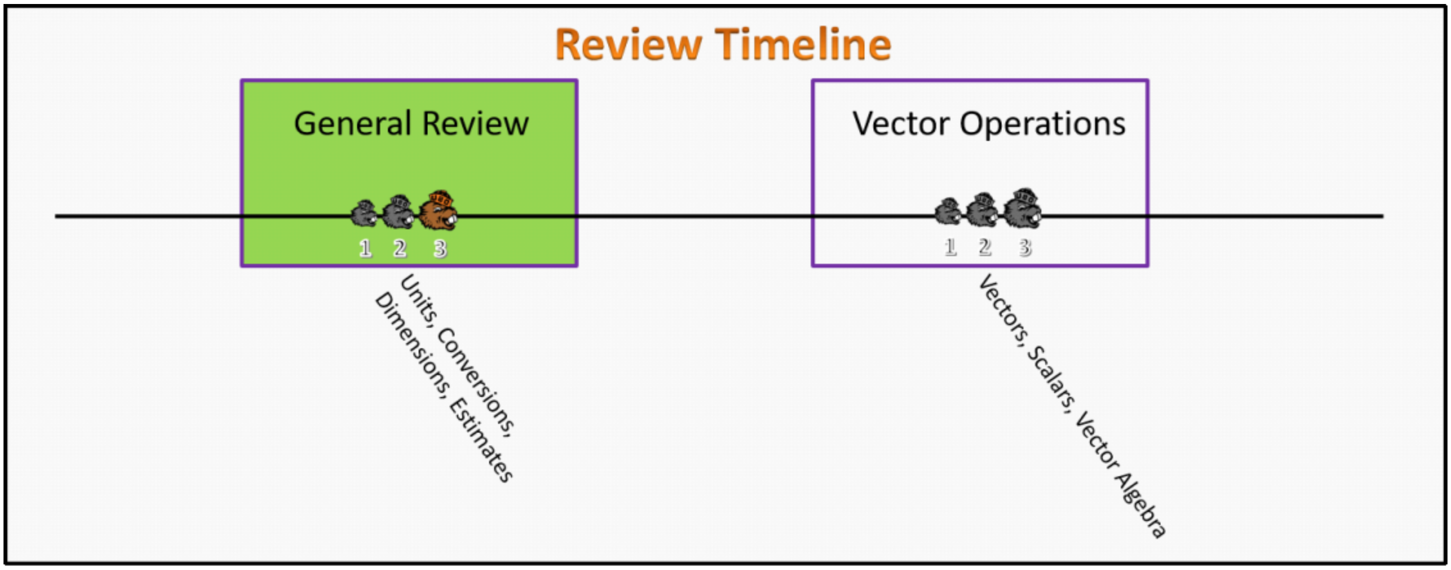
(GR.L1.3-sols) Practice Stage Solutions

Thursday, March 29, 2018 8:34 PM

General Review
Practice Stage (GR.L1.3)

Post-Lecture 1
Units, Conversions, Dimensions, Estimates



Questions

GR.L1.3-01

Description: Unit conversions

Learning Objectives: [x]

Problem Statement: Convert the following. Note: 1 m = 3.28 ft, 1 kg = 2.204 lbs.

(a) 32.2 ft per second squared to m/s^2.

(1) 19.6 m/s^2
(2) 12.2 m/s^2
(3) 10.9 m/s^2
(4) 9.82 m/s^2
(5) 5.43 m/s^2

$$32.2\ \frac{ft}{s^2} \times \frac{1\ m}{3.28\ ft} = 9.81707\ \frac{m}{s^2}$$

$$32.2\ \frac{ft}{s^2} = \boxed{9.82\ \frac{m}{s^2}}$$

(b) 12 square miles to square meters.

(1) 2.59 x 10^6 m^2
(2) 3.12 x 10^7 m^2
(3) 1.93 x 10^4 m^2
(4) 42.9 m^2
(5) 1609 m^2

$$12\ mi^2 \times \left(\frac{5280\ ft}{1\ mi}\right)^2 \times \left(\frac{1\ m}{3.28\ ft}\right)^2 \approx 31095776.32\ m^2$$

$$12\ mi^2 = \boxed{3.12 \times 10^7\ m^2}$$

(c) 32.2 ft/s^2 to centimeters per day squared.

(1) 7.33 x 10^{12} cm/day^2
(2) 7.33 x 10^9 cm/day^2
(3) 9.81 x 10^{12} cm/day^2
(4) 9.81 x 10^9 cm/day^2
(5) 42 cm/day^2

$$32.2\ \frac{ft}{s^2} \times \frac{1\ m}{3.28\ ft} \times \frac{100\ cm}{1\ m} \times \left(\frac{60\ s}{1\ min}\right)^2 \times \left(\frac{60\ min}{1\ hr}\right)^2 \times \left(\frac{24\ hr}{1\ day}\right)^2$$

$$= 7.328405 \times 10^{12}\ \frac{cm}{DAY^2}$$

$$32.2\ \frac{ft}{s^2} = \boxed{7.33 \times 10^{12}\ \frac{cm}{DAY^2}}$$

GR.L1.3-02

Description: Algebra: linear equations in two variables

Learning Objectives: [x]

Problem Statement: Solve the following set of equations for the two unknown variables.

(a)
$$\left.\begin{aligned} 4\overset{?}{x} + \overset{?}{y} &= 5 \\ 2\overset{?}{x} - 3\overset{?}{y} &= 13 \end{aligned}\right\} \text{2 EQNS, 2 UNKNOWNS}$$

$4x + y = 5$ $\quad\quad$ $2x - 3y = 13$

(1) **x** = 3, **y** = -2
(2) **x** = 2, **y** = 3

(1) **x** = 3, **y** = -2
(2) **x** = 2, **y** = 3
(3) **x** = 2, **y** = -3
(4) **x** = 4, **y** = -3
(5) **x** = 4, **y** = 2

$4x + y = 5$ $\qquad$ $2x - 3y = 13$

$y = 5 - 4x$

$2x - 3(5 - 4x) = 13$

$2x - 15 + 12x = 13$

$14x = 28$

$\boxed{x = 2}$

$y = 5 - 4x$

$y = 5 - 4(2)$

$\boxed{y = -3}$

(b) $$\left.\begin{aligned} 2\overset{?}{x} - 3\overset{?}{y} &= 4 \\ 4\overset{?}{x} + 5\overset{?}{y} &= 3 \end{aligned}\right\} \begin{array}{l} \text{2 Eqs} \\ \text{2 unknowns} \end{array}$$

(1) **x** = 24/13, **y** = 32/12
(2) **x** = 29/22, **y** = -5/11
(3) **x** = 22, **y** = -9
(4) **x** = 14, **y** = -43
(5) **x** = 33/21, **y** = 6/13

$2x - 3y = 4$ $\qquad$ $4x + 5y = 3$

$2x = 4 + 3y$

$x = 2 + \frac{3}{2}y$

$4(2 + \frac{3}{2}y) + 5y = 3$

$8 + 6y + 5y = 3$

$11y = -5$

$\boxed{y = -\frac{5}{11}}$

$x = 2 + \frac{3}{2}y$

$x = 2 + \frac{3}{2}(-\frac{5}{11})$

$x = 2 - \frac{15}{22}$

$x = \frac{44}{22} - \frac{15}{22} \longrightarrow \boxed{x = \frac{29}{22}}$

GR.L1.3-03

Description: Mathematization of two linear equations

Learning Objectives: [x]

$O \qquad S \qquad O + S = 24 \qquad O - S = 6$

Problem Statement: The sum of Orion and Sagan's age is 24, and the difference between their ages is 6. Find their ages given that Orion is older than Sagan. (**hint**: *do not guess and check, you won't be able to do that on exams. Instead write out the two*

Learning Objectives: [x]

Problem Statement: The sum of Orion and Sagan's age is 24, and the difference between their ages is 6. Find their ages given that Orion is older than Sagan. (**hint:** *do not guess and check, you won't be able to do that on exams. Instead write out the two equations and solve for the two unknown variables*)

(1) Orion is 16 and Sagan is 10
(2) Orion is 18 and Sagan is 12
(3) Orion is 14 and Sagan is 10
(4) Orion is 20 and Sagan is 14
(5) Orion is 15 and Sagan is 9

$O + S = 24 \qquad O - S = 6$

$O = 6 + S$

$6 + S + S = 24$

$6 + 2S = 24$

$2S = 18$

$\boxed{S = 9}$

$O = 6 + S$

$O = 6 + 9$

$\boxed{O = 15}$

GR.L1.3-04

Description: Mathematization of two linear equations

Learning Objectives: [x]

$\frac{\$}{b} = B \qquad \frac{\$}{t} = T$

Problem Statement: A landscaping company placed two orders with a nursery. The first order was for 13 bushes and 4 trees, and totaled \$487. The second order was for 6 bushes and 2 trees, and totaled \$232. The bills do not list the per-item price. What were the costs of one bush and one tree?

(1) Trees are \$47 and bushes are \$23
(2) Trees are \$52 and bushes are \$17
(3) Trees are \$32 and bushes are \$33
(4) Trees are \$67 and bushes are \$12
(5) Trees are \$55 and bushes are \$21

1ST ORDER

$13B + 4T = 487$

2ND ORDER

$6B + 2T = 232$

$2T = 232 - 6B$

$T = 116 - 3B$

$13B + 4(116 - 3B) = 487$

$13B + 464 - 12B = 487$

$\boxed{B = 23 \frac{\$}{\text{Bush}}}$

$T = 116 - 3B$

$T = 116 - 3(23)$

$\boxed{T = 47 \frac{\$}{\text{TREE}}}$

$\boxed{T = 47 \frac{\$}{\text{TREE}}}$

GR.L1.3-05

Description: Solve the quadratic equations

Learning Objectives: [x]

Problem Statement: Solve for the unknown variable.

(a) $\mathbf{x}^2 + \mathbf{x} = 42$, (**hint**: *use factoring*)

(1) $\mathbf{x} = 6$ or -9
(2) $\mathbf{x} = 11$ or -7
(3) $\mathbf{x} = -11$ or 6
(4) $\mathbf{x} = -7$ or 6
(5) $\mathbf{x} = -7$ or -11

$$x^2 + x - 42 = 0$$

$$(x+7)(x-6) = 0$$

$$\boxed{x = -7} \text{ OR } \boxed{x = 6}$$

(b) $\mathbf{x}^2 + 10\mathbf{x} = -25$, (**hint:** *use the Quadratic Formula*)

(1) $\mathbf{x} = -10$
(2) $\mathbf{x} = 15$
(3) $\mathbf{x} = -5$
(4) $\mathbf{x} = -5$ or 5
(5) $\mathbf{x} = -10$ or 15

$$\underbrace{1}_{a}x^2 \underbrace{+10}_{b}x \underbrace{+25}_{c} = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-10 \pm \sqrt{10^2 - 4(1)(25)}}{2(1)}$$

$$= \frac{-10 \pm \sqrt{100 - 100}}{2}$$

$$x = \frac{-10 \pm 0}{2}$$

$$\boxed{x = -5}$$

GR.L1.3-06

Description: Solve an applied quadratic equation

Learning Objectives: [x]

Problem Statement: An object is launched at 19.6 m/s from a 58.8 meter tall platform. The equation for the object's height **x** at time **t** in seconds after the launch is **x**(**t**) = -4.9**t**^{2} + 19.6**t** +58.8, where **x** is in meters. You'll learn the origin of this equation in the next few weeks. When does the object strike the ground? (**Hint:** *we have set **x** = 0 m to the ground*)

(1) -2 s
(2) 2 s
(3) -6 s
(4) 6 s
(5) 19.6 s
(6) 58.8 s

$$0 = \underbrace{-4.9t^2}_{a} + \underbrace{19.6t}_{b} + \underbrace{58.8}_{c}$$

QUADRATIC EQN.

$$t = \cancel{-2\,s} \text{ or } \boxed{6\,s}$$

GR.L1.3-07

Description: Dimensional analysis

Learning Objectives: [x]

Problem Statement: While solving a kinematics problem you derive an equation for the acceleration as a function time **t**, and position **x**, and velocity **v**. Which of the following expressions for the acceleration could be correct considering the dimensions of acceleration are length per time squared.

X (1) **x**/**t**^{2} - **v**/**t**^{2}
X (2) 2(**t**^{2}/**x** - **t**/**v**)
X (3) 2(**x**/**t** - **v**/**t**^{2})
X (4) **x**/**t** - **v**/**t**
(5) 2(**x**/**t**^{2} - **v**/**t**)

$$[a] = \frac{[L]}{[T]^2} \qquad [v] = \frac{[L]}{[T]} \qquad [x] = [L]$$

$$\frac{x}{t^2} \rightarrow \frac{[L]}{[T]^2} \checkmark$$

$\frac{}{t^2}$ $\frac{}{[T]^2}$

$\frac{v}{t} \rightarrow \frac{[L]}{[T]} \cdot \frac{1}{[T]} = \frac{[L]}{[T]^2}$ ✓

GR.L1.3-08

Description: Proportional reasoning problem

Learning Objectives: [15,16,17,18]

Problem Statement:
Two identical planets have a gravitational force F between them defined by the function

$$F = \frac{Gmm}{r^2}$$

where **m** is the mass of a planet, **r** is the distance between the centers of the planets and **G** is a constant.

(a) What is the new gravitational force if the distance between the planets is doubled?

(1) The force is one-eighth the original force
(2) The force is one-fourth the original force
(3) The force is one-half the original force
(4) The force is the same as the original force
(5) The force is twice the original force
(6) The force is four times the original force
(7) The force is eight times the original force

$F \propto \frac{1}{r^2}$

If $r \rightarrow 2r$

then $F \rightarrow \frac{1}{4} F$

(b) What is the new gravitational force if the mass of the planets is doubled and the distance between them is halved?

(1) The force is one-eighth the original force
(2) The force is one-fourth the original force
(3) The force is one-half the original force
(4) The force is the same as the original force
(5) The force is twice the original force
(6) The force is four times the original force
(7) The force is eight times the original force
(8) The force is sixteen times the original force

$$F \propto \frac{m^2}{r^2}$$

IF $m \rightarrow 2m$

AND $r \rightarrow \frac{1}{2}r$

$$F \rightarrow \frac{(2)^2}{(\frac{1}{2})^2} F = \boxed{16F}$$