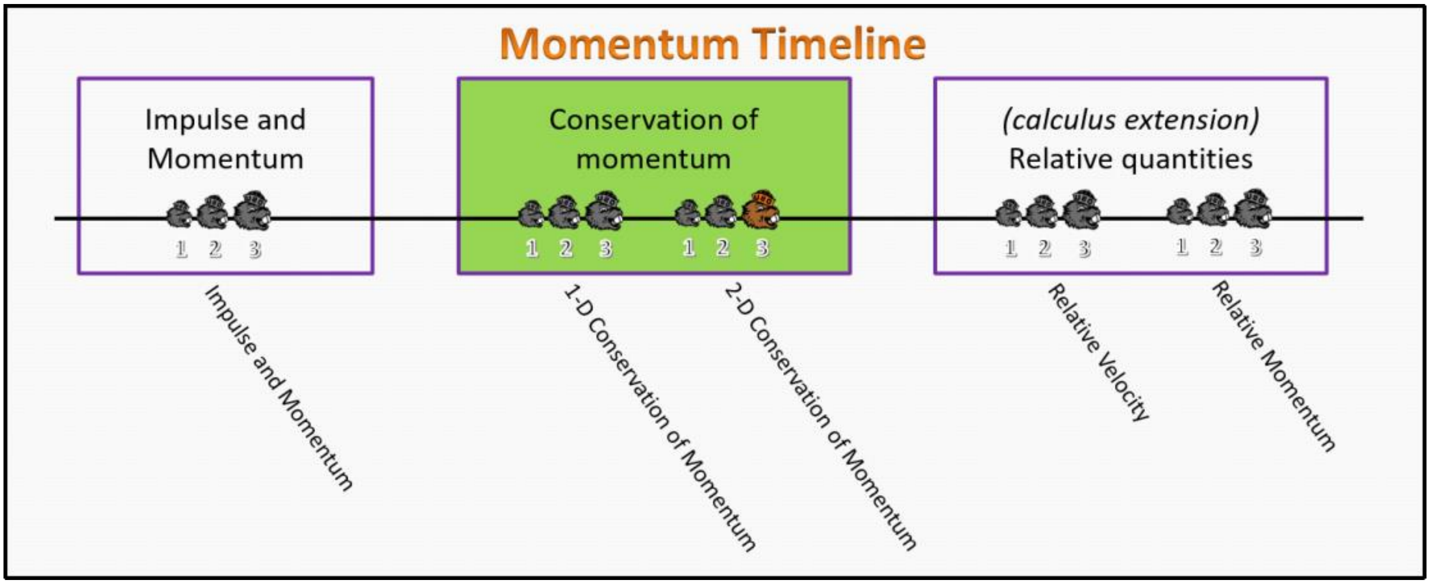
(CM.L2.3-sols) Practice Stage Solutions

Thursday, March 29, 2018 8:34 PM

Conservation of Momentum

Foundation Stage (CM.L2.3)

Post-Lecture 2
2-D Conservation of Momentum



Questions

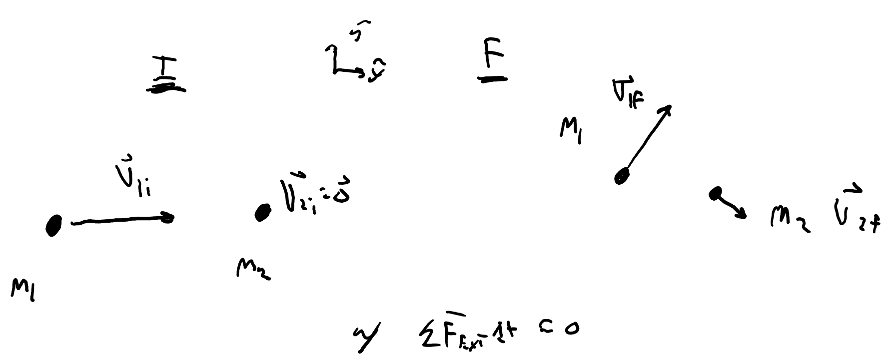
CM.L2.3-01

Description: Identifying number of unknowns in a 2D conservation of momentum problem

Learning Objectives: [x]

Problem Statement: Suppose a particle of known mass and velocity collides with a second particle of known mass that is at rest. You try to solve for the final velocities of both particles in the plane of collision. How many unknowns could you possibly have to solve this problem?

(1) One unknown, you only have one conservation of momentum equation to use
(2) Two, you can create two non-trivial equations from the two directions, x, y, that define the plane
(3) Three, there is an equation for x, y, and z
(4) Four, you can always solve for four unknowns with conservation of momentum



M_l

$\sum \vec{F}_{ext} \Delta t = 0$

$P_{ix} = P_{fx}$ 1

$P_{fy} = P_{fy}$ 2

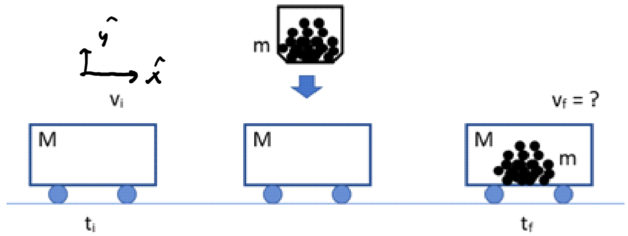
CM.L2.3-02

Description: Determine the final velocity of an object that undergoes a collision in 2-D
Learning Objectives: [x]

Problem Statement: A train car of mass **M** is moving at an initial velocity $\mathbf{v_i}$. The car moves under a chute, which dumps coal of mass m into the car.

(a) What happens to the speed of the car during this loading.

(1) Slows down
(2) Stays the same
(3) Speeds up



(b) What is the final velocity of the car + coal?

(1) $\mathbf{v_f} = \mathbf{v_i}\,\mathbf{M/(M+m)}$
(2) $\mathbf{v_f} = \mathbf{v_i}\,\mathbf{(M+m)/M}$
(3) $\mathbf{v_f} = \mathbf{v_i}\,\mathbf{m/(M+m)}$
(4) $\mathbf{v_f} = \mathbf{v_i}\,\mathbf{(M+m)/m}$
(5) $\mathbf{v_f} = \mathbf{M/v_i}\,\mathbf{(M+m)}$
(6) $\mathbf{v_f} = \mathbf{m/v_i}\,\mathbf{(M+m)}$

$\sum F_x \Delta t = 0$

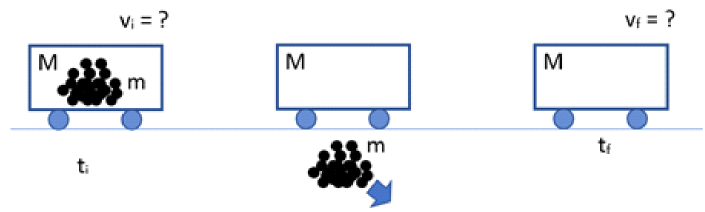
$\sum P_{ix} = \sum P_{fx}$

$M v_{ix} + m(0) = (M+m) v_{fx}$

$$\boxed{v_{fx} = \frac{M}{M+m} v_{ix}}$$

(c) The train car carrying the coal now rolls over a pit and drops the coal, and rolls on. What happens to the speed of the car when it dumps the coal?

(1) Slows down
(2) Stays the same
(3) Speeds up



CM.L2.3-03

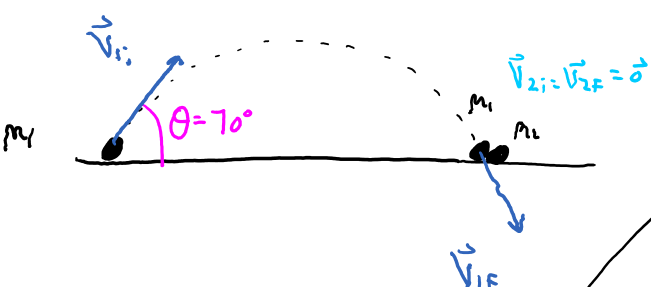
Description: Conservation of momentum in one direction

Learning Objectives: [x]

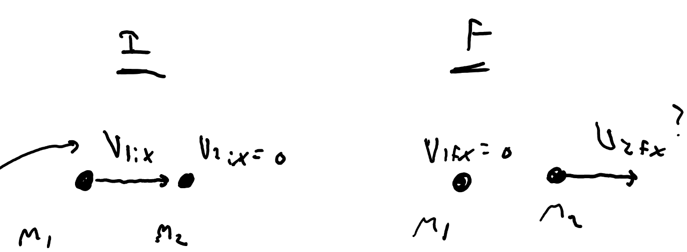
Problem Statement: A steel ball is thrown in the air with a speed of 3.6 m/s at an angle of 70° from the horizontal. It drops on another steel ball of 1.4 times its mass resting on a sandy surface. If the original ball comes to a rest after the collision and the resting ball bounces, find the horizontal component of its velocity.

(1) 0.437 m/s
(2) 0.165 m/s
(3) 10.5 m/s
(4) 3.38 m/s
(5) 5.04 m/s
(6) 0.879 m/s

KINEMATICS



COM X-ANALYSIS



Assume $\Sigma F_x \Delta t \approx 0$

$$\Sigma P_{ix} = \Sigma P_{Fx}$$

$$m_1 V_{1ix} + m_2 \cancel{V_{2ix}}^{0} = m_1 \cancel{V_{1Fx}}^{0} + m_2 V_{2Fx}$$

$$m_1 V_{1ix} = 1.4\, m_1 V_{2Fx}$$

$$V_{1ix} = 3.6\ m/s \cos 70°$$

$$V_{1ix} \approx 1.23127\ m/s = V_{1Fx}$$

$$m_1 V_{1ix} = 1.4\, m_1 V_{2fx}$$

$$V_{F2x} = \frac{1}{1.4} V_{1ix}$$

$$\boxed{V_{2Fx} \approx 0.879 \text{ m/s}}$$

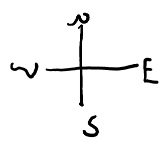
CM.L2.3-04

Description: 2D conservation of momentum problem with a sticky collision

Learning Objectives: [x]

Problem Statement: A 7.77 kg mass is moving due west at 7.77 m/s. A second mass of 8.88 kg is moving due south at 8.88 m/s. What is their speed after they collide and stick together?

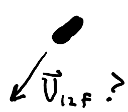
(1) 4.36 m/s
(2) 9.99 m/s
(3) 11.8 m/s
(4) 16.7 m/s
(5) 5.95 m/s
(6) 0.481 m/s



I $\quad$ $\hat{y}$ $\hat{x}$ $\quad$ F

w/ $\Sigma \vec{F}_{ext} = \vec{0}$

$\Sigma \vec{P}_i = \Sigma \vec{P}_f$



$$\Sigma P_{xi} = \Sigma P_{fx} \qquad \Sigma P_{iy} = \Sigma P_{fy}$$

$$m_1 V_{1ix} + \cancel{m_2 V_{2ix}}^{\,0} = (m_1 + m_2) V_{12fx} \qquad \cancel{m_1 V_{1iy}}^{\,0} + m_2 V_{2iy} = (m_1 + m_2) V_{12fy}$$

$$m_1 V_{1ix} = (m_1 + m_2) V_{12fx} \qquad m_2 V_{2iy} = (m_1 + m_2) V_{12fy}$$

$$(7.77\text{kg})(-7.77\text{ m/s}) = (7.77 + 8.88) V_{12fx} \qquad (8.88\text{kg})(-8.88\text{ m/s}) = (7.77 + 8.88) V_{12fy}$$

$$V_{12fx} = -3.626 \text{ m/s} \qquad V_{12fy} \approx -4.736 \text{ m/s}$$

$$|\vec{V}_{12f}| = \sqrt{3.626^2 + 4.726^2} = \boxed{5.96\ m/s}$$

CM.L2.3-05

Description: 2D conservation of momentum experiment

Learning Objectives: [x]

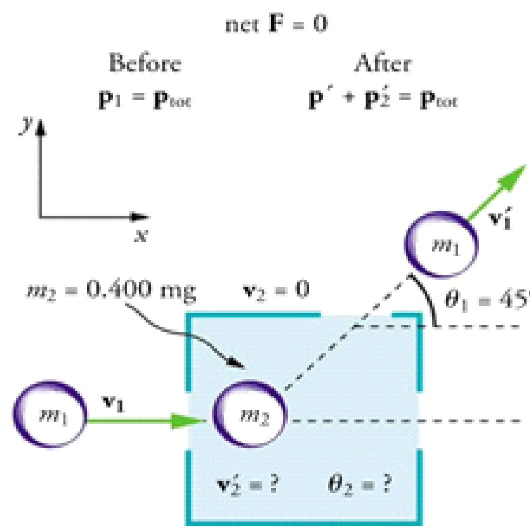
Problem Statement: Suppose the following experiment is performed. A 0.250 kg object (**m₁**) is slid on a frictionless surface into a dark room, where it strikes an initially stationary object with mass of 0.400 kg (**m₂**). The 0.250 kg object emerges from the room at an angle of 45° with its incoming direction. The speed of the 0.250 kg object is originally 2.00 m/s and is 1.50 m/s after the collision. Calculate the magnitude and direction of the velocity (**v'₂** and **θ₂**) of the 0.400 kg object after the collision.

(1) $\mathbf{v'_2}$ = 0.701 m/s and $\mathbf{\theta_2}$ = 19.1°

(2) $\mathbf{v'_2}$ = 0.937 m/s and $\mathbf{\theta_2}$ = 75°

(3) $\mathbf{v'_2}$ = 0.892 m/s and $\mathbf{\theta_2}$ = 26°

(4) $\mathbf{v'_2}$ = 0.886 m/s and $\mathbf{\theta_2}$ = 48°



$$\Sigma P_{xi} = \Sigma P_{fx}$$

$$m_1 V_{1ix} + m_2 \cancel{V_{2ix}}^{0} = m_1 V_{1fx} + m_2 V_{2fx}$$

$$(0.250\ kg)(2\ m/s) = (0.25\ kg)(1.5\cos 45) + (0.40) V_{2fx}$$

$$V_{2fx} = 0.5870873926\ m/s$$

Vectors

$$V_{1fx} = 1.5 \cos 45°$$

$$V_{1fy} = 1.5 \sin 45$$

$$\Sigma P_{iy} = \Sigma P_{fy}$$

$$m_1 \cancel{V_{1iy}}^{0} + m_2 \cancel{V_{2iy}}^{0} = m_1 V_{1fy} + m_2 V_{2fy}$$

$$0 = (0.25)(1.5 \sin 45) + (0.4) V_{2fy}$$

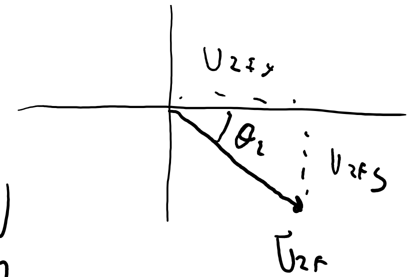
$$V_{2fy} = -0.6629126074\ m/s$$

$$|\vec{V}_{2f}| = \sqrt{V_{2fx}^2 + V_{2yf}^2}$$

$$\boxed{|\vec{V}_{2F}| \approx 0.886\ m/s}$$

V_{2fy}

$V_{2fy} = -0.6629126074$ m/s



$\theta_2 = \tan^{-1}\left(\frac{V_{2fy}}{V_{2fx}}\right)$

$\boxed{\theta_2 \approx 48.5°}$

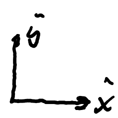
CM.L2.3-06

Description: Determine the final velocity of an object that undergoes a collision in 2-D

Learning Objectives: [x]

Problem Statement: A rocket with total mass m is in deep space, traveling with velocity **v**$_i$ in the x-direction. The rocket suddenly shoots some of its fuel out in the y-direction. The mass of the fuel is 1/5 the total mass of the rocket. Which of the following could be the final velocity of the rocket?

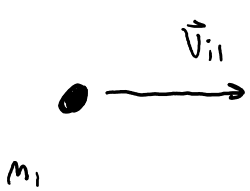
(1) **v**$_f$ = **v**$_i$ < -1/2 , 5/4 >
(2) **v**$_f$ = **v**$_i$ < 1/2 , -5/4 >
(3) **v**$_f$ = **v**$_i$ < 4/5 , -1/2 >
(4) **v**$_f$ = **v**$_i$ < -4/5 , 1/2 >
(5) **v**$_f$ = **v**$_i$ < 5/4 , -1/2 >
(6) **v**$_f$ = **v**$_i$ < -5/4 , 1/2 >



$* \; V_i = V_{ix1}$

0

I



w/ $\sum \vec{F}_{ext} = \vec{0}$

$\sum \vec{P}_i = \sum \vec{P}_f$

F

$\vec{V}_{2f}$ $\quad m_2 = \frac{1}{5} m_1$

$\vec{V}_{3f}$? $\quad m_3 = \frac{4}{5} m_1$

$\sum P_{ix} = \sum P_{fx}$

$\sum P_{iy} = \sum P_{fy}$

$m_1 V_{1ix} = m_3 V_{3fx}$

$0 = m_2 V_{2fy} + m_3 V_{3fy}$

$0 = \frac{1}{5}\cancel{m_1} V_{2fy} \quad \frac{4}{...}$

$$m_1 v_{1ix} = \frac{4}{5} m_1 v_{3fx}$$

$$v_{3fx} = \frac{5}{4} v_{1ix}$$

$$v_{3fx} = \frac{5}{4} v_i$$

$$0 = \frac{1}{5} \cancel{m_1} v_{2fy} - \frac{4}{5} \cancel{m_1} v_{3fy}$$

$$0 = \frac{1}{5} v_{2fy} - \frac{4}{5} v_{3fy}$$

$$v_{3fy} = \frac{1}{4} v_{2fy}$$