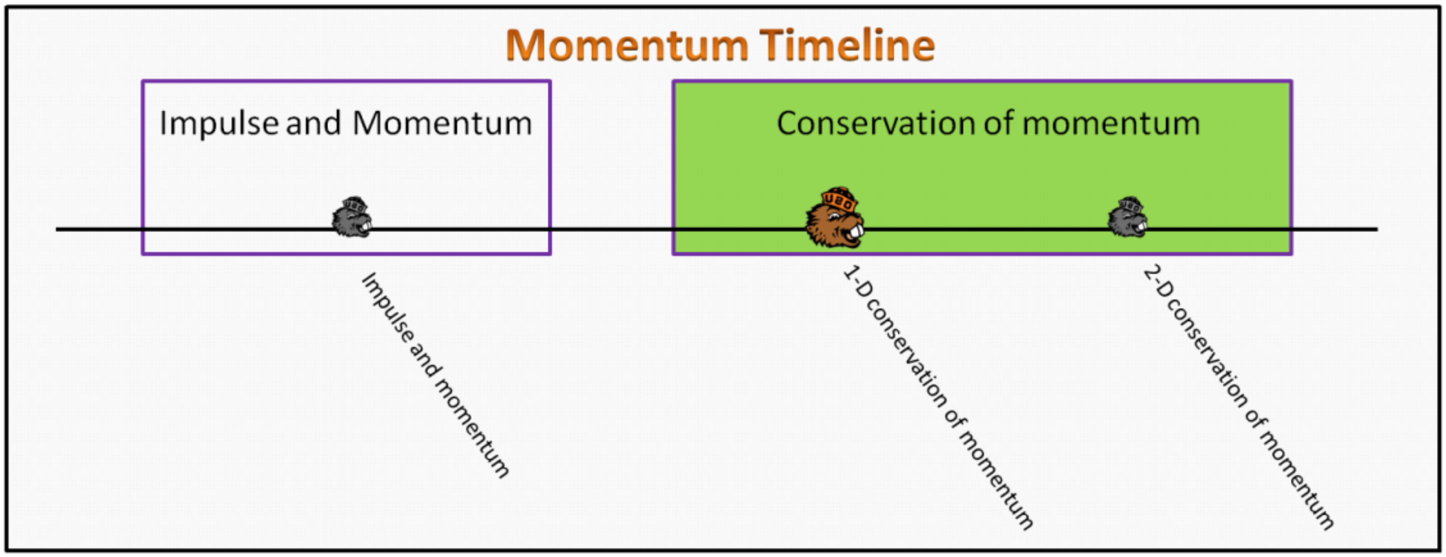
CM.2.L1.sols: 1-D Conservation of Momentum

Monday, January 22, 2018 5:44 PM

Conservation of momentum
Foundation Stage (CM.2.L1)

lecture 1
1-D conservation of momentum



Textbook Chapters

- **BoxSand** :: KC videos (Conservation Of Momentum)
- **Giancoli** (Physics Principles with Applications 7th) :: 7-2 ; 7-6 ; 7-7
- **Knight** (College Physics : A strategic approach 3rd) :: 9.4 ; 9.5 ; 9.6
- **Knight** (Physics for Scientists and Engineers 4th) :: 11.2 ; 11.3 ; 11.4 ; 11.5

Warm up

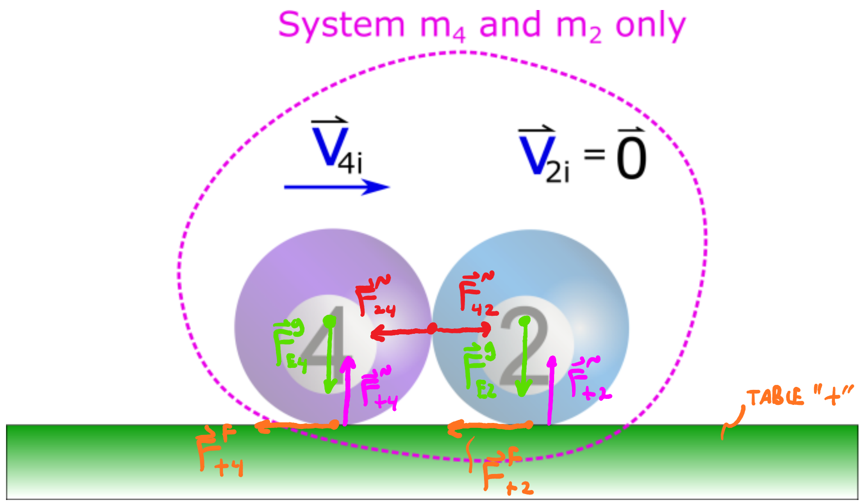
CM.2.L1-1:

Description: Calculate momentum of a single object.

Learning Objectives: [?] - Can you identify the objectives from the previous lecture, and this lecture, that this question is relevant to?

Problem Statement: Consider a two billiard balls: ball 4 is initially moving to the right when it collides with a stationary ball 2. The moment they collide is shown in the image below. One possible system can be defined as containing both billiard balls only.

(a) On the image, draw all of the forces acting on each individual billiard ball; place the tail of each force vector at the location the force is acting on. Gravitational forces can be thought of as acting at the center of each billiard ball.



(b) Are there any external forces acting on the ball 4 + ball 2 system? If so, list them below.

YES ... SINCE EARTH + TABLE ARE NOT PART OF OUR SYSTEM, THEN THOSE INTERACTIONS W/ m_4 + m_2 ARE EXTERNAL.

$$\boxed{\vec{F}^N_{+4} \quad \vec{F}^N_{+2} \quad \vec{F}^g_{E4} \quad \vec{F}^g_{E2} \quad \vec{F}^f_{+4} \quad \vec{F}^f_{+2}}$$

(c) Are there any internal forces acting within the ball 4 + ball 2 system? If so, list them below.

YES... SINCE M_4 AND M_2 ARE IN OUR SYSTEM, THEN THE INTERACTIONS BETWEEN M_4 AND M_2 ARE INTERNAL

$$\underbrace{\boxed{\vec{F}^N_{42} \quad \vec{F}^N_{24}}}_{\text{NOT INCLUDED IN } \Sigma \vec{F}_{EXT}}$$

RECALL...

$$\Sigma \vec{F}_{EXT} \Delta t = \Delta \vec{P}_{SYSTEM}$$

$$\left(\underbrace{\vec{F}^N_{+4} + \vec{F}^g_{E4}}_{\text{CANCEL}} + \underbrace{\vec{F}^N_{+2} + \vec{F}^g_{E2}}_{\text{CANCEL}} + \vec{F}^f_{+2} + \vec{F}^f_{+4}\right) \Delta t = \Delta \vec{P}_{SYS}$$

$$\underbrace{\left(\vec{F}^f_{+2} + \vec{F}^f_{+4}\right)}_{\text{BUT VERY SMALL FOR BILLIARD BALLS}} \Delta t = \Delta \vec{P}_{SYS}$$

SO ... $\boxed{\vec{0} \approx \Delta \vec{P}_{SYSTEM}}$

Selected Learning Objectives

1. Identify collisions in physical phenomena.
2. Define that a quantity is conserved when the change in that quantity is zero.
3. Identify whether the forces are internal or external to a system and if the net external force is zero.
4. Show that momentum is conserved for systems where the net external force is zero.

5. (UPMF) Justify that momentum conservation can be assumed when the impulse on the system is negligible.
6. Draw an appropriate physical representation including the initial and final momentum vectors and a wise coordinate system.
7. Draw a vector operation diagram with initial, final, and change in momentum vectors.
8. Apply a 1-D momentum analysis in the mathematical representation when appropriate.
9. Construct, in the mathematical representation, a conservation of momentum vector equation in 2-D.
10. Determine when momentum is conserved in one direction but not another.

Key Terms

- Conservation of momentum
- Isolated
- Collision
- Explosion
- Center of mass
- Zero impulse approximation

Key Equations

Change in momeuntum of the system

Null vector

$$\Delta\vec{p}_{system} = \vec{0}$$

In words: The change in momentum of an isolated system is equal to the null vector (i.e. <0,0,0>).

Equivalent definitions

<--------------------->

Initial momeuntum

Final momeuntum

$$\Sigma\vec{p}_i = \Sigma\vec{p}_f$$

Net (i.e. add up all of the __)

In words: The net initial momentum of a system is equal to the net final momentum of the system.

Key Concepts

- If momentum is conserved, then the initial momentum vector of the system will have the same length and point in the same direction as the final momentum of the system.
- When there is no net external force on a system, the center of mass of that system will have an acceleration of zero.
- If there is a non-zero net external force on the system, then momentum is not conserved for that system.
- An isolated mechanical system is a system with no external forces, or a system where the external forces add up to zero.
- The center of mass of an object/system is the point that the object/system will rotate around when there is no net force acting on the object/system. Mathematically, the center of mass is a mass-weighted average of the geometric center of an object/system. Mass-weighted average of the geometric center what??...For example, if two equal masses are located a distance d apart, the center of mass is at the geometric center, d/2. Now if the object on the left is more massive, the center of mass is closer to the left object.
- Almost no system is truly isolated, but a lot of systems satisfy the zero impulse approximation which states that if the impulse on a system is very small, then momentum is conserved immediately before and immediately after a collision/explosion.

Act I: 1-D conservation of momentum

Questions

CM.2.L1-2:

Description: Identify collisions and explosions that are candidates for a conservation of momentum analysis. (3 minutes)

Learning Objectives: [1]

C E

Problem Statement: Which of the following scenarios describes a collision or explosion, and thus a good candidate for a conservation of momentum analysis?

C (1) A billiard ball strikes another billiard ball.
C (2) A baseball bat hits a baseball.

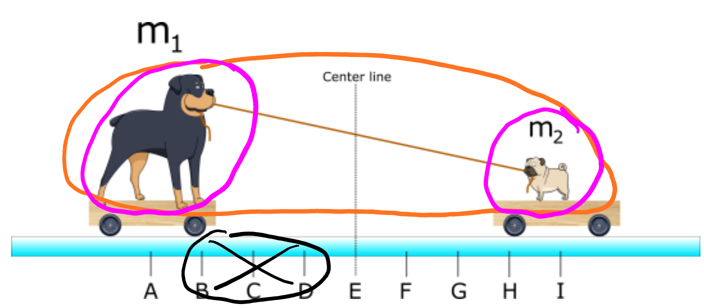
C (1) A billiard ball strikes another billiard ball.
C (2) A baseball bat hits a baseball.
E (3) A firecracker explodes into multiple pieces.
C (4) An asteroid passes by the earth close enough that the trajectory of the asteroid changes due to gravitational effects.
E (5) A person standing on ice throws a heavy ball away from themselves.
C (6) A person gets a running start and jumps onto a stationary skateboard.

CM.2.L1-3:

Description: Rank final speeds given a system at rest and relative masses of objects. (6 minutes + 2 minutes + 4 minutes)

Learning Objectives: [1,2,3,4]

Problem Statement: Two dogs are playing tug-of-war with each other on low friction carts as shown in the image below. They quickly pull on the rope connecting them.



(a) Rank the final speed of each dog.

(1) $v_{1f} > v_{2f}$
(2) $v_{1f} < v_{2f}$
(3) $v_{1f} = v_{2f}$

INDIVIDUAL SYSTEMS

FORCE ANALYSIS: $\Sigma \vec{F}_{Ext} = m\,\vec{a}_{cm}$

$\Sigma \vec{F}_1 = -\Sigma \vec{F}_2$ w/ $m_1 > m_2$

THEN $\vec{a}_{1cm} < \vec{a}_{2cm}$

IMPULSE-MOMENTUM ANALYSIS: $\Sigma \vec{F}_{Ext} \Delta t = \Delta \vec{P}_{sys}$

$\Sigma \vec{F}_1 = -\Sigma \vec{F}_2$ w/ $\Delta t_1 = \Delta t_2$

THEN $\Delta \vec{P}_1 = -\Delta \vec{P}_2$

AND $m_1 > m_2$

SO $v_1 < v_2$

SYSTEM $m_1 + m_2$

IMPULSE-MOMENTUM ANALYSIS: $\cancel{\Sigma \vec{F}_{Ext}}^{\,0} \Delta t = \Delta \vec{P}_{sys}$

$\Sigma \vec{F}_{Ext} = \vec{0}$ so... $\Sigma \vec{P}_i = \vec{P}_f$

$m_1 \cancel{\vec{v}_{1i}}^{\,\vec{0}} + m_2 \cancel{\vec{v}_{2i}}^{\,\vec{0}} = m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f}$

(b) Where is the center of mass of the two dogs located?

⊗

(c) After they pull on the rope connecting them, where do you expect them to meet? Talk to your neighbor about why you think they will meet at the location you picked.

@ CM B/C w/ $m_1 + m_2$ System

$\cancel{\Sigma \vec{F}_{Ext}}^{\,\vec{0}} = M_{sys} \vec{a}_{cm}$

$\vec{0} = M_{sys} \vec{a}_{cm}$

$\sum \vec{F}_{Ext} = \vec{0}$ so... $\sum P_i = P_f$

$$M_1 \cancel{\vec{V}_{1i}}^{\vec{0}} + M_2 \cancel{\vec{V}_{2i}}^{\vec{0}} = M_1 \vec{V}_{1f} + M_2 \vec{V}_{2f}$$

$$\vec{0} = M_1 \vec{V}_{1f} + M_2 \vec{V}_{2f}$$

$$\vec{V}_{2f} = -\underbrace{\left(\frac{M_1}{M_2}\right)}_{\text{LARGER THAN } 1} \vec{V}_{1f}$$

$$\vec{0} = M_{sys} \vec{a}_{cm}$$

$$\vec{a}_{cm} = \vec{0}$$

CM.2.L1-4:

Description: Conceptual question explaining an observation with known physics analysis tools. Apply 1-D conservation of momentum in the mathematical representation. (4 minutes + 5 minutes)

Learning Objectives: [1,3,4,8]

Problem Statement: D.O.G. the dog, starting from rest, walks to the right on a sled which is on top of slippery ice. Oskar is standing stationary off to the side observing this motion. Oskar notices that as D.O.G walks to the right, the sled moves to the left.

(a) Which of the following statements correctly describes the observation made by Oskar?

T (1) Since there is no net external force on the dog + sled system, the center of mass does not accelerate. Also, everything was initially at rest, thus the center of mass is not moving at all.

$$\cancel{\sum \vec{F}_{Ext}}^{0} = M \vec{a}_{cm}$$

$$\vec{a}_{cm} = \vec{0}$$

T (2) As the dog walks to the right, there must be a frictional force from the sled on the dog to the right. By Newton's 3rd law, there then is an equal in magnitude frictional force from the dog on the sled in the opposite direction, to the left.

$$\underbrace{\vec{F}^{f}_{sd}}_{\rightarrow} = -\underbrace{\vec{F}^{f}_{ds}}_{\leftarrow}$$

T (3) Since there is no net external force on the dog + sled system, momentum is conserved. Thus as the dog increases their momentum to the right, the sled must increase its momentum to the left to maintain the original zero momentum of the system.

$$\cancel{\sum \vec{F}_{Ext}}^{\vec{0}} \Delta t = \Delta \vec{P}_{sys}$$

$$\vec{0} = \Delta \vec{P}_{sys}$$

$$\Delta \vec{P}_d + \Delta \vec{P}_s = \vec{0}$$

$$\underbrace{\Delta \vec{P}_d}_{\rightarrow} = -\underbrace{\Delta \vec{P}_s}_{\leftarrow}$$

(b) The 30 kg dog and the 20 kg sled are initially at rest. The dog then begins to move to the right and Oskar measures the dog's velocity as 2 m/s in the positive x direction. Oskar forgot to measure the sled's velocity. What is the velocity of the sled that Oskar would have measured as the dog was moving?

SYSTEM $M_d + M_s$

$\Sigma \vec{F}_{EXT} = \vec{0}$

$\therefore \quad \Sigma \vec{P}_i = \Sigma \vec{P}_f$

$\Sigma \vec{P}_i = \Sigma \vec{P}_f$

$M_d \cancel{V_{idx}}^{0} + M_s \cancel{V_{isx}}^{0} = M_d V_{fdx} + M_s V_{fsx}$

$0 = M_d V_{fdx} + M_s V_{fsx}$

$V_{fsx} = -\dfrac{M_d}{M_s} V_{fdx} = -\dfrac{30}{20}(2\ m/s)$

$= \boxed{-3\ m/s}$

$\boxed{\vec{V}_{fs} = -3\ m/s\ \hat{x}}$

CM.2.L1-5:

Description: Rank the final speed of objects that stick after they collide. Apply 1-D conservation of momentum in the mathematical representation. (4 minutes + 5 minutes)

Learning Objectives: [3,6,8]

Problem Statement: Two hockey pucks are both moving to the right. Puck **1** catches up with puck **2** and collides with it. There is sticky tape on the first puck so after they collide they are stuck together and continue with a speed $\mathbf{v}_{12f}$.

(a) Which of the following statements are true regarding this scenario?

(1) $\mathbf{v}_{1f} = \mathbf{v}_{2i}$
(2) $\mathbf{v}_{12f}$ is less than $\mathbf{v}_{2i}$
(3) $\mathbf{v}_{12f}$ is greater than $\mathbf{v}_{2i}$, but less than $\mathbf{v}_{1i}$
(4) $\mathbf{v}_{12f} = \mathbf{v}_{1i}$
(5) $\mathbf{v}_{12f}$ is greater than $\mathbf{v}_{1i}$

Before collision $\hat{y}$ $\hat{x}$ After collision

$M_1 = M_2$

SYSTEM $m_1 + m_2$

$\Sigma \vec{F}_{EXT} = \vec{0}$

SO $\Sigma \vec{P}_i = \Sigma \vec{P}_f$

COM VECTOR OP

$M_1 = M_2$

$\vec{P}_{1i}$ $\vec{P}_{2i}$

ΣP_i

$\vec{P}_{1f}$ $\vec{P}_{2f}$

$\Sigma \vec{P}_f$

(b) Find the final speed of the stuck hockey pucks if the first puck had a speed of 10 m/s and the second puck had a speed of 3 m /s. Both pucks have a mass of 0.165 kg.

$$\Sigma \vec{P}_i = \Sigma \vec{P}_f$$

$$M_1 V_{1ix} + M_2 V_{2ix} = (M_1 + M_2) V_{12fx}$$

$$V_{12fx} = \frac{M_1 V_{1ix} + M_2 V_{2ix}}{(M_1 + M_2)}$$

$M_1 = M_2 \equiv M$

$$= \frac{M(V_{1ix} + V_{2ix})}{2M} = \frac{1}{2}(V_{1ix} + V_{2ix})$$

$$= \frac{1}{2}(10\,m/s + 3\,m/s)$$

$$V_{2f} = \boxed{6.5\ m/s}$$

CM.2.L1-6:

Description: Conceptual question regarding an object in free fall and momentum. (4 minutes + 2 minutes)

Learning Objectives: [2,3,4]

Problem Statement: An apple is in free-fall near the surface of the Earth.

(a) Which one of the following statements is true?

SYSTEM APPLE

F (1) The apple is in equilibrium. ~ $\Sigma \vec{F}_{EXT} = \vec{0}$
F (2) The momentum of the apple is conserved.
F (3) The apple is isolated. ~ $\Sigma \vec{F}_{EXT} = \vec{0}$
F (4) The impulse acting on the apple is zero.
F (5) The change in momentum of the apple is zero.
T (6) The change in momentum during the 1st second is the same as during the 2nd second.

FREE FALL... $\vec{a} = \vec{g}$ ← CONSTANT

$$\vec{g} = \frac{\Delta \vec{V}}{\Delta t}$$

$$\Delta \vec{V} = \Delta t\, \vec{g}$$

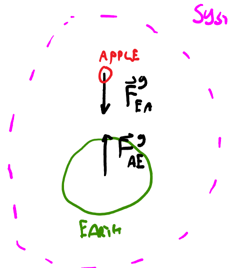
CONSTANT FOR EQUAL Δt

$$\Delta \vec{v} = \underbrace{\Delta t}_{\text{CONSTANT FOR EQUAL } \Delta t} \vec{g}$$

$$\Delta \vec{p} = m \Delta \vec{v} = \underbrace{m \Delta t}_{\text{CONSTANT FOR EQUAL } \Delta t} \vec{g}$$

(b) What can you do to make the momentum conserved?

(1) Change your system to the earth.
(2) Add the earth to your system.
(3) Let the apple be in free fall next to the moon where there is no atmosphere.
(4) Nothing can be done, the momentum is already conserved; it is always conserved.



SYSTEM E+A

$\sum \vec{F}_{EXT} = \vec{0}$

$\therefore \Delta \vec{P}_{SYS} = \vec{0}$

$\Delta \vec{P}_A + \Delta \vec{P}_E = \vec{0}$

$\Delta \vec{P}_A = \underbrace{-\Delta \vec{P}_E}$

EARTH "MOVES" TOWARDS APPLE!

BUT HOW MUCH?

$M_E >>>>> \cdots >> M_A$

SO

$V_E <<<< \cdots <<< V_A$

CM.2.L1-7:

Description: Apply 1-D conservation of momentum in physical representation. Apply 1-D conservation of momentum in mathematical representation. (3 minutes + 2 minutes + 6 minutes + 3 minutes + 5 minutes)

Learning Objectives: [7,11]

Problem Statement: A ferocious dragon of mass $\mathbf{m_A}$ rolls to the right with a speed of **v** when it collides with a dog of mass $\mathbf{m_B}$ traveling with the same speed in the opposite direction. The dragon is 3 times a massive as the dog. After the collision, the dragon's speed has decreased by a factor of 3. We eventually wish to determine the ratio of their final speeds after the collision.

(a) Which of the following represent a correct set of known relationships?

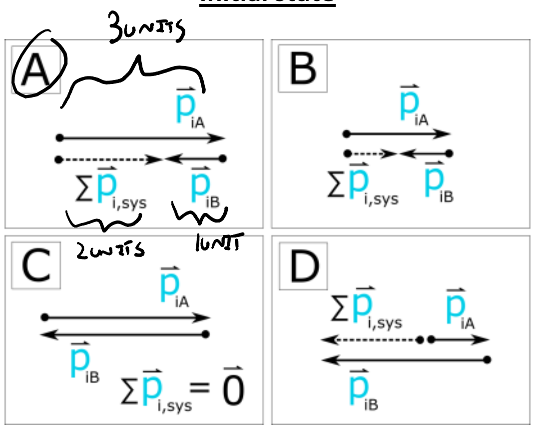
(b) Complete the physical representation of the initial and final states.

(1)	(2)
$3\,m_A = m_B$ $\vec{v}_{iA} = \vec{v}_{iB}$ $\vec{v}_{fA} = \frac{1}{3}\,\vec{v}_{iA}$	$m_A = 3\,m_B$ $\vec{v}_{iA} = \vec{v}_{iB}$ $\vec{v}_{fA} = \frac{1}{3}\,\vec{v}_{iA}$

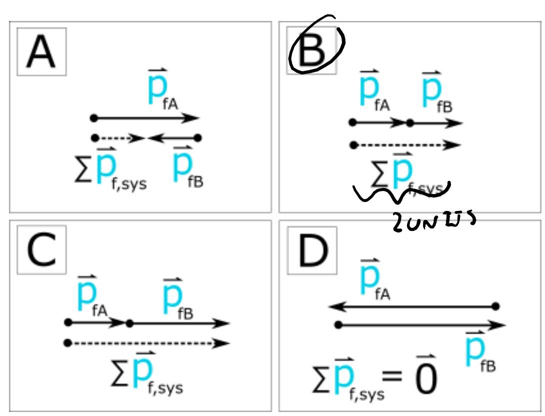
$\vec{v}_{fA} = \frac{1}{3}\,\vec{v}_{iA}$	$\vec{v}_{fA} = \frac{1}{3}\,\vec{v}_{iA}$
(3) $m_A = \frac{1}{3}\,m_B$ $\vec{v}_{iA} = -\,\vec{v}_{iB}$ $\vec{v}_{fA} = 3\,\vec{v}_{iA}$	(4) $m_A = 3\,m_B$ $\vec{v}_{iA} = -\vec{v}_{iB}$ $\vec{v}_{fA} = \frac{1}{3}\,\vec{v}_{iA}$

(c) Let's find the ratio of the final speeds using a physical representation. To do this, pick which of the following vector ope ration diagrams represent the initial and final states.

Initial state



Final state



(d) What is the ratio of the final speeds (**v_{fB}/v_{fA}**)?

(1) 1/3
(2) 1
(3) 3
(4) 9

$P_{fAx} = P_{fBx}$

$m_A V_{Af} = m_B V_{Bf}$

$\frac{V_{Bf}}{V_{Af}} = \frac{m_A}{m_B} = 3$

(e) Check that your answer to part **(d)** makes sense by reproducing the answer in the mathematical representation.

$\rightarrow \hat{x}$

$\Sigma P_{ix} = \Sigma P_{fx}$

$m_A V_{iA} + m_B V_{iB} = m_A V_{fA} + m_B V_{fB}$

$m_A = 3m_B$

$3\cancel{m_B} V_{iA} + \cancel{m_B} V_{iB} = 3\cancel{m_B} V_{fA} + \cancel{m_B} V_{fB}$

$3V_{iA} + V_{iB} = 3V_{fA} + V_{fB}$

$$3v_{iA} + v_{iB} = 3v_{fA} + v_{fB} \qquad v_{iA_x} = -v_{iB_x}$$

$$3v_{iA} - v_{iA} = 3v_{fA} + v_{fB}$$

$$2v_{iA} = 3v_{fA} + v_{fB} \qquad v_{fA} = \frac{1}{3} v_{iA}$$

$$2(3v_{fA}) = 3v_{fA} + v_{fB}$$

$$3v_{fA} = v_{fB}$$

$$\boxed{\frac{v_{fB}}{v_{fA}} = 3}$$

Act II: Breaking our idealizations

Zero impulse approximations

If $\Delta t \approx 0$

$$\underbrace{\underbrace{\underbrace{\sum \vec{F}_{ext}\,\Delta t = \Delta \vec{p}_{sys}}_{\vec{0} \approx \Delta \vec{p}_{sys}}}_{\sum \vec{p}_i \approx \sum \vec{p}_f}}$$

If $\sum \vec{F}_{ext} \approx \vec{0}$

$$\underbrace{\underbrace{\sum \vec{F}_{ext}\,\Delta t = \Delta \vec{p}_{sys}}_{\vec{0} \approx \Delta \vec{p}_{sys}}}_{\sum \vec{p}_i \approx \sum \vec{p}_f}$$

If $\Delta t \approx 0$ and $\sum \vec{F}_{ext} \approx \vec{0}$

$$\underbrace{\underbrace{\sum \vec{F}_{ext}\,\Delta t = \Delta \vec{p}_{sys}}_{\vec{0} \approx \Delta \vec{p}_{sys}}}_{\sum \vec{p}_i \approx \sum \vec{p}_f}$$

CM.2.L1-8:

Description: Identify systems with negligible impulse. (Infinity minutes)

Learning Objectives: [3, 5]

Problem Statement: In each scenario below, determine if the system is isolated or not. If the system is not isolated identify the external forces, and also identify if one of the zero impulse approximations be invoked to claim that momentum is approximately conserved. The first row is filled out as an example.

Scenario	System	Isolated (yes/no)	External forces	Zero impulse approximation
Two billiard balls collide.	Both billiard balls.	No	Gravity and normal force from table on balls, but they cancel each other out. Also friction from table on balls which do not cancel out.	Δt≈0 and $\Sigma\vec{F}_{ext} \approx \vec{0}$
Two cars collide.	Both cars.	No	$F^g + F^N$ BUT THEY CANCEL F^f	$\Delta t \approx 0$
A person gets a running start and jumps on a stationary skateboard.	Person and skateboard.	NO	$F^g + F^N$ BUT THEY CANCEL F^f	$\Delta t \approx 0$ $\Sigma\vec{F}_{Ext} \approx \vec{0}$
Two asteroids in our solar system collide.	Both asteroids.	NO	F^G FROM OTHER PLANETS AND SUN	$\Sigma\vec{F}_{EXT} \approx \vec{0}$
A person standing in a boat begins to walk to the front of the boat.	Both the person and the boat.	NO	$F^g + F^B$ BUT THEY CANCEL F^D	$\Sigma\vec{F}_{EXT} \approx \vec{0}$ BUT DEPENDS ON BOAT
A train moving at some constant velocity along straight tracks is slowly filled with water as it rains.	Train and water from rain.	NO	$F^N + F^g$ BUT THEY CANCEL F^f	$\Sigma\vec{F}_{EXT} \approx \vec{0}$
A fire cracker on its way upwards suddenly explodes into 3 pieces.	Firecracker.	NO	$F^g + F^D$	$\Delta t \approx 0$
A tennis player hits a tennis ball with a tennis racket.	Tennis racket and tennis ball.	NO	F^D ON BALL F^N FROM HAND ON RACKET	$\Delta t \approx 0$

Conceptual questions for discussion

1. Can you always use conservation of momentum for collisions? If so, why? If not, provide an example of a collision that you cannot use conservation of momentum for.
2. Consider a system consisting of 4 objects that all collide at some point while there are no external forces acting on them. Is the individual momentum of each object conserved?
3. If momentum is conserved, why do objects eventually stop moving?
4. Consider filling up a balloon but not tying up the open end, just pinch it with your fingers. When you release your fingers, why does

the balloon begin to fly around the room?
5. Explain why you are propelled forwards when swimming using both a force analysis and a momentum analysis.
6. If you are floating out in space, far away from gravitational bodies, can you change your own momentum? If you can, provide an example of what you could do. If you can't, explain why.

Hints

CM.2.L1-1: Remember that external forces are interactions with objects that are not included with the system.

CM.2.L1-2: No hints.

CM.2.L1-3: No hints.

CM.2.L1-4: No hints.

CM.2.L1-5: Try sketching a vector operation diagram for the initial and final momentum. Remember that if momentum is conserved, the initial momentum vector representing the system must be the same length and in the same direction of the final momentum vector representing the system.

CM.2.L1-6: No hints.

CM.2.L1-7: Remember that if momentum is conserved, the initial momentum vector representing the system must be the same length and in the same direction of the final momentum vector representing the system.

CM.2.L1-8: No hints.