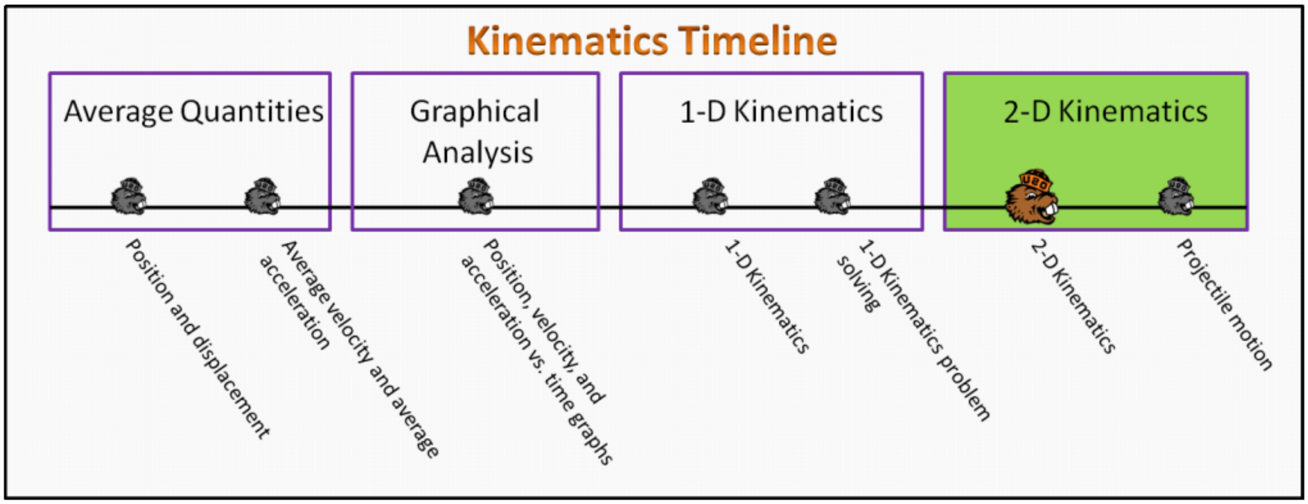
2-D Kinematics
Foundation Stage (K1.2)

lecture 1
2-D Kinematics



Textbook Chapters

- **BoxSand** :: KC videos (2D Kinematics)
- **Giancoli** (Physics Principles with Applications 7th) :: N/A
- **Knight** (College Physics : A strategic approach 3rd) :: N/A
- **Knight** (Physics for Scientists and Engineers 4th) :: 4.1

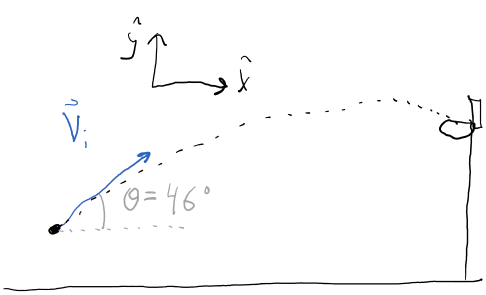
Warm up

K1.2-1:

Description: Find the components of an initial velocity.

Learning Objectives: [?] - Can you identify the objectives from the previous lecture, and this lecture, that this question is relevant to?

Problem Statement: Bernice shoots a 3 pointer releasing the basketball around 8.10 m/s at an angle of 46.0° from the horizontal to the vertical. When the ball is in the air, its acceleration is -**g** assuming a standard coordinate system. One of the first steps when solving a 2-D kinematics problem is to break vectors into components along the axis you chose for the problem. What are the initial x and y components of the basketball assuming a standard coordinate system?



$V_{ix} = |\vec{V}_i| \cos\theta$

$= 8.10\,m/s \cos(46°)$

$V_{ix} \approx 5.63\,m/s$

$V_{iy} = |\vec{V}_i| \sin\theta$

$= 8.10\,m/s \sin(46°)$

$V_{iy} \approx 5.83\,m/s$

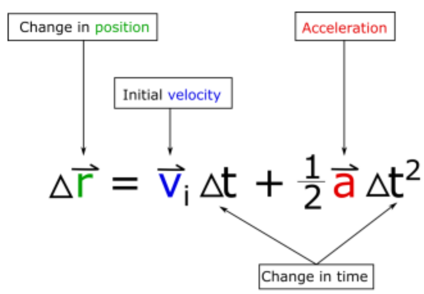
Selected Learning Objectives

1. Identify that the motion occurs in more than 1-dimension and requires a 2-D analysis.
2. Define a coordinate system that simplifies the complexity of the vector analysis.
3. Construct a physical representation the involves multiple dimensions and show a representation of the vector components.
4. Demonstrate the ability find the Cartesian components of a vector in the mathematical representation.
5. Identify known and unknown quantities for each object, stage, and dimension.
6. Solve for a desired unknown in the mathematical representation using a set of kinematic equations for each dimension. Use the problem solving skills developed in 1-D kinematics
7. Identify which quantities are the same when comparing two different dimensions, objects, or stages, e.g. elapsed time is the same for both x and y motion.
8. Define projectile motion.
9. Show that in projectile motion the acceleration has a magnitude of $g = 9.8\ m/s^2$ and points downward.
10. Show that in projectile motion time of flight is determined in an analysis of the vertical motion.
11. Show that in projectile motion the horizontal motion can be the same between two cases even when the vertical is not.
12. Show that in projectile motion range depends on both the horizontal speed and the time of flight, thus dependent on both the vertical and horizontal analysis.
13. Show that in projectile motion the range is the same for complementary angles.
14. Show that in projectile motion any system can be analyzed using only the fundamental kinematics equations for constant acceleration, e.g. you do not need specially derived equations like the *range* equation.
15. Apply limiting cases sense-making procedures to check their solutions.

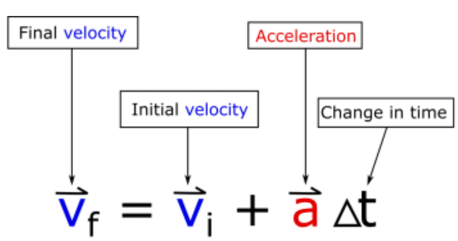
Key Terms

- Trajectory

Key Equations



In words: The change in position is equal to the initial velocity multiplied by the change in time plus one-half of the acceleration multiplied by the change in time squared.



In words: The final velocity is equal to the initial velocity plus the acceleration multiplied by the change in time.

Final x-component of velocity · Initial x-component of velocity · x-component of acceleration · Change in x-component of position

$$v_{fx}^2 = v_{ix}^2 + 2\,a_x\,\Delta x$$

In words: The final x-compoent of velocity squared is equal to the initial x-compoent of velocity squared plus two times the x-component of acceleration multiplied by the change in the x-component of position.

Final y-component of velocity · Initial y-component of velocity · y-component of acceleration · Change in y-component of position

$$v_{fy}^2 = v_{iy}^2 + 2\,a_y\,\Delta y$$

In words: The final y-compoent of velocity squared is equal to the initial y-compoent of velocity squared plus two times the y-component of acceleration multiplied by the change in the y-component of position.

Key Concepts

- The kinematics problem solving techniques from 1-D are directly applicable to 2-D problems.
- If an object is moving along a line that is not horizontal or vertical, you can simplify its motion as a 1-D problem rather than a 2-D problem.
- Known and unknown lists help organize kinematic information as well as your thoughts.
- It is highly recommended to not attempt to do algebra (i.e. re-arrange kinematic equations and/or plug them into each other) until you have identified the same number of equations as you have unknowns.
- Recall that time is a scalar; there is no x-component of time or y-component of time, there is only one time which is the same value for both the x and y kinematic analysis.

Act I: 2-D Kinematics

Questions

K2.2-2:

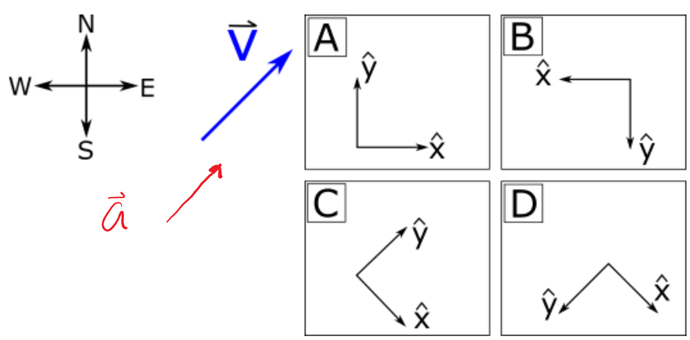
Description: Choose a coordinate system that simplifies the mathematical representation. (3 minutes + 3 minutes)

Learning Objectives: [1, 2]

Problem Statement: Benny is in a scooty puff jr spaceship and moving in the galactic northeast direction, when it speeds up in the same direction.

(a) If we were asked to solve for how far Benny travels in some given amount of time, which coordinate system would simplify the mathematical analysis the most?

(1) A.
(2) B.
(3) C. $a_y = +$ $a_x = 0$
(4) D. $a_y = -$ $a_x = 0$



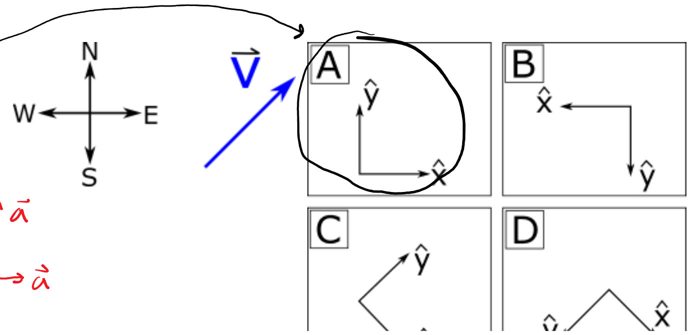
$\vec{a}$

(b) Which of the following cases would make a standard coordinate system also a wise choice for Benny?

F (1) The acceleration is in the same direction of motion.
F (2) The acceleration is in the opposite direction of motion.
T (3) The acceleration is in the galactic north direction.
T (4) The acceleration is in the galactic east direction.

$\uparrow \vec{a}$

$\rightarrow \vec{a}$



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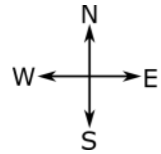
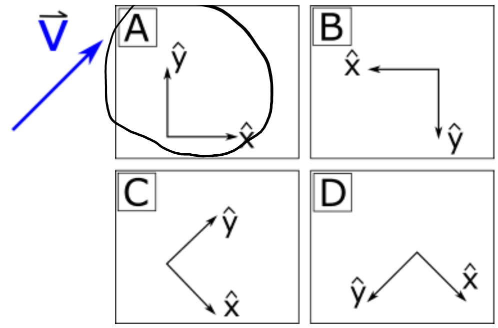


$\uparrow \vec{a}$

$\rightarrow \vec{a}$



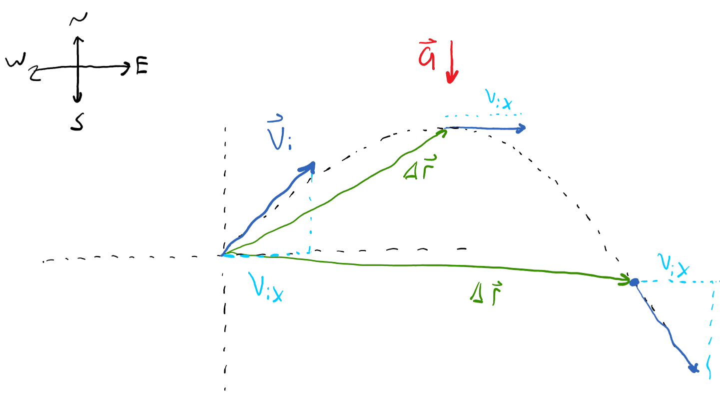
K2.2-3:

Description: Construct a physical representation from a written description. (5 minutes)

Learning Objectives: [1, 3]

Problem Statement: A spaceship is traveling in the galactic northeast direction. The ship's thrusters then create a large constant acceleration in the galactic south direction. Which of the following statements are *necessarily* true regarding the time after the thrusters have fired?

? (1) The ship will be moving in the southeast direction.
F (2) The ship will eventually be moving in the galactic south direction.
F (3) The ship's displacement will be in the galactic south direction.
T (4) The ship will eventually be moving with both southern and eastern components.
F (5) The change in the ship's velocity will be in the southern direction.

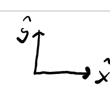


K2.2-4:

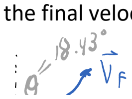
Description: 2-D kinematics problem solving for displacement. (6 minutes + 2 minutes + 2 minutes + 5 minutes + 1 min + 4 minutes + 2 minutes + 5 minutes)

Learning Objectives: [1, 3, 4, 5, 6, 7, 15]

Problem Statement: A spaceship's controls fail for 5.00 s and during this time the thrusters on the ship give it an acceleration of 4.472 m/s^2 in a direction 63.43° from the positive x-direction towards the positive y-direction. They know that after the incident the ship was traveling with a speed of 15.81 m/s in a direction 18.43° from the positive y-direction towards the positive x-direction.



(a) Find the final velocity and the average acceleration in standard Cartesian coordinates.



$V_{fy} = |\vec{V}_f| \cos\theta$

$a_y = |\vec{a}| \sin\phi$

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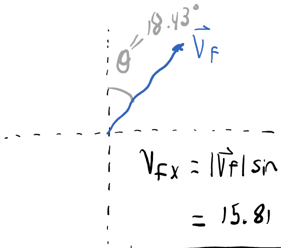


$V_{fy} = |\vec{V}_f| \cos\theta$

$= 15.81 \cos(18.43°)$

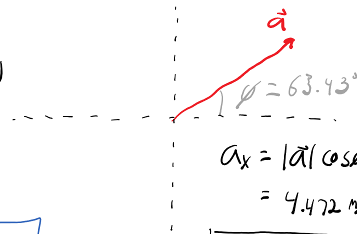
$\boxed{V_{fy} \simeq 15.0 \text{ m/s}}$

$V_{fx} = |\vec{V}_f| \sin\theta$

$= 15.81 \text{ m/s} \sin(18.43°)$

$\boxed{V_{fx} \simeq 5.00 \text{ m/s}}$

$\boxed{\vec{V}_f = \langle 5, 15 \rangle \text{ m/s}}$



$a_y = |\vec{a}| \sin\phi$

$= 4.472 \text{ m/s}^2 \sin(63.43°)$

$\boxed{a_y \simeq 4.00 \text{ m/s}^2}$

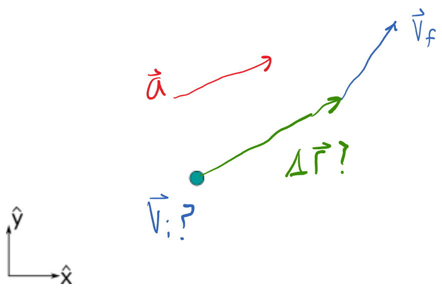
$a_x = |\vec{a}| \cos\phi$

$= 4.472 \text{ m/s}^2 \cos(63.43°)$

$\boxed{a_x \simeq 2.00 \text{ m/s}^2}$

$\vec{a} = \langle 2, 4 \rangle \text{ m/s}^2$

(b) Draw a physical representation.



(c) Identify which of the following quantities are known.

? (1) **Δx**
? (2) $\mathbf{v}_{ix}$
✓ (3) $\mathbf{v}_{fx}$
✓ (4) $\mathbf{a}_x$
✓ (5) **Δt**

(d) Identify which of the following quantities are known.

? (1) **Δy**
? (2) $\mathbf{v}_{iy}$
✓ (3) $\mathbf{v}_{fy}$
✓ (4) $\mathbf{a}_y$

(e) Find **Δx**.

$V_{fx} = V_{ix} + a_x \Delta t$

$5 \text{ m/s} = V_{ix} + 2 \text{ m/s}^2 (5 s)$

$V_{ix} = -5 \text{ m/s}$

$\Delta X = V_{ix} \Delta t + \frac{1}{2} a_x \Delta t^2$

$= -5 \text{ m/s}(5 s) + \frac{1}{2}(2 \text{ m/s}^2)(5)^2$

$\boxed{\Delta X = 0}$

x				
Δx	v_{ix}	v_{fx}	a_x	Δt

K	UK
$V_{fx} = 5 \text{ m/s}$	ΔX
$a_x = 2 \text{ m/s}^2$	V_{ix}
$\Delta t = 5 s$	

$\Delta x = v_{ix} \Delta t + \frac{1}{2} a_x \Delta t^2$ } 2 Eqns 2 UKNS

$v_{fx} = v_{ix} + a_x \Delta t$

$v_{fx}^2 = v_{ix}^2 + 2\, a_x \Delta x$ } 2 Eqns 2 UKNS

(f) Find **Δy**.

$V_{fy} = V_{iy} + a_y \Delta t$

$15 \text{ m/s} = V_{iy} + (4 \text{ m/s}^2)(5 s)$

$V_{iy} = -5 \text{ m/s}$

$\Delta y = V_{iy} \Delta t + \frac{1}{2} a_y \Delta t^2$

$= -5 \text{ m/s}(5 s) + \frac{1}{2}(4 \text{ m/s}^2)(5)^2$

$\boxed{\Delta y = 25 \text{ m}}$

y				
Δy	v_{iy}	v_{fy}	a_y	Δt

K	UK
$V_{fy} = 15 \text{ m/s}$	Δy
$a_y = 4 \text{ m/s}^2$	V_{iy}
$\Delta t = 5 \text{ sec}$	

$\Delta x = v_{ix} \Delta t + \frac{1}{2} a_x \Delta t^2$

$v_{fx} = v_{ix} + a_x \Delta t$

$v_{fx}^2 = v_{ix}^2 + 2\, a_x \Delta x$

(g) Which of the following figures are a correct physical representation including trajectory (dotted line)?

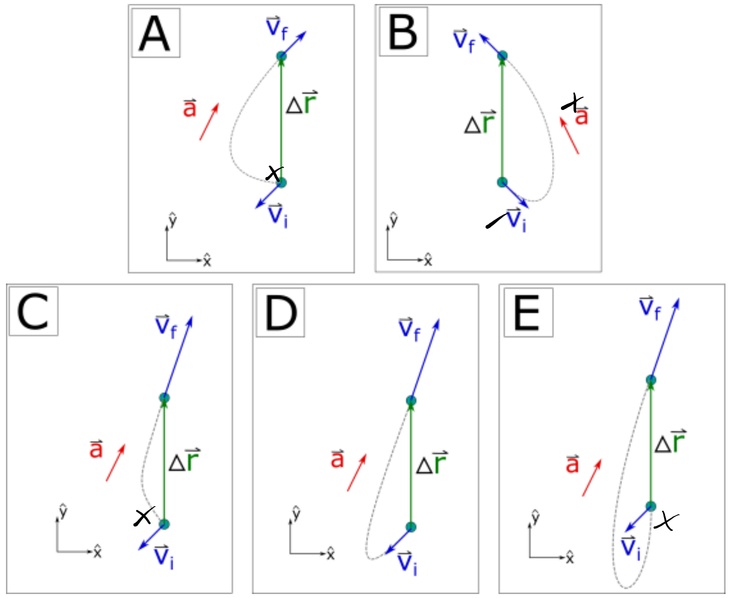
(1) A
(2) B
(3) C
(4) D
(5) E

$\vec{a} = \langle 2, 4 \rangle \, m/s^2$

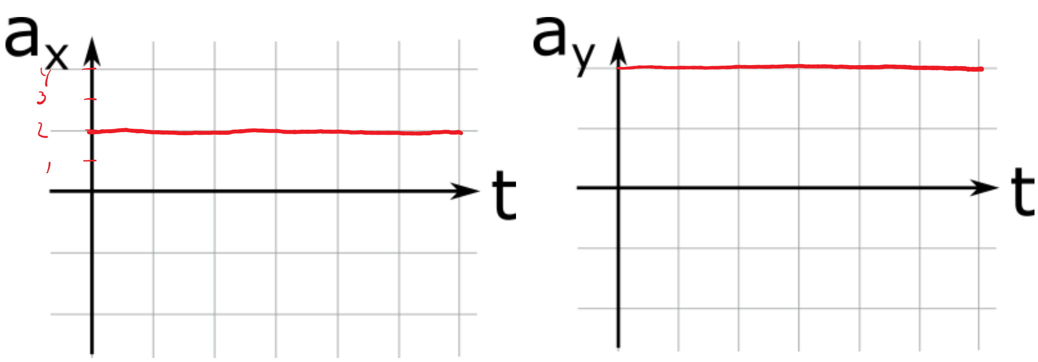
$\vec{V}_f = \langle 5, 15 \rangle \, m/s$

$\vec{V}_i = \langle -5, -5 \rangle \, m/s$

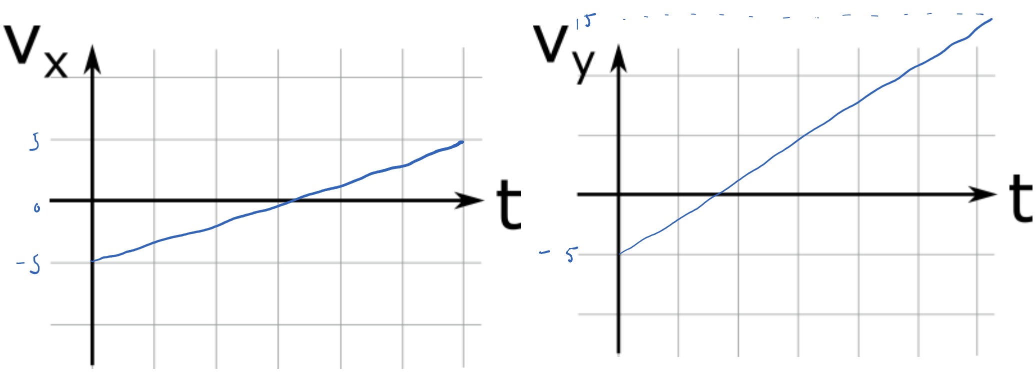
$\Delta\vec{r} = \langle 0, 25 \rangle \, m$



(h) Sketch the acceleration as a function of time for both the x and y components.



(i) Sketch the velocity as a function of time for both the x and y components.



Conceptual questions for discussion

1. Do you agree with the following statement? *If the velocity of an object is not horizontal or vertical then a 2-D kinematic analysis is required.*
2. If a spaceship has an initial velocity in the galactic north direction and a thruster that can only provide an acceleration in the galactic east direction, can the spaceship ever travel in the galactic east direction?

Hints

K2.2-1: No hints.

K2.2-2: Can you choose a coordinate system to avoid breaking vectors into components?

K2.2-3: Draw a physical representation. If an object has a velocity in the x-direction and no acceleration in the x-direction, does the velocity in the x-direction ever change?

K2.2-4: No hints.