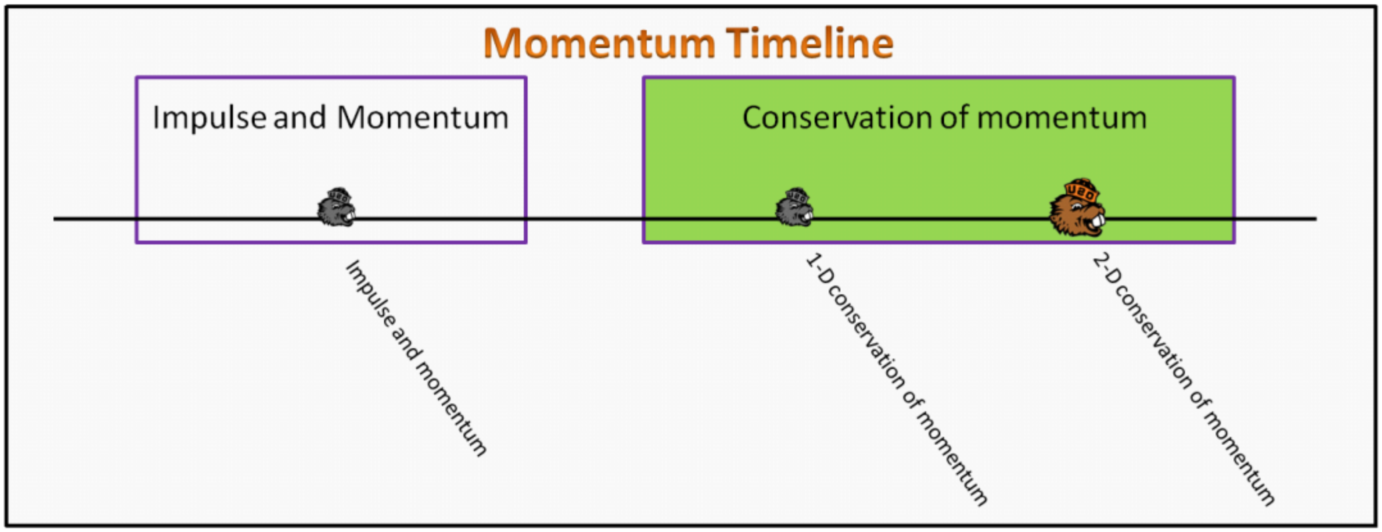
CM.2.L2.sols: 2-D Conservation of Momentum

Monday, January 22, 2018 5:44 PM

Conservation of momentum
Foundation Stage (CM.2.L2)

lecture 2
2-D conservation of momentum



Textbook Chapters

- **BoxSand** :: KC videos (conservation of momentum)
- **Giancoli** (Physics Principles with Applications 7th) :: 7-7
- **Knight** (College Physics : A strategic approach 3rd) :: 9.6
- **Knight** (Physics for Scientists and Engineers 4th) :: 11.5

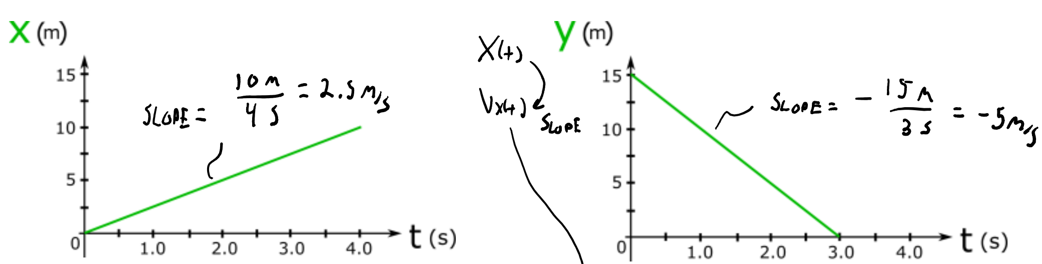
Warm up

CM.2.L2-1:

Description: Calculate momentum of a single object given two position vs time graphs.

Learning Objectives: [?] - Can you identify the objectives from the previous lecture, and this lecture, that this question is relevant to?

Problem Statement: An 8 kg object is floating through space. Below are the position vs time graphs for this object. What is its momentum?



$\vec{p} = m\vec{v}$

$\vec{p} = m\langle v_x, v_y \rangle$

$$\vec{p} = m\langle v_x, v_y \rangle$$

$$= 8\,\text{kg}\,\langle 2.5\,\tfrac{m}{s}, -5\,\tfrac{m}{s} \rangle$$

$$\boxed{\vec{p} = \langle 20, -40 \rangle\,\tfrac{\text{kg}\,m}{s}}$$

Selected Learning Objectives

1. Identify collisions in physical phenomena.
2. Define that a quantity is conserved when the change in that quantity is zero.
3. Identify whether the forces are internal or external to a system and if the net external force is zero.
4. Show that momentum is conserved for systems where the net external force is zero.
5. (UPMF) Justify that momentum conservation can be assumed when the impulse on the system is negligible.
6. Draw an appropriate physical representation including the initial and final momentum vectors and a wise coordinate system.
7. Draw a vector operation diagram with initial, final, and change in momentum vectors.
8. Apply a 1-D momentum analysis in the mathematical representation when appropriate.
9. Construct, in the mathematical representation, a conservation of momentum vector equation in 2-D.
10. Determine when momentum is conserved in one direction but not another.

Key Terms

- No new key terms.

Key Equations

Change in momeuntum of the system

Null vector

$$\Delta\vec{p}_{\text{system}} = \vec{0}$$

In words: The change in momentum of an isolated system is equal to the null vector (i.e. <0,0,0>).

Equivalent definitions

<--------------------->

Initial momeuntum

Final momeuntum

$$\Sigma\vec{p}_i = \Sigma\vec{p}_f$$

Net (i.e. add up all of the __)

In words: The net initial momentum of a system is equal to the net final momentum of the system.

Key Concepts

- Momentum is a vector, thus you must split up the net initial and final momentum into x and y components when applying conservation of momentum.
- Momentum isn't always conserved in both x and y directions, thus you must identify which direction momentum is conserved in and only apply conservation of momentum in that direction. Drawing a physical representation of the net initial and final momentum vectors is a good way to see if the momentum is conserved in both coordinate directions.

Act I: 2-D conservation of momentum

Questions

CM.2.L2-2:

Description: Identify vector nature of mathematical representation. (2 minutes)

Learning Objectives: []

Problem Statement: When there is no net external force on a system, the momentum of the system is conserved. The mathematical representation of this statement is show below. How many equations is the mathematical representation of conservation of momentum?

$$\Sigma\vec{p}_i = \Sigma\vec{p}_f$$

(1) 1
(2) 2
(3) 3
(4) 4

$\Sigma P_{ix} = \Sigma P_{fx}$ $\quad$ $\Sigma P_{iy} = \Sigma P_{fy}$ $\quad$ $\Sigma P_{iz} = \Sigma P_{fz}$

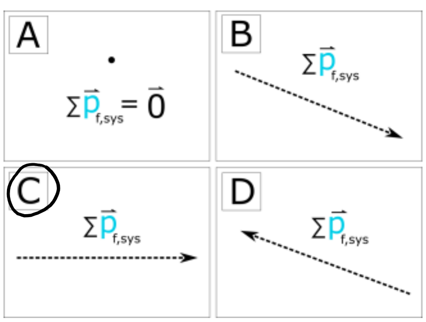
CM.2.L2-3:

Description: Find the final speed of a piece of an object that undergoes an explosion in 2-D. (2 minutes + 4 minutes + 3 minutes + 5 minutes)

Learning Objectives: [2, 6, 7, 9]

Problem Statement: Benny the beaver is riding his spacescooter and the total momentum of Benny and his scooter is 3,000 N·s in the positive x-direction. The antimatter reactor on the scooter fails, blowing the scooter into two pieces. Once piece of the scooter is shot directly backwards in the - x-direction with a momentum of 1,000 N·s. The second piece of the scooter is flung directly in the positive y-direction with a momentum of 3,000 N·s. We wish to find poor Benny's momentum when he is thrown off the scooter after the explosion.

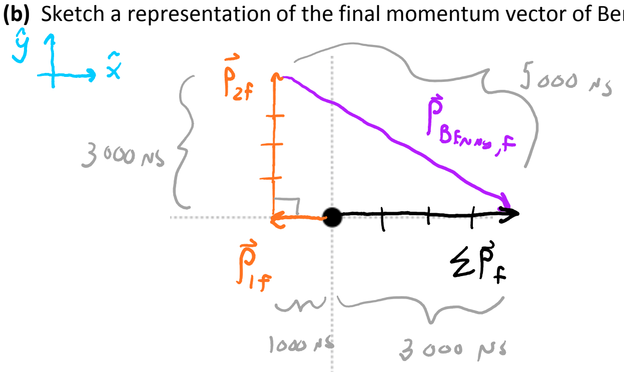
(a) Which of the following vectors could represent the final total momentum of the Benny + spacescooter system?



w/ $\Sigma\vec{F}_{ext} = \vec{0}$

$\Sigma\vec{P}_i = \Sigma\vec{P}_f$

(b) Sketch a representation of the final momentum vector of Benny



(c) What is the magnitude of Benny's final momentum?

3-4-5 :) $\quad$ 5000 NS $= |\vec{P}_{FB}|$

(d) The original mass of the Benny + Spacescooter system was 125 kg. Scooter chunk 1, which was thrown backwards, has as mass of 95 kg. Scooter chunk 2, which was thrown upwards, has a mass of 10 kg. What is the final speed of Benny after the explosion?

$\vec{P}_B = m_B \vec{V}_B$

$|\vec{P}_{fB}| = m_B |\vec{V}_{Bf}|$

$M_{total} = m_1 + m_2 + m_B$

$125\ kg = 95\ kg + 10\ kg + m_B$

$m_B = 20\ kg$

$$|\vec{V}_{Bf}| = \frac{|\vec{P}_{FB}|}{m_B}$$

$$= \frac{5000\ Ns}{20\ kg}$$

$$\boxed{|\vec{V}_{FB}| = 250\ m/s}$$

YIKES!! ... THAT'S FAST.

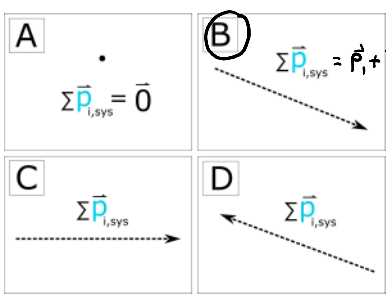
CM.2.L2-4:

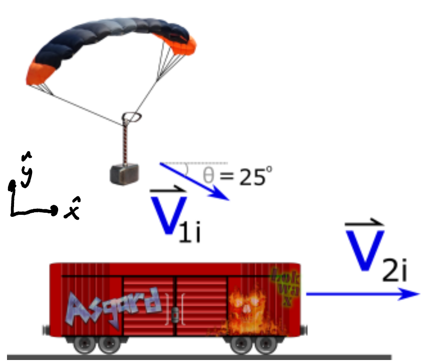
Description: Determine the mass of an object that undergoes a collision in 2-D. (2 minutes + 2 minutes + 4 minutes + 8 minutes)

Learning Objectives: [2, 6, 7, 9, 10]

Problem Statement: A train cart with a mass of 5000 kg is rolling at a speed of 12 m/s on a horizontal track as shown in the figure below. A payload attached to a worthy parachute is dropped from a plane and is descending at a speed of 8 m/s at an angle of 25 degrees with respect to the horizontal. The payload lands on the train and the resulting velocity of train and payload is 10 m/s horizontally to the right in the positive x-direction.

(a) Which of the following vectors could represent the initial momentum of the payload + train system?





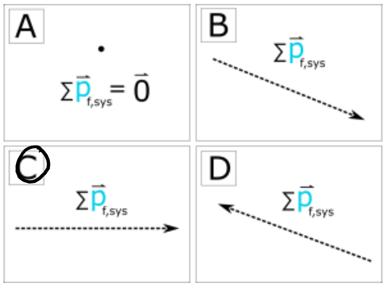
(b) Which of the following vectors could represent the final momentum of the payload + train system?

$\vec{V}_{12f}$ ⟶

(c) The net momentum of the payload + train system appears to be down and to the right before the coal lands in the cart and towards the right after. Which of the following statements about the payload + train system can account for this phenomena?

F (1) The momentum is conserved.

train system? $V_{12f} \longrightarrow$



after. Which of the following statements about the payload + train system can account for this phenomena?

F (1) The momentum is conserved.
F (2) The system is isolated.
T (3) The momentum is not conserved.
T (4) The system is not isolated.
T (5) The system is not large enough to justify conservation of momentum.

$\Sigma P_{y_i} \neq \Sigma P_{f_y}$

B/c Earth...

OK ... But $M_E >>> \dots >> M_{sys}$

So $V_{fy} \approx 0$

However... $\Sigma F_{x\,Ext} \approx 0$

So

$\Sigma P_{ix} = \Sigma P_{fx}$

(d) A train cart with a mass of 5000 kg is rolling at a speed of 12 m/s on a horizontal track as shown in the figure below. A payload attached to a worthy parachute is dropped from a plane and is descending at a speed of 8 m/s at an angle of 25 degrees with respect to the horizontal. The payload lands on the train and the resulting velocity of train and payload is 10 m/s horizontally to the right in the positive x-direction. **Determine the mass of the payload.**

$\longrightarrow \hat{x}$

$$\Sigma P_{ix} = \Sigma P_{fx}$$

$$M_1 V_{1ix} + M_2 V_{2ix} = (M_1 + M_2) V_{12fx}$$

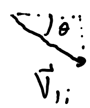
$$M_1 V_{1ix} + M_2 V_{2ix} = M_1 V_{12fx} + M_2 V_{12fx}$$

$$M_1 V_{1ix} - M_1 V_{12fx} = M_2 V_{12fx} - M_2 V_{2ix}$$

$$M_1 (V_{1ix} - V_{12fx}) = M_2 (V_{12fx} - V_{2ix})$$

$$M_1 = \frac{M_2 (V_{12fx} - V_{2ix})}{(V_{1ix} - V_{12fx})}$$

Vector Stuff



$V_{1ix} = |\vec{V}_{1i}| \cos\theta$

≈ 7.2505 m/s

$\approx 5000\text{ kg} \left(\frac{10\text{ m/s} - 12\text{ m/s}}{7.2505\text{ m/s} - 12\text{ m/s}} \right)$

$\boxed{\approx 3640\text{ kg}}$

CM.2.L2-5:

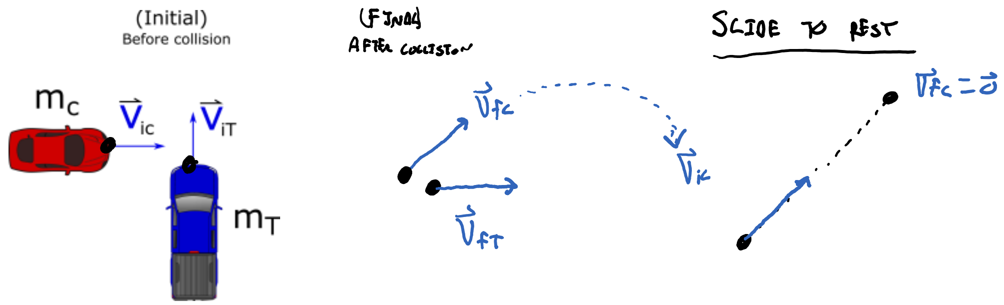
Description: Find final speed of an object that undergoes a 2-D collision. (2 minutes + 3 minutes + 3 minutes + 3 minutes + 2 minutes + 8 minutes)

Learning Objectives: [3, 5, 9, 10]

Problem Statement: A car of mass 1300 kg traveling horizontally to the right as seen from above collides with a 2300 kg truck which was traveling 9.0 m/s vertically upwards as seen from above. The moment after the collision the truck is moving horizontally to the right at a speed of 7 m/s. The moment after the collision the car is traveling up and to the right and slides 22.5 m before coming to rest. The truck also slides to rest. The speed limit at the intersection is 25 mph. We are studying the claim that the car was speeding.

(a) To study the claim that the car was speeding we need to study the motion from before the collision to afterwards when it slides to rest. How many stages should we break this problem up into?

2) Collision + Slide to rest



(b) During the collision stage, which of the following types of physics could be used to analyze the system?

X (1) Kinematics
X (2) Mechanics (force analysis)
X (3) Impulse
(4) Conservation of momentum
X (5) Conservation of energy

(c) Which of the following reasons most justifies your answer to part (b)?

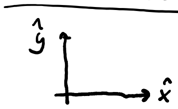
F (1) The energy of the collision is conserved.
F (2) No external forces act on the two during the collision.
F (3) No net external forces act on the two during the collision.
T (4) The collision is assumed to happen so quickly that the impulse from friction from the road can be neglected during the collision.
F (5) The two bounce off each other.

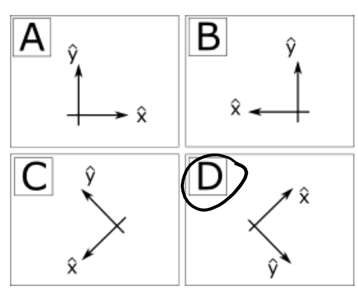
(d) During the sliding to rest stage, identify all of the types of physics that could be used to analyze the system.

(1) Kinematics
(2) Mechanics (force analysis)
(3) Impulse
X (4) Conservation of momentum
(5) Work and energy

(e) We wish to see if the car was actually speeding and will have to study the sliding to rest stage. Which coordinate system would simplify this part of the analysis into a 1-D problem?

Collision coord. sys.





(f) A car of mass 1300 kg traveling horizontally to the right as seen from above collides with a 2300 kg truck which was traveling 9.0 m/s vertically upwards as seen from above. The moment after the collision the truck is moving horizontally to the right at a speed of 7 m/s. The moment after the collision the car is traveling up and to the right and slides 22.5 m before coming to rest. The coefficient of kinetic friction between the road and the car's tires is 0.70. The truck also slides to rest. The speed limit at the intersection is 25 mph. **What was the initial speed of the car?**

Stage A: Collision ($M_1 + M_2$ system)

$|\vec{V}_{ci}| = \sqrt{V_{ci,x}^2 + V_{...}^{2}} = V_{ci...}$

STAGE A : COLLISION ($M_1 + M_2$ SYSTEM)

$\Sigma \vec{F}_{EXT} \approx \vec{0}$

$\Sigma P_{ix} = \Sigma P_{fx}$

$M_c V_{cix_A} + M_T \cancel{V_{Tix_A}}^{0} = M_c V_{cfx_A} + M_T V_{Tfx_A}$

$\checkmark \quad ? \quad\quad ? \quad \checkmark \quad \checkmark$

$M_c V_{cix_A} = M_c V_{cfx_A} + M_T V_{Tfx_A} \quad \boxed{I}$

STUCK.... TRY

STAGE B

$|\vec{V}_{ci_A}| = \sqrt{V_{cix_A}^2 + \cancel{V_{ciy_A}^2}^{0}} = V_{cix_A}$

$\Sigma P_{iy} = \Sigma P_{fy}$

$M_c \cancel{V_{ciy_A}}^{0} + M_T V_{Tiy_A} = M_c V_{cfy_A} + M_T \cancel{V_{Tfy_A}}^{0}$

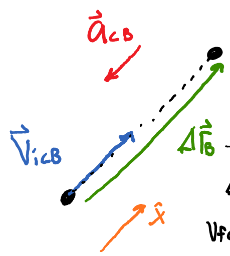
$M_T V_{Tiy_A} = M_c V_{cfy_A}$

$V_{cfy_A} = \frac{M_T}{M_c} V_{Tiy_A}$

$V_{cfy_A} = 15.923 \text{ m/s}$

STAGE B : SLIDE TO REST

KINEMATICS



K	UK
$\Delta X_B = 22.5\,m$	V_{icxB}
$V_{fcxB} = 0$	
$a_{cxB} = -6.86 \text{ m/s}^2$	Δt_B

$\cancel{V_{fcxB}^2}^{0} = V_{icxB}^2 + 2 a_{cxB} \Delta X_B$

$0 = V_{icxB}^2 - 2\mu_k g \Delta X_B$

$V_{icxB} = \sqrt{2\mu_k g \Delta X_B} = |\vec{V}_{icB}| \approx 17.5699 \text{ m/s}$

BUT REMEMBER ... THIS IS THE FINAL

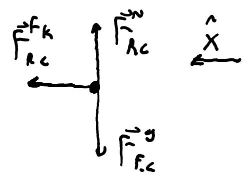
SPEED OF THE COLLISION STAGE

CONNECTION

$|\vec{V}_{icB}| = |\vec{V}_{fcA}|$

FORCE ANALYSIS

FBD M_c



$\Sigma F_x = m a_{xB}$ $\quad\quad$ $\Sigma F_y = m \cancel{a_{yB}}^{0}$

$\mu_k F^N_{Rc} = M_c a_{cxB}$ $\quad\quad$ $F^N_{Rc} - M_c g = 0$

$\mu_k M_c g = M_c a_{cxB}$

$a_{cxB} = \mu_k g$

$a_{cxB} = 6.86 \text{ m/s}^2$

$|\vec{V}_{fcA}|^2 = V_{fcxA}^2 + V_{fcyA}^2$

$2\mu_k g \Delta X_B = V_{fcxA}^2 + \left(\frac{M_T}{M_c}\right)^2 V_{TiyA}^2$

$V_{fcxA} = \sqrt{2\mu_k g \Delta X_B - \left(\frac{M_T}{M_c}\right)^2 V_{TiyA}^2} \quad \boxed{II}$

$\approx 7.42668 \text{ m/s}$

FINALLY W/ I + II

$M_c V_{cixA} = M_c V_{fcx} + M_T V_{fTx}$

$1300\,kg\,(V_{cixA}) = 1300\,kg\,(7.42668 \text{ m/s}) + 2300\,kg\,(7 \text{ m/s})$

$$V_{cixA} = \frac{M_c \sqrt{2\mu_k g \Delta X_B - \left(\frac{M_T}{M_c}\right)^2 V_{TiyA}^2} + M_T V_{fTx}}{M_c} = |\vec{V}_{ciA}|$$

$$V_{cixA} = \frac{m_c \sqrt{2\mu_k g \Delta x_B - \left(\frac{m_T}{m_c}\right)^2 V_{TiyA}^2} + m_T V_{FTx}}{m_c} = |\vec{V}_{icA}|$$

$$\boxed{|\vec{V}_{iAc}| \approx 19.8 \text{ m/s}} \approx 44 \text{ mph}$$

Conceptual questions for discussion

1. As a particle physicist, you collide two particles head on in an environment where there are no external forces. After the collision you observe two particles, one traveling along the same initial path, and another one at some angle relative to their initial path. After this collision, are there only two particles, or is there a possible third particle that you didn't observe? Support your answer using physics principles.
2. Consider a tennis player hitting a tennis ball and a system that contains just the tennis racket and the tennis ball. Is the momentum before the collision equal to the momentum after the collision? Is the momentum before the collision equal to the momentum 3 seconds after the collision?

Hints

CM.2.L2-1: No hints.

CM.2.L2-2: No hints.

CM.2.L2-3: No hints.

CM.2.L2-4: No hints.

CM.2.L2-5: No hints.