} + gy_f$$

$$\frac{1}{2}v_i^2 = \frac{1}{2}v_f^2 + \frac{\mu_k g y_f}{\tan\theta} + gy_f$$

$$v_i^2 = v_f^2 + 2gy_f\left(\frac{\mu_k}{\tan\theta} + 1\right)$$

$$v_f = \sqrt{v_i^2 - 2gy_f\left(\frac{\mu_k}{\tan\theta} + 1\right)}$$

FBD on incline for person



$$\Sigma F_y = m\cancel{a_y}^{0}$$

$$F^N - mg\cos\theta = 0$$

$$F^N = mg\cos\theta$$

Geometry

$$\sin\theta = \frac{y_f}{\Delta r}$$

$$\Delta r = \frac{y_f}{\sin\theta}$$

$$v_f = \sqrt{v_i^2 - 2 g y_f \left(\frac{\mu_k}{\tan\theta} + 1 \right)}$$

$$v_f = \sqrt{12^2 - 2(9.8)(2.5)\left(\frac{0.08}{\tan 35} + 1 \right)}$$

$$\boxed{v_f = 9.455 \text{ m/s}}$$

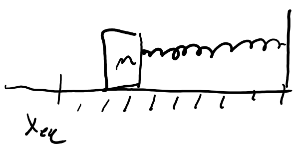
CE.L2.3-03

Description: Block slides into spring with friction

Learning Objectives: [x]

Problem Statement: A 1.80-kg block slides on a rough, horizontal surface. The block hits a spring with a speed of 2.00 m/s and compresses it a distance of 11 cm before coming to rest. The coefficient of kinetic friction between the block and the surface is 0.56, what is the force constant of the spring?

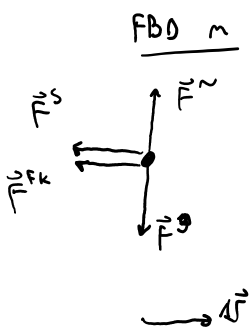
(1) 112 N/m
(2) 274 N/m
(3) 310 N/m
(4) 415 N/m
(5) 568 N/m



Energy system: m + surface + spring

$$KE_i + \cancel{U_i^s} + E_i^{th} + \cancel{\Sigma W_{ext}} = \cancel{KE_f} + U_f^s + E_f^{th}$$

FBD m



$$KE_i = U_f^s + \Delta E^{th,sf}$$

$$\frac{1}{2} m v_i^2 = \frac{1}{2} k x_f^2 - F^{fk} \Delta r \cos(180)$$

$$\frac{1}{2} m v_i^2 = \frac{1}{2} k x_f^2 + \mu_k F^N x_f$$

$$\frac{1}{2} m v_i^2 = \frac{1}{2} k x_f^2 + \mu_k m g x_f$$

$$\frac{1}{2}(1.8)(2)^2 = \frac{1}{2} k (.11)^2 + (0.56)(1.8)(9.8)(.11)$$

$$k = 415.4 \ \frac{N}{m}$$

CE.L2.3-04

Description: Calculate final displacement using conservation of energy for a system with a falling object attached to a bungee cord.

Learning Objectives: [x]

Problem Statement: A 85 kg student stands on a bridge with a 12-m-long bungee cord tied to her feet. You can assume that the bungee cord is massless and has a spring constant of 250 N/m. The student jumps off the bridge and falls until the bungee cord is fully stretched, where she comes to a stop. How far below the bridge does the student fall before coming to a stop?

(1) 12 m
(2) 16 m
(3) 20 m
(4) 25 m
(5) 32 m
(6) 40 m

$$\cancel{KE_i}^{\,0} + U^g_i + \cancel{U^s_i}^{\,0} + \cancel{E_{ext}}^{\,0} = \cancel{KE_f}^{\,0} + \cancel{U^g_f}^{\,0} + U^s_f$$

$$U^g_i = U^s_f$$

$$mgy_i = \frac{1}{2}kx_f^2$$

$$mgd = \frac{1}{2}k(d-L)^2$$

$$mgd = \frac{1}{2}k(d^2 - 2Ld + L^2)$$

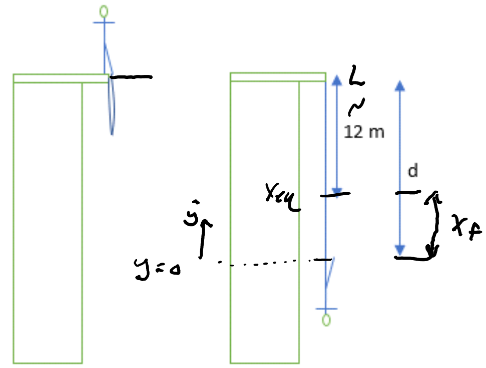
$$mgd = \frac{1}{2}kd^2 - kLd + \frac{1}{2}kL^2$$

$$0 = \frac{1}{2}kd^2 - kLd - mgd + \frac{1}{2}kL^2$$

$$0 = \underbrace{\frac{1}{2}k}_{a}d^2 \underbrace{-(kL+mg)}_{b}d + \underbrace{\frac{1}{2}kL^2}_{c}$$

$$0 = 125d^2 - 3833d + 18000$$

$$d = \underline{\underline{24.9\ m}} \quad \text{or} \quad 5.79\ m$$



CE.L2.3-05

Description: Energy and position plots

Learning Objectives: [x]

Problem Statement: The figure shows several plots, one of which labeled as representing the position of a mass on a frictionless surface, connected to a horizontal spring that is vibrating back and forth. The other two plots are number **I** and **II**. Which of the following statements are true regarding these plots and this situation?

(1) Plot **I** could represent the mass's velocity as a function of time.

(2) Plot **I** represents the mass's acceleration as a

function of time.

(3) Plot **II** represents the mass's acceleration as a function of time.

(4) The mass has the greatest amount of potential energy at points **A**, **C**, and **E**.

(5) The mass has the greatest amount of kinetic energy at points **A**, **C**, and **E**.

(6) The potential energy of the mass is maximum when its magnitude of acceleration is greatest.

(7) The kinetic energy of the mass is greatest when its position is zero.