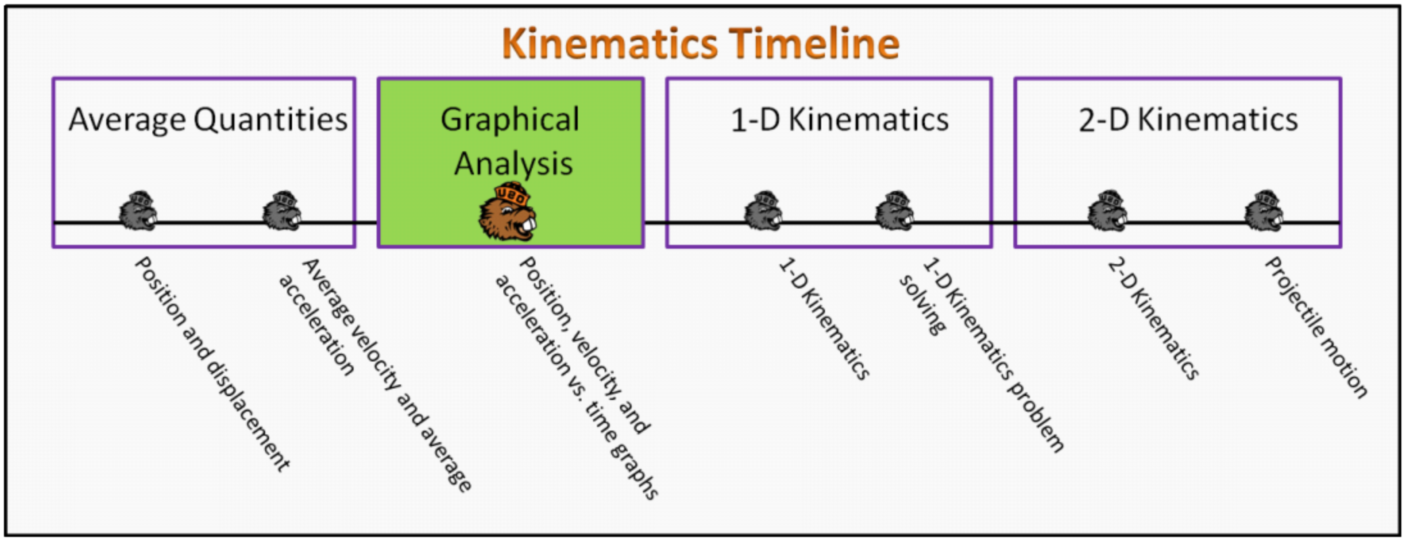
(GA.L1.2-sols) Foundation Stage Solutions

Monday, January 22, 2018   5:44 PM

Graphical Analysis
Foundation Stage (GA.2)

lecture 1
Position, velocity, and acceleration vs time graphs



Textbook Chapters

- **BoxSand** :: KC videos (Graphical Analysis)
- **Giancoli** (Physics Principles with Applications 7th) :: 2-8
- **Knight** (College Physics : A strategic approach 3rd) :: 2.1 ; 2.2 ; 2.3 ; 2.4
- **Knight** (Physics for Scientists and Engineers 4th) :: 1.6 ; 2.1 ; 2.4

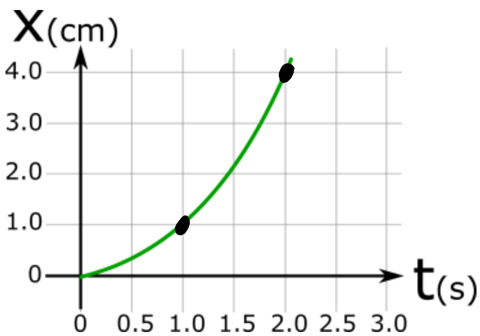
Warm up

GA.2-1

Description: Given a position vs time graph, calculate average velocity.

Learning Objectives: [?] - Can you identify the objectives from the previous lecture, and this lecture, that this question is relevant to?

Problem Statement: Below shows a graph of position vs time for Terry the Termite as he walks along a straight line. Recall the definition for average velocity: $\bar{v}_x = \frac{\Delta x}{\Delta t}$. What is Terry's average velocity between 1.0 s and 2.0 s?



$$x_i = x(t=1) = 1\ cm$$

$$x_f = x(t=2) = 4\ cm$$

$$\bar{\vec{v}} = \frac{\Delta \vec{r}}{\Delta t} \qquad \bar{v}_x = \frac{\Delta x}{\Delta t}$$

$$\bar{v}_x = \frac{x_f - x_i}{t_f - t_i}$$

$$= \frac{4-1}{2-1}\ \frac{cm}{s}$$

$$\boxed{\bar{v}_x = 3\ cm/s}$$

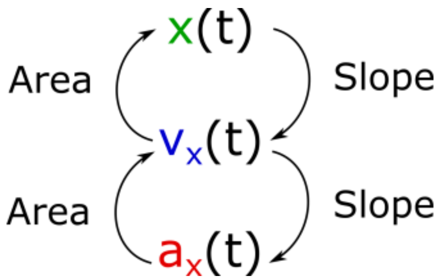
Selected Learning Objectives

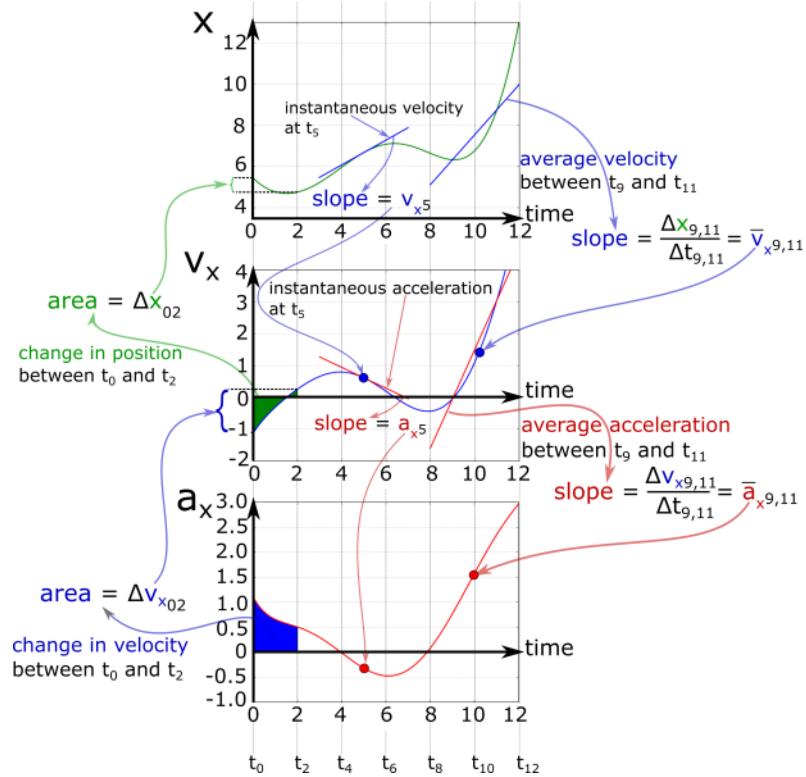
1. Interpret a position versus time graph to determine the position of the object at any given time, is it positive, negative, o r zero.
2. Interpret a position versus time graph to determine if it is traveling in the positive or negative direction.
3. Interpret a position versus time graph to determine if the object is speeding up or slowing down.
4. Interpret a velocity versus time graph to determine what the velocity of the object is at any given time.
5. Interpret a velocity versus time graph to determine if an object is traveling in the positive or negative direction.
6. Interpret a velocity versus time graph to determine if the object is speeding up or slowing down.
7. Interpret an acceleration versus time graph to determine what the acceleration of the object is at any given time.
8. Interpret an acceleration versus time graph to determine if the object is speeding up or slowing down.
9. Show on a graph an example of instantaneous velocity and instantaneous acceleration.
10. Show on a graph an example of average velocity and average acceleration.
11. Differentiate between the magnitude and direction when interpreting plots.
12. Demonstrate the ability to estimate an area under a non-linear curve by breaking the complex curve into more simple well known shapes.
13. Recognize what the area under the curve determines, i.e. it represents in changes in position and velocity and not the actual position or velocity, you need to add the change to the initial conditions to find the final state.
14. Differentiate between the features of positive and *negative area*.
15. Take a graph of position, velocity, or acceleration vs time and be able to sketch all other graphs from the original. Here is where slopes, areas, and initial conditions are involved.
16. Determine if a set of position, velocity, and accelerations could be describing the motion of the same object.
17. If given a graph of position, be able to describe the acceleration of the object by analyzing the slope of the slope of posit ion vs. time and/or determining the curvature and concavity.
18. Create graphs of position, velocity, and acceleration for the motion of an object given a written description or experimental demonstration.
19. If given a shape of a graph translate the nature of the function into the math representation, e.g. a straight line on a grap h is a linear function of the form $y = mx + b$.
20. Recognize when to use the graphical representation to analyze systems that would be more difficult in the other representatio ns
21. Use data tables to create graphs and analyze the position, velocity or acceleration of the an object.
22. Demonstrate making approximations to slopes or areas when analyzing position, velocity, and acceleration vs. time graphs.

Key Terms

- Slope
- Area
- Instantaneous velocity
- Initial conditions

Key Equations





Key Concepts

- Identify what each axis on a graph is representing before trying to extract information from the graph.
- The value of the slope of a position vs time graph between two times is the average velocity between those two times.
- The value of the slope of a velocity vs time graph between two times is the average acceleration between those two times.
- The area under a velocity vs time graph between two times is the change in position between those two times.
- The area under an acceleration vs time graph between two times is the change in velocity between those two times.
- If the velocity (or acceleration) curve is above the origin, the area represents a positive change in position (or velocity), if the curve is below the origin then the area represents a negative change in position (or velocity).
- The instantaneous velocity at a time $\mathbf{t_0}$ is the value of the slope of a linear line that is tangent to the position vs time curve at $\mathbf{t_0}$.

Act I: Position vs Time

Questions

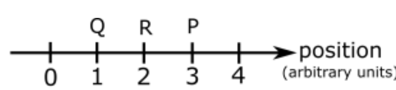
GA.2-2:

Description: Match the position vs time graph that correctly represents a written description of motion. (5 minutes + 1.5 minutes)

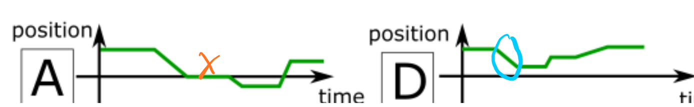
Learning Objectives: [1, 2, 3]

Problem Statement: A person initially at point **P** in the illustration stays there a moment and then moves along the axis to **Q** and stays there for a moment. She then runs quickly to **R**, stays there a moment, and then strolls slowly back to **P**.

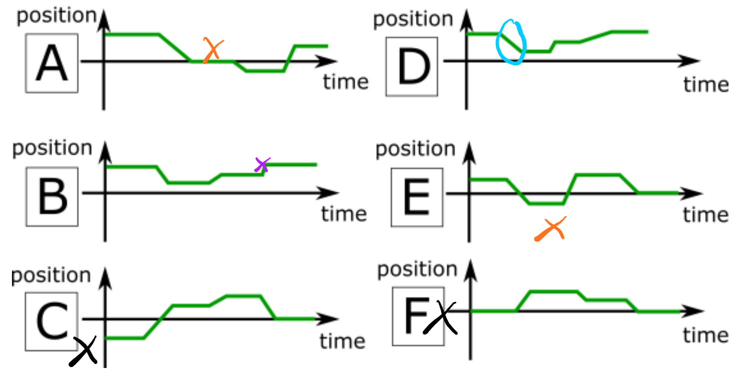
(a) Which of the position versus time graphs below correctly represents this motion?



(1) A
(2) B
(3) C



(1) A
(2) B
(3) C
(4) D
(5) E
(6) F



(b) Using the graph you identified in part (a), circle the region(s) of the graph where the person is traveling in the negative direction.

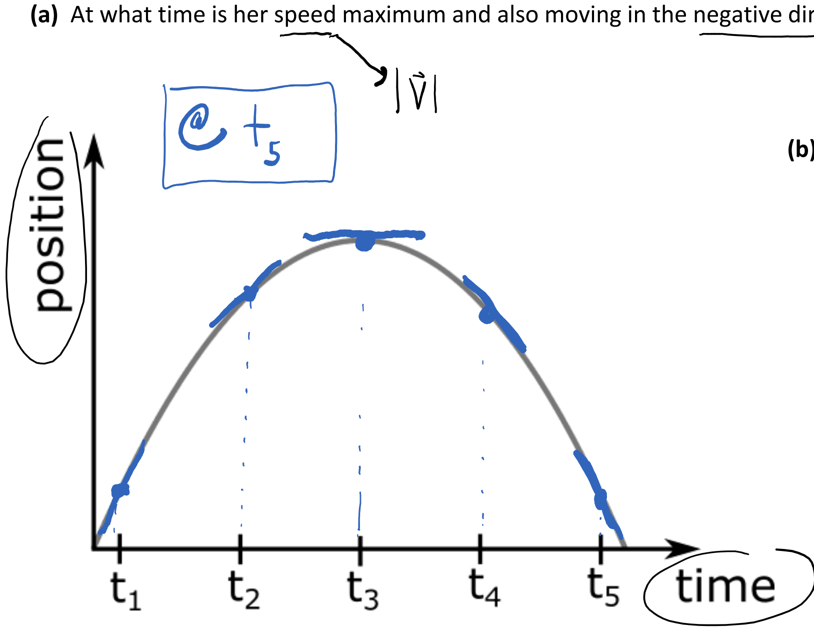
GA.2-3:

Description: Use a position vs time graph to determine information about speed and direction. (2 minutes + 2 minutes)

Learning Objectives: [2, 3, 11]

Problem Statement: The below graph represents the position vs time for Fiona the Flea as she moves along a linear line.

(a) At what time is her speed maximum and also moving in the negative direction?



(b) Over what time interval is Fiona speeding up?

(1) **t_1 - t_3**
(2) **t_3 - t_5**
(3) **t_1 - t_5**
(4) Fiona never speeds up.

$X(t)$ SLOPE $V_x(t)$

GA.2-4:

Description: Use a position vs time graph to determine information about average velocity, instantaneous velocity, and displacement. (8 minutes)

Learning Objectives: [2, 3, 9, 10]

Problem Statement: The graph shows position as a function of time for two trains running on parallel tracks. Which of the following statements are true?

F (1) At time **t_1**, both trains have the same velocity.

F (2) Both trains speed up at all times.

F (1) At time t_1, both trains have the same velocity.

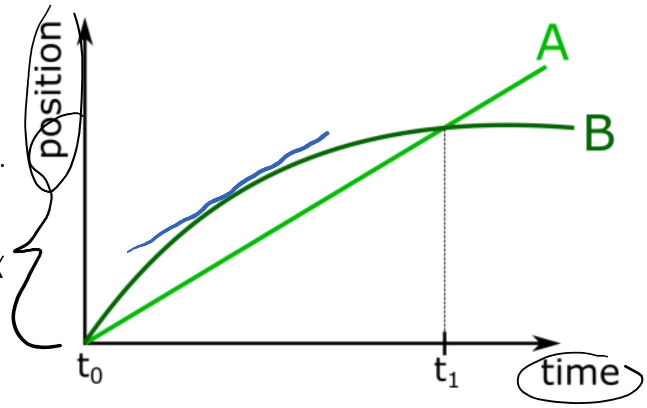
F (2) Both trains speed up at all times.

T (3) Both trains have the same velocity at some time before t_1.

F (4) Train **B** has a greater average velocity between t_0 and t_1.

F (5) At some instant, both trains have the same acceleration.

T (6) At time t_1, both trains have the same magnitude of displacement from t_0.

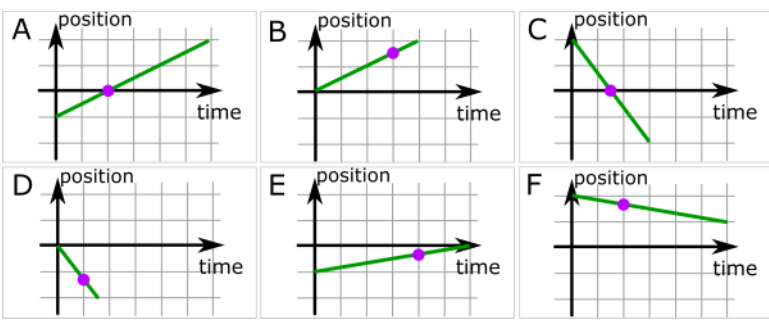


GA.2-5:

Description: Use position versus time graphs to rank the magnitude of velocity at an indicated position. (5 minutes)

Learning Objectives: []

Problem Statement: The graphs below show position versus time for six boats traveling along a straight, narrow channel. The scales on both axes are the same for all of these graphs (meters and seconds). In each graph, a point is marked with a purple dot. Rank the magnitude of the velocity of the boat at the point indicated.



$X(t)$, $V_x(t)$: slope = rise/run = $\frac{\Delta x}{\Delta t}$

$V_{Ax} = \frac{3}{6}$ m/s $= \frac{1}{2}$ m/s

$V_{Bx} = \frac{2}{4}$ m/s $= \frac{1}{2}$ m/s

$V_{Cx} = \frac{-4}{3}$ m/s

$V_{Dx} = \frac{-2}{1.5}$ m/s $= \frac{4}{3}$ m/s

$V_{Ex} = \frac{1}{6}$ m/s

$V_{Fx} = \frac{-1}{6}$ m/s

$|V_{Fx}| = |V_{Ex}| < |V_{Ax}| < |V_{Bx}| < |V_{Cx}| = |V_{Dx}|$

GA.2-6:

Description: Use a position vs time graph to determine information about velocity and acceleration. (2 minutes + 2 minutes + 2 minutes)

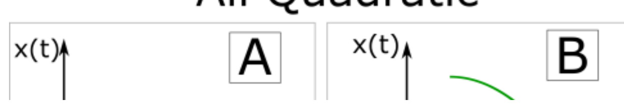
Learning Objectives: [2, 3, 17]

Problem Statement: Below are four graphs representing the motion of a car. Which of the following position as a function of time graphs…

(a) …shows that the car has the direction of its velocity change during the motion plotted.

D

All Quadratic

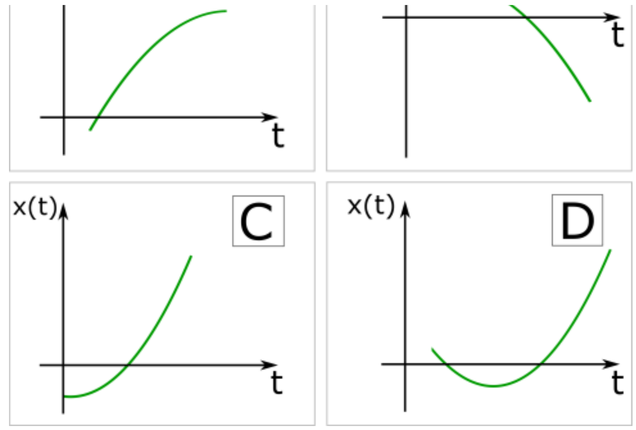


(b) …shows that the car is speeding up during the entire time plotted.

B, C

(c) …shows that the car has an acceleration with a constant direction and magnitude.

A, B, C, D



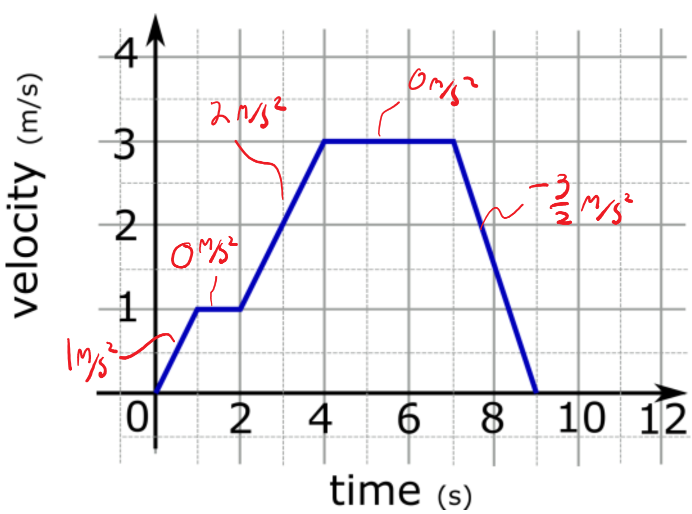
Act 2: Velocity vs Time

GA.2-7:

Description: Rank magnitude of acceleration during different time intervals from a velocity vs time graph. (4 minutes)

Learning Objectives: [7]

Problem Statement: The figure shows a plot of Jean-François the Jellyfish's velocity as a function of time. What is the value, in m/s^2, of the greatest magnitude acceleration Jean-François undergoes?



$V_x(t)$, $a_x(t)$: SLOPE $= \frac{\text{RISE}}{\text{RUN}} = \frac{\Delta V_x}{\Delta t}$

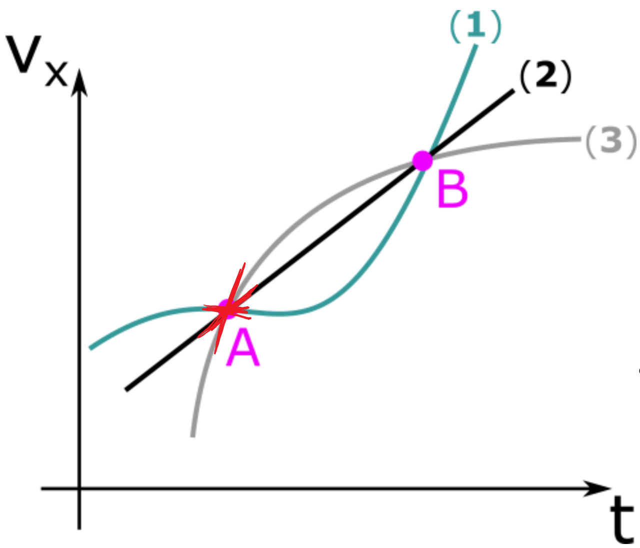
$|a_x|_{max} = 1.5 \ m/s^2$

GA.2-8:

Description: Use a velocity versus time graph to determine information about velocity and average acceleration. (1.5 minutes + 1.5 minutes + 2 minutes)

Learning Objectives: [4, 7, 9, 10]

Problem Statement: The figure below shows the velocities as a function of time for three different awesomely named animals moving in a straight line. All three curves pass through points **A** and **B**.



(a) Order the magnitudes of the velocity at point **B**.

$|V_{1x}| = |V_{2x}| = |V_{3x}|$

(b) Order the magnitudes of acceleration at point **A**.

$V_x(t)$ } SLOPE → $a_x(t)$

$|a_{1x}| < |a_{2x}| < |a_{3x}|$

(c) Order the magnitudes of the average acceleration between points **A** and **B**.

$\bar{a}_x = \frac{\Delta V_x}{\Delta t}$

SAME ΔV_x BETWEEN A & B

SAME Δt BETWEEN A & B

$|\bar{a}_{x1}| = |\bar{a}_{x2}| = |\bar{a}_{x3}|$

GA.2-9:

Description: Use a velocity versus time graph to determine information about position and displacement. (6 minutes)

Learning Objectives: [13, 14]

Problem Statement: The graph below shows the velocity as a function of time for two turtles. Which of the following statements are necessarily true?

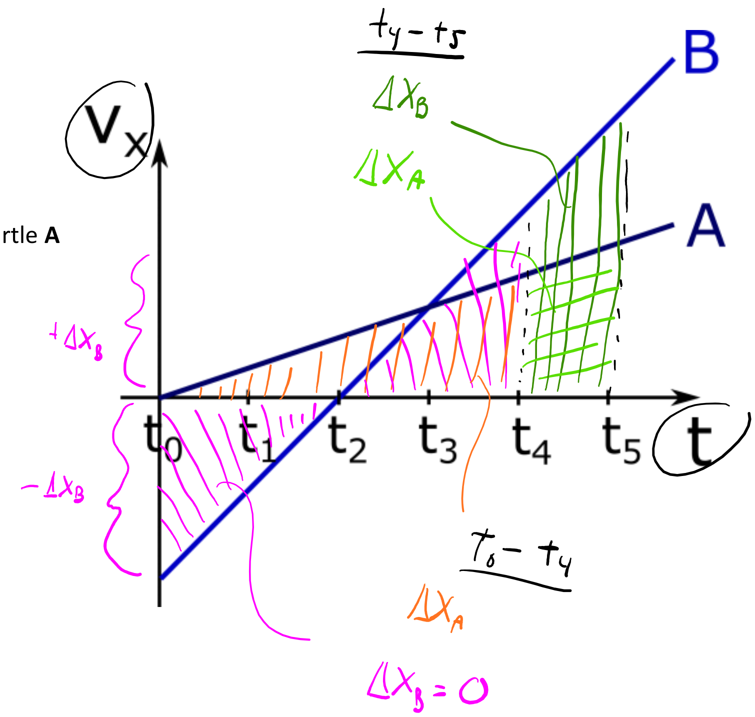
? (1) At t_0: $x_A = X_B$

? (2) At t_3: $x_A = x_B$

? (3) At t_5: the position of turtle **B** is greater than turtle **A**

F (4) Between the time interval t_4 to t_5: $\Delta x_A = \Delta x_B$

T (5) Between the time interval t_0 to t_4: $\Delta x_B < \Delta x_A$



Act 3: Acceleration vs Time

GA.2-10:

Description: From an acceleration vs time graph, sketch both the velocity vs time and position vs time graphs. (1.5 minutes + 4 minutes + 4 minutes)

Learning Objectives: [15]

Problem Statement: Below shows a graph of acceleration vs time for Sally the Seagull as she flies along a straight line path.

(a) Before sketching the velocity vs time graph for Sally's motion, what initial condition(s) must be known that will allow us to sketch the velocity vs time graph for Sally's motion?

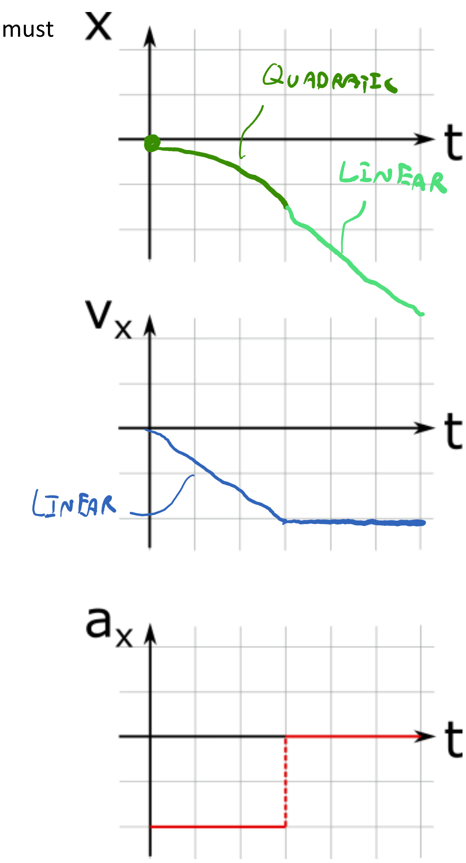
(1) x_i
(2) x_f
(3) v_i
(4) v_f
(5) mass
(6) taste

V_{ix} OR V_{fx}

(b) Sketch the velocity vs time graph for Sally's motion. ASSUME $V_{ix}=0$

(c) Sketch the position vs time graph for Sally's motion.

NEED X_i ... ASSUME $X_i=0$



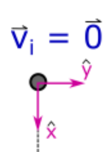
GA.2-11:

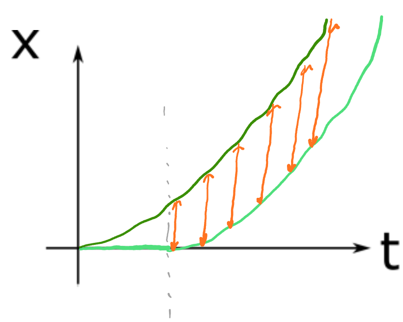
Description: Apply the rules of vector addition and subtraction to construct a vector equation. (5 minutes)

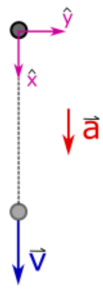
Learning Objectives: [15, 20]

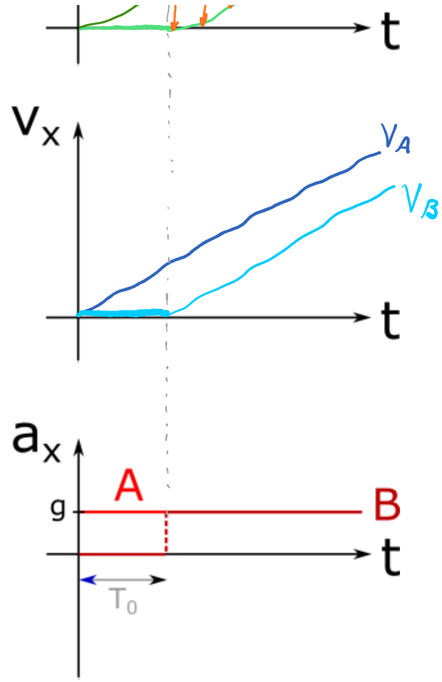
Problem Statement: Two balls are released from rest and undergo free-fall. Ball **A** is released first, followed by ball **B** a moment later. What happens to the distance between the balls while they are in free-fall?

(1) Increases.
(2) Decreases.
(3) Stays the same.



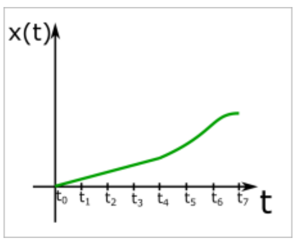






Conceptual questions for discussion

1. Describe a scenario where your position is negative but your velocity is positive. Sketch a graph of this position vs time g raph.
2. Construct a written description that matches the motion shown in the graphical representation below.



3. The area under a velocity vs time graph for a car traveling along a straight line is the car's position. Do you agree with t his statement?.
4. The area under an acceleration vs time graph for a car traveling along a straight line is the car's velocity. Do you agree w ith this statement?
5. Two cars, **A** and **B**, start from rest and move for an equal amount of time along a straight line. Car **A** has an acceleration **a** for the first half of the total time, and **2a** for the second half. Car **B** has an acceleration of **2a** for the first half of the total time and **a** for the second half. Do the cars travel the same distance?

Hints

GA.2-1: No hints.

GA.2-2: **(a)** Can you eliminate any of the graphs by looking at the starting position at t=0? Ask your neighbors about what quickly and strolls slowly describe.

GA.2-3: Use a straight edge, like a ruler, to help visualize slopes.

GA.2-4: Use a straight edge, like a ruler, to help visualize slopes.

GA.2-5: What is the mathematical definition of average velocity?

GA.2-6: Use a straight edge, like a ruler, to help visualize slopes.

GA.2-7: What is the mathematical definition of average acceleration?

GA.2-8: Use a straight edge, like a ruler, to help visualize slopes.

GA.2-9: Talk to your neighbors about the area under the horizontal time axis; are areas always positive?

GA.2-10: After you sketch a velocity vs time graph, take the slope of your sketch to confirm it gives you back the acceleration vs tim e graph. Do the same for your sketch of position vs time.

GA.2-11: Draw a few lines on the position vs time graph that represent the distance between the balls. Also, remember that as you move vertically upwards on any of the three graphs, time is constant.