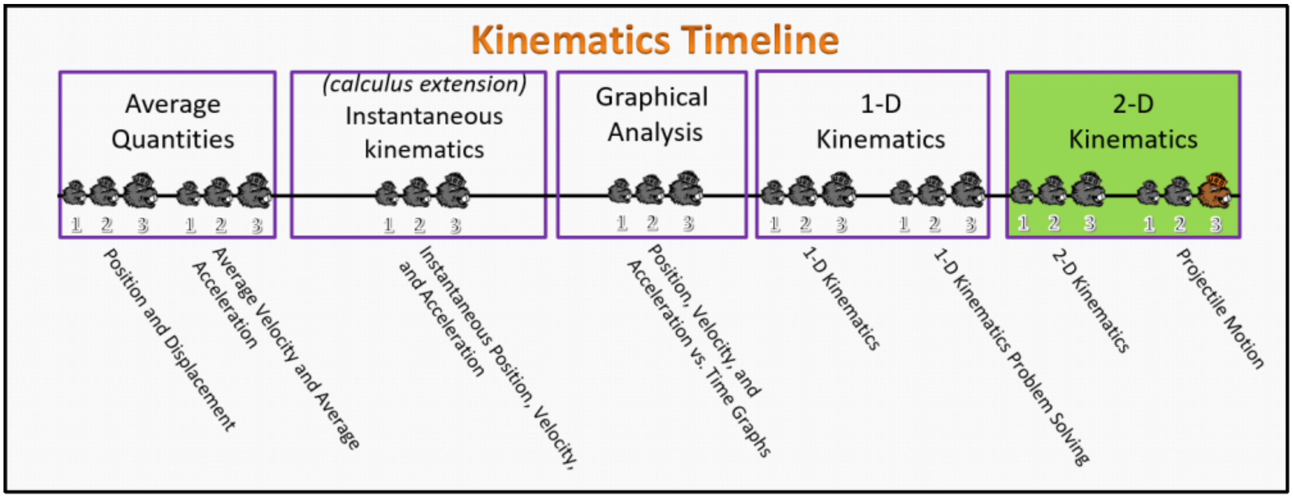
(K2.L2.3-sols) Practice Stage Solutions

Thursday, March 29, 2018 8:34 PM

2-D Kinematics
Foundation Stage (K2.L2.3)

Post-Lecture 2
Projectile Motion



Questions

K2.L2.3-01

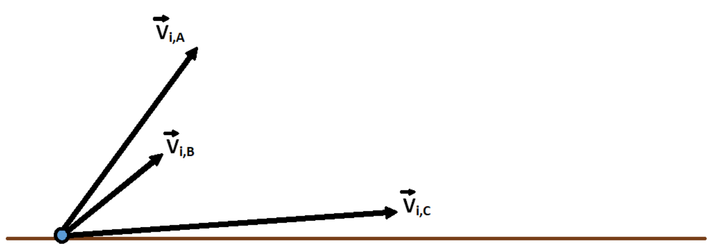
Description: 2-D projectile motion of 3 objects with different trajectories

Learning Objectives: [x]

Problem Statement: Three projectiles, **A**, **B**, and **C**, are launched as shown. The arrows represent the initial velocities of the projectiles, drawn to scale. All three projectiles eventually land on the ground. Neglect air resistance.

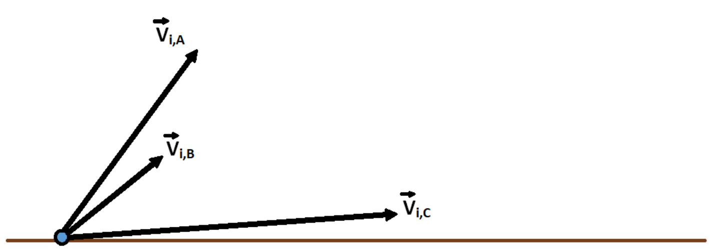
(a) Which projectile has the greatest initial speed?

(1) A
(2) B
(3) C
(4) Same for all three projectiles



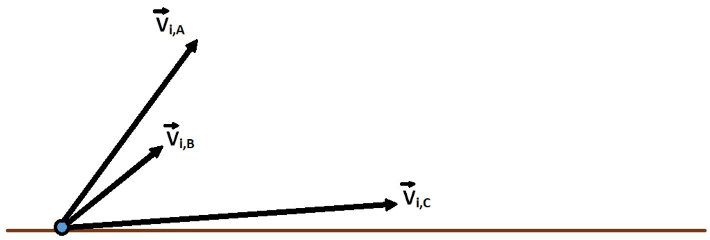
(b) Which projectile has the smallest speed immediately before it impacts the ground?

(1) A
(2) B
(3) C
(4) Same for all three projectiles



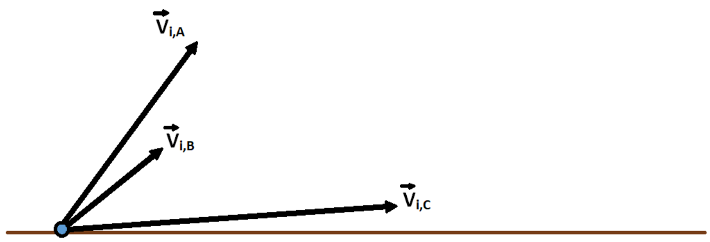
(c) Which projectile has the greatest initial horizontal component of the velocity?

(1) A
(2) B
(3) C
(4) Same for all three projectiles



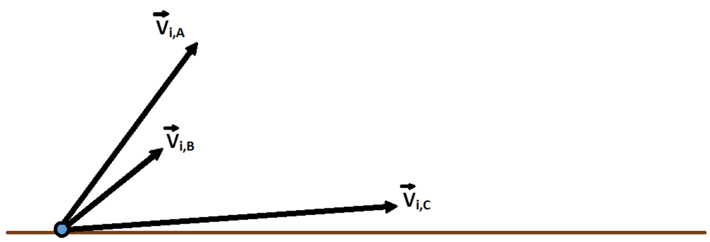
(d) Which projectile has the smallest final horizontal component of the velocity?

(1) A
(2) B
(3) C
(4) Same for all three projectiles



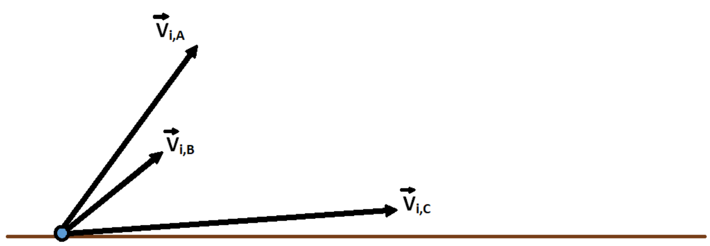
(e) Which projectile has the greatest acceleration at the top point of the flight?

(1) A
(2) B
(3) C
(4) Same for all three projectiles



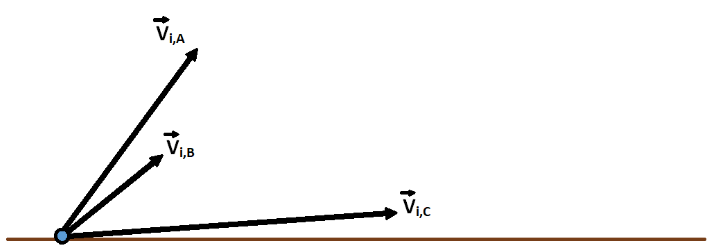
(f) Which projectile has the greatest speed at the top point of the flight?

(1) A
(2) B
(3) C
(4) Same for all three projectiles



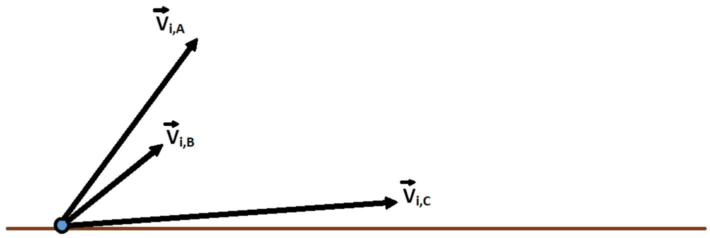
(g) Which projectile reaches the greatest maximum height?

(1) A
(2) B
(3) C
(4) Same for all three projectiles



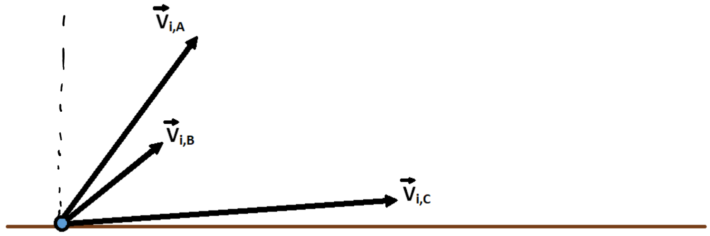
(h) Which projectile spends the longest time in the air?

(1) A
(2) B
(3) C
(4) Same for all three projectiles



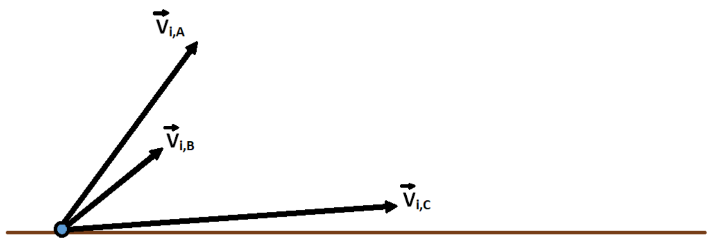
(i) Which projectile has the greatest horizontal range?

(1) A
(2) B
(3) C
(4) Same for all three projectiles



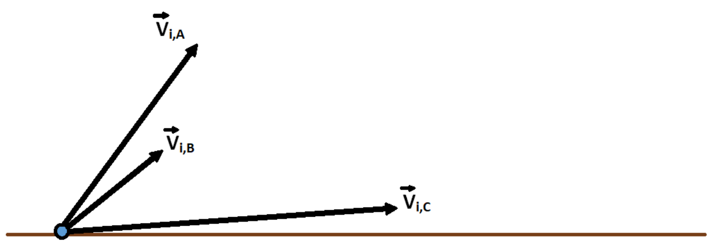
(j) Assuming that all three projectiles are still in the air three seconds into the flight, which projectile has the greatest height above the ground at that moment?

(1) A
(2) B
(3) C
(4) Same for all three projectiles



(k) Assuming that all three projectiles are still in the air three seconds into the flight, which projectile has the greatest horizontal displacement at that moment?

(1) A
(2) B
(3) C
(4) Same for all three projectiles



K2.L2.3-02

Description: Projectile frog basics

Learning Objectives: [x]

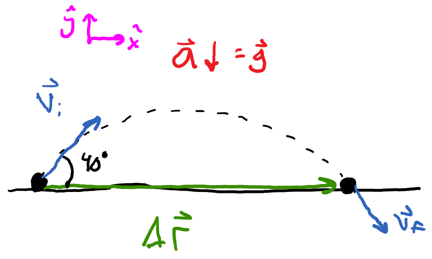
Problem Statement: A frog leaps from flat ground with a speed of 2.00 m/s at an angle 40.0° up from the horizontal. Use g = 9.82 m/s^2

(a) What is the time of flight, in seconds, for the frog?

x y

Problem Statement: A frog leaps from flat ground with a speed of 2.00 m/s at an angle 40.0° up from the horizontal. Use g = 9.82 m/s²

(a) What is the time of flight, in seconds, for the frog?



x: K	x: UK
$v_{ix} = 1.53209$	Δx
$a_x = 0$	v_{fx}
	Δt

y: K	y: UK
$\Delta y = 0$	v_{fy}
$v_{iy} = 1.28558$	Δt
$a_y = -9.82$	

$v_{ix} = 2\cos 40° = 1.53209$ m/s

$v_{iy} = 2\sin 40° = 1.28558$ m/s

$$\Delta y = v_{iy}\Delta t + \tfrac{1}{2}a_y\Delta t^2$$

$$0 = 1.28558\,\Delta t + \tfrac{1}{2}(-9.82)\Delta t^2$$

$$0 = 1.28558 - 4.91\,\Delta t$$

$$\Delta t = 0.26183 \text{ sec}$$

$$\boxed{\Delta t = 0.262 \text{ sec}}$$

(b) What is the range, in centimeters, of the frog's flight?

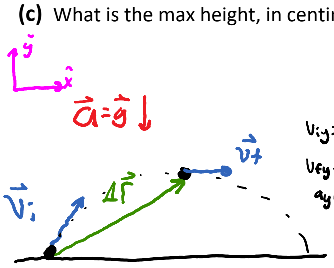
$$\Delta x = v_{ix}\Delta t + \cancel{\tfrac{1}{2}a_x\Delta t^2}$$

$$\Delta x = 1.53209(0.26183) \text{ m}$$

$$\Delta x = 0.40115 \text{ m}$$

$$\Delta x = 0.401 \text{ m} \rightarrow \boxed{40.1 \text{ cm}}$$

(c) What is the max height, in centimeters, the frog achieves?



y: K	y: UK
$v_{iy} = 1.28558$	Δy
$v_{fy} = 0$	Δt
$a_y = -9.82$	

$$v_{fy}^2 = v_{iy}^2 + 2a_y\Delta y$$

$$0^2 = 1.28558^2 + 2(-9.82)(\Delta y)$$

$$\Delta y = 0.0841505 \text{ m}$$

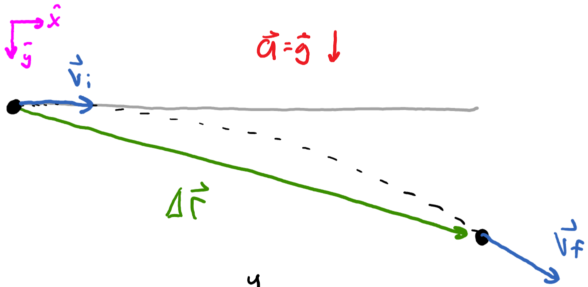
$$\Delta y = 0.0842 \text{ m} \rightarrow \boxed{8.42 \text{ cm}}$$

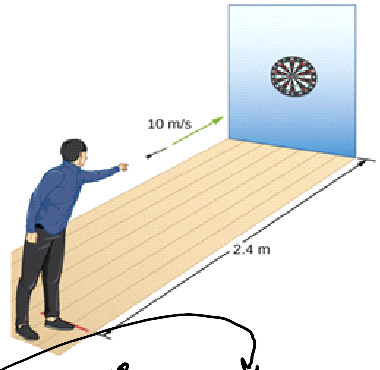
K2.L2.3-03

Description: Projectile dart missing the target

Learning Objectives: [x]

Problem Statement: A dart is thrown horizontally at a speed of 10 m/s at the bull's-eye of a dartboard 2.4 m away, as in the following figure. How far below the intended target, in centimeters, does the dart hit? Use g = 9.82 m/s²





x

K	UK
$\Delta x = 2.4$	
$V_{ix} = 10$ m/s	V_{fx}
$a_x = 0$	Δt

y

K	UK
$V_{iy} = 0$	Δy
	V_{fy}
$a_y = 9.82$	Δt

$\Delta x = V_{ix}\Delta t + \cancel{\frac{1}{2}a_x\Delta t^2}$

$2.4 = 10\,\Delta t$

$\Delta t = 0.24$ sec

$\Delta y = \cancel{V_{iy}\Delta t} + \frac{1}{2}a_y\Delta t^2$

$\Delta y = \frac{1}{2}(9.82)(0.24)^2$

$\Delta y \approx 0.28816$ m

$\boxed{\Delta y \approx 28.3 \text{ cm}}$

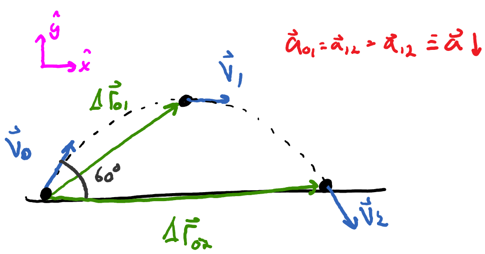
K2.L2.3-04

Description: Projectile motion on a different planet

Learning Objectives: [x]

Problem Statement: Somewhere in the vast flat tundra of planet Tehar, a projectile is launched from the level ground at an angle of 60 degrees. It reaches the maximum height of 15 m. The acceleration due to gravity is 30 m/s².

(a) Find the time in seconds the projectile spends in the air.



$V_{0x} = V_0 \cos 60$

$V_{0y} = V_0 \sin 60$

$0 \to 1$

y

K	UK
$\Delta y_{01} = 15$ m	$V_{0y} = V_0 \sin 60$
$V_{1y} = 0$	
$a_y = -30$ m/s²	Δt_{01}

$V_{1y}^2 = V_{0y}^2 + 2a_y\Delta y_{01}$

$0 = V_{0y}^2 + 2(-30)(15)$

$V_{0y} = 30$ m/s

$0 \to 2$

y

K	UK
$\Delta y_{02} = 0$	
$V_{0y} = 30$	V_{1y}
$a_y = -30$	Δt_{02}

$\Delta y_{02} = V_{0y}\Delta t_{02} + \frac{1}{2}a_y\Delta t_{02}^2$

$0 = 30\,\Delta t_{02} - 15\,\Delta t_{02}^2$

$0 = 30 - 15\,\Delta t_{02}$

$\boxed{\Delta t_{02} = 2 \text{ sec}}$

(b) Find the initial speed of the projectile in m/s.

$$V_{oy} = |\vec{V}_o| \sin 60°$$

$$|\vec{V}_o| = \frac{V_{oy}}{\sin 60}$$

$$|\vec{V}_o| = 34.64102 \text{ m/s} \rightarrow \boxed{34.6 \text{ m/s}}$$

(c) Find the minimum speed of the projectile in m/s.

@ MAX HEIGHT

$$|\vec{V}_1| = V_{ox}$$

$$V_{ox} = V_o \cos 60$$

$$= 34.64102 \cos 60$$

$$V_{ox} = 17.32051 \text{ m/s} \rightarrow \boxed{17.3 \text{ m/s}}$$

(d) Find the horizontal range of the projectile in meters.

$$\Delta X_{02} = V_{ox} \Delta t_{02} + \tfrac{1}{2} \cancel{a_x} \Delta t_{02}$$

$$= 17.32051 (2)$$

$$\Delta X_{02} = 34.64102 \text{ m} \rightarrow \boxed{34.6 \text{ m}}$$

K2.L2.3-05

Description: A golfer hitting uphill

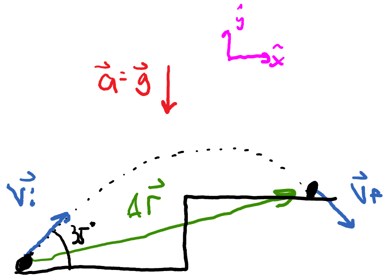
Learning Objectives: [x]

Problem Statement: A golfer hits the ball off the tee at an angle of thirty-five degrees from the horizontal with a speed of 46 m/s. It lands on the green while on the way down, which is elevated 5.50 m higher than the tee. How much time elapsed from when the ball was hit to when it landed on the green? Use g = 9.82 m/s^2

(1) 0.217 s
(2) 5.16 s
(3) 3.47 s
(4) 1.97 s
(5) 7.87 s

$\hat{y}$, $\hat{x}$

y	
K	UK
$\Delta y = 5.5$	V_{fy}
$V_{iy} = 46 \sin 35$	



K	UK
$\Delta y = 5.5$	V_{fy}
$V_{iy} = 46\sin 35$	
$a_y = -9.82$	Δt

$$\Delta y = V_{iy}\Delta t + \tfrac{1}{2} a_y \Delta t^2$$

$$5.5 = 46\sin 35\,\Delta t - 4.91\Delta t^2$$

$$-4.91\Delta t^2 + 46\sin 35\,\Delta t - 5.5 = 0$$

$\Delta t = 0.21724$ sec on way up

or

$\Delta t = 5.15639$ sec on way down

$$\boxed{\Delta t = 5.16 \text{ sec}}$$

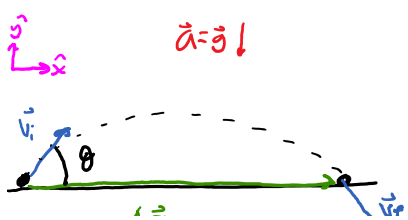
K2.L2.3-07

Description: Result of doubling initial conditions of kinematic variables

Learning Objectives: [x]

Problem Statement: In the absence of air resistance, an old textbook is launched from and returns to ground level. It follows a parabolic trajectory and has a horizontal range of 25 m. Suppose the launch speed is doubled, and the projectile is fired at the same angle above the ground. What is the new range of the launched book?

(1) 12.5 m
(2) 25 m
(3) 50 m
(4) 75 m
(5) 100 m
(6) 225 m
(7) 625 m



x

K	UK
	Δx
V_{ix}	V_{fx}
$a_x = 0$	Δt

$$\Delta x = V_{ix}\Delta t + \cancel{\tfrac{1}{2} a_x \Delta t^2}^{\,0}$$

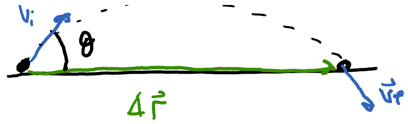
$$\Delta x = |\vec{V}_i|\cos\theta\,\Delta t$$

y

K	UK
$\Delta y = 0$	
V_{iy}	V_{fy}
a_y	Δt

$$\Delta y = V_{iy}\Delta t + \tfrac{1}{2} a_y \Delta t^2$$

$$0 = |\vec{V}_i|\sin\theta\,\Delta t - \tfrac{1}{2} g \Delta t^2$$



$\Delta x = |\vec{v}_i| \cos\theta \, \Delta t$

$0 = |\vec{v}_i| \sin\theta \, \Delta t - \frac{1}{2} g \Delta t^2$

$0 = |\vec{v}_i| \sin\theta - \frac{1}{2} g \Delta t$

$\Delta t = \frac{2|\vec{v}_i| \sin\theta}{g}$

$\Delta x = \frac{|\vec{v}_i| \cos\theta \, 2|\vec{v}_i| \sin\theta}{g}$

$\Delta x = \frac{2|\vec{v}_i|^2 \cos\theta \sin\theta}{g}$

If $\theta = \text{const}$

$\Delta x = |\vec{v}_i|^2$

If $|\vec{v}_i| \to 2|\vec{v}_i|$

Then $\Delta x \to 4 \Delta x$

$= 4(25) = \boxed{100 \text{ m}}$

K2.L2.3-08

Description: Given an initial height, a projectile is fired from the horizontal and hits the floor a distance D away. Find the initial velocity, and use that to find the angle necessary to graze the ceiling.

Learning Objectives: [x]

Problem Statement: When a bb gun is positioned such that its barrel is a height h above the floor and its barrel is parallel to the floor, a bb fired from the gun hits the floor a distance D from the end of the barrel.

(a) What is the muzzle velocity for the gun?

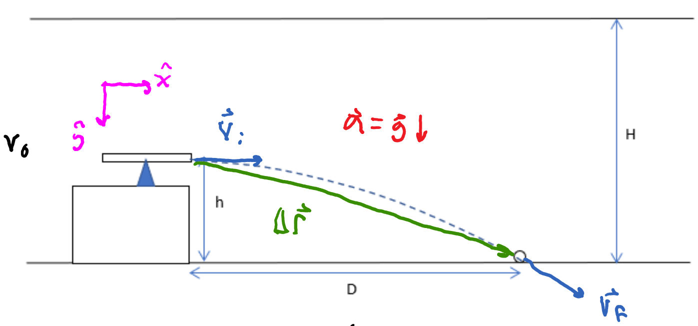
(1) $v_0 = h\sqrt{\frac{g}{2D}}$

(2) $v_0 = D\sqrt{\frac{gD}{2h^2}}$

(3) $v_0 = D\sqrt{\frac{g}{2h}}$

(4) $v_0 = \frac{1}{2}\sqrt{gh}$

$* \; |\vec{v}_i| = v_0$



x

K	ok
$\Delta x = D$	$v_{ix} = v_0$
$a_x = 0$	

y

K	ok
$\Delta y = h$	
$v_{iy} = 0$	

(4) $v_0 = \frac{1}{2}\sqrt{gh}$

(5) $v_0 = h\sqrt{\frac{gD}{2}}$

$\Delta x = D$	$V_{ix} = V_0$
$a_x = 0$	$\cancel{V_{fx}}$
	Δt

$\Delta y = h$	
$V_{iy} = 0$	V_{fy}
$a_y = g$	Δt

$$\Delta x = V_{ix}\Delta t + \cancel{\tfrac{1}{2}a_x\Delta t^2}$$

$$D = V_0 \Delta t$$

$$\Delta y = V_{iy}\Delta t + \tfrac{1}{2}a_y\Delta t^2$$

$$h = 0 + \tfrac{1}{2}g\,\Delta t^2$$

$$h = \tfrac{1}{2}g\,\Delta t^2$$

$$\Delta t = \sqrt{\frac{2h}{g}}$$

$$D = V_0\sqrt{\frac{2h}{g}}$$

$$\boxed{V_0 = D\sqrt{\frac{g}{2h}}}$$

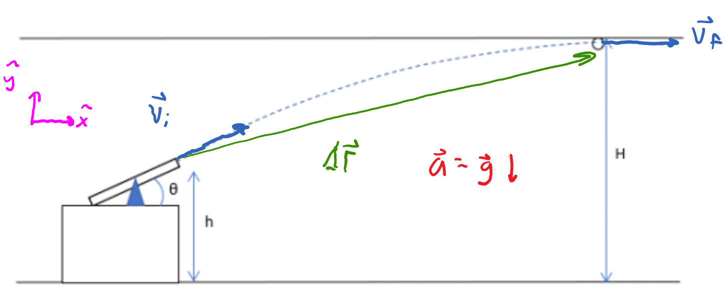
(b) You rotate the gun such that the end of the barrel is still h above the floor, but now the barrel makes an angle θ with the horizontal. What angle θ will allow you to just graze the ceiling at height H above the floor? Use: H = 2.5 m, h = 1.2 m, D = 3.6 m, g = 9.8 m/s². Note: The horizontal distance the bb travels is not necessarily the same as the distance D in the first part.

(1) 21°
(2) 23°
(3) 26°
(4) 29°
(5) 44°

$$V_0 = D\sqrt{\frac{g}{2h}}$$

$$= 3.6\sqrt{\frac{9.8}{2(1.2)}}$$

$$V_0 = 7.27461 \text{ m/s}$$



x

K	UK
	Δx
	$V_{ix} = V_0\cos\theta$
	$\cancel{V_{fx}}$
$a_x = 0$	Δt

y

K	UK
$\Delta y = H - h$	
	$V_{iy} = V_0\sin\theta$
$V_{fy} = 0$	
$a_y = -g$	Δt

V_f eqns

$$V_{iy} = V_0\sin\theta$$

$$\sqrt{2g(H-h)} = V_0\sin\theta$$

$$\sqrt{2g(H-h)}$$

$v_{fy} = 0$

$a_y = -g \quad | \, \Delta t$

$$v_{fy}^2 = v_{iy}^2 + 2a_y \Delta y$$

$$0 = v_{iy}^2 + 2(-g)(H-h)$$

$$v_{iy} = \sqrt{2g(H-h)}$$

$$\sin\theta = \frac{\sqrt{2g(H-h)}}{v_0}$$

$$\theta \approx 43.93875^\circ$$

$$\boxed{\theta \approx 43.9^\circ}$$