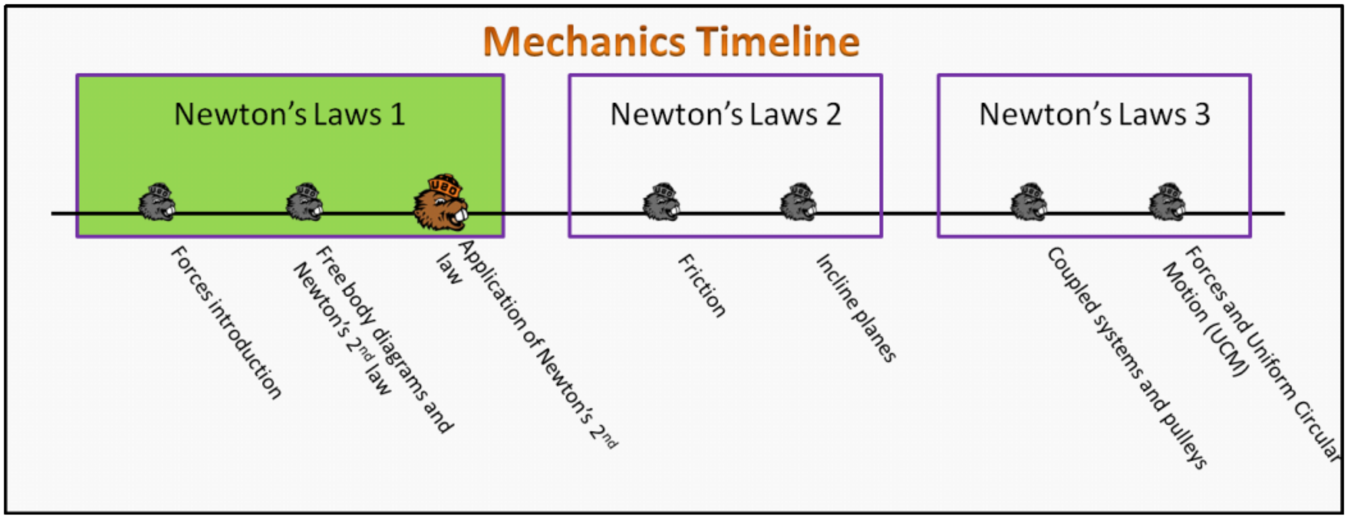
(N1.L3.2-sols) Foundation Stage Solutions

Monday, January 22, 2018 5:44 PM

Newton's Laws 1

Foundation Stage (N1.2)

lecture 3
Application of Newton's 2nd law



Textbook Chapters

- **BoxSand** :: KC videos ([Newton's Second Law](#))
- **Giancoli** (Physics Principles with Applications 7th) :: 4-4
- **Knight** (College Physics : A strategic approach 3rd) :: 5.2 ; 5.3
- **Knight** (Physics for Scientists and Engineers 4th) :: 6.2

Warm up

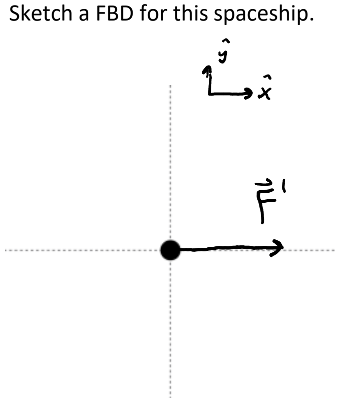
N1.2-1:

Description: Given net force, mass, and initial velocity, find acceleration and distance travelled.

Learning Objectives: [?] - Can you identify the objectives from the previous lecture, and this lecture, that this question is relevant to?

Problem Statement: A 4000-kg spaceship has a single force acting on it: $\vec{\mathbf{F}}^1 = < 1000\,, 0 > N$.

(a) Sketch a FBD for this spaceship.



(b) What is the acceleration of this spaceship?

Force Analysis

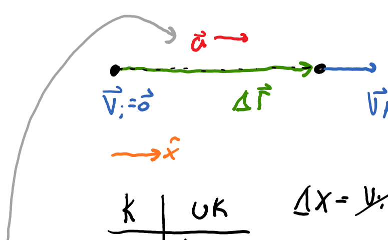
$\Sigma F_x = m_1 a_{1x}$

$|\vec{F}^1| = m_1 a_{1x}$

$1000\ N = 4000\ kg\ a_{1x}$

(c) If the ship starts from rest, how far does it travel in 30 seconds?

KINEMATICS



K	UK

$\Delta x = \cancel{v_{ix}\Delta t}^{\,0} + \frac{1}{2} a_x \Delta t^2$

$\xrightarrow{\quad} x$

$1000\ N = 4000\ kg\ a_{1x}$

$a_{1x} = 0.25\ m/s^2$

K	UK
$v_{ix}=0$	Δx
$a_x = .25\ m/s^2$	v_{fx}
$\Delta t = 30\ s$	

$\Delta x = \cancel{v_{ix}\Delta t}^{\,0} + \frac{1}{2} a_x \Delta t^2$

$\Delta x = \frac{1}{2}(.25)(30)^2$

$\boxed{\Delta x \simeq 113\ m}$

Selected Learning Objectives

1. Demonstrate the fact that a force is inherently an interaction between two objects or two systems.
2. Explain Newton's 1st law as it relates to inertia.
3. Explain Newton's 1st law and how it relates to the 2nd law.
4. Define static and dynamic equilibrium.
5. Explain Newton's 2nd Law.
6. Explain Newton's 3rd Law force pairs.
7. Define the point particle model.
8. Define a system(s) boundary and adhere their analysis to that boundary(s).
9. Identify the type of forces interacting with your system and the direction they are applied.
10. Differentiate between contact and non-contact forces.
11. Demonstrate that the normal force is always perpendicular to the surface.
12. Draw a free-body-diagram (FBD) for the system(s).
13. Demonstrate the ability to draw a *properly scaled* FBD.
14. (UPMF) Draw the coordinate system that reduces the complexity of the vector analysis next to the FBD.
15. Apply geometry to determine appropriate angles for the given coordinate system.
16. Find the components of a force in the chosen coordinate system.
17. Differentiate between *A* force and a *NET* force.
18. Apply Newton's 2nd law in the mathematical representation.
19. Differentiate between static and dynamic equilibrium.
20. Differentiate between weight and apparent weight.
21. Synthesize a force and kinematics analysis via the acceleration.
22. Demonstrate that the net force points in the same direction as the acceleration.
23. Demonstrate the fact that the net force can be in the opposite direction of the motion.

Key Terms

- Review N1 lecture 1 and lecture 2 key terms

Key Equations

Newton's 2nd law

Force external to the system

Mass of the system

Acceleration of the center of mass of the system

$$\sum \vec{F}_{\substack{\text{external}\\ \text{on system}}} = m_{\text{system}}\, \vec{a}_{cm}$$

Net (i.e. add up all of the __)

In words: The net force external to the system is equal to the mass of the system multiplied by the acceleration of the center of mass of the system.

Key Concepts

- Force analysis and kinematics are related to each other via acceleration.
- If a component of the net force is plotted as a function of acceleration, the slope is the mass of the system.
- Review how to move between position vs time, velocity vs time, and acceleration vs time graphs.
- If net force for 1-D motion is plotted as a function of time, the area divided by mass gives velocity as a function of time.
- Normal forces are always perpendicular to the two surfaces in contact.

Act I: Graphical analysis with forces

Questions

N1.2-2:

Description: Given a plot of net force as a function of acceleration find the mass. (3 minutes)

Learning Objectives: []

Problem Statement: Haley the hyena adjusts the net force on her spacesuit and measures the corresponding acceleration while traveling in 1-D. A plot of the net force vs acceleration of Haley is shown below. What is the mass of Haley and her spacesuit?

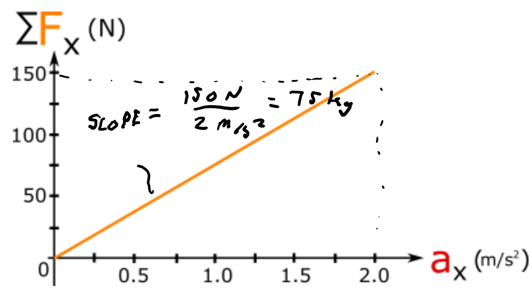
(1) 75 kg
(2) 150 kg
(3) 300 kg
(4) 150 lbs
(5) Need more information to determine.

$\Sigma F_x = m\, a_x$

"y" = "m" "x"

SLOPE = m

M = 75 kg



N1.2-3:

Description: Given a plot of velocity as a function of time, and the mass, determine the net force. (4 minutes)

Learning Objectives: [21]

Problem Statement: Caine the coyote has a mass of 50 kg. The velocity vs time graph for Caine as he ran onto a sheet ice is shown below. His motion is 1-D. What is the net force acting on Caine between 2.0 and 4.0 seconds?

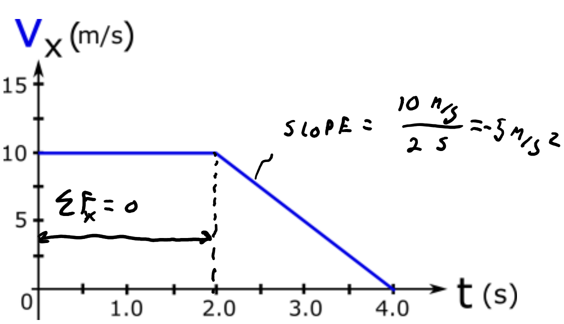
(1) 0 N $\hat{x}$
(2) 2.5 N $\hat{x}$
(3) 5.0 N $\hat{x}$
(4) - 10 N $\hat{x}$
(5) - 250 N $\hat{x}$

$\Sigma F_x = m a_x$

$= 50\text{kg}(-5\ \text{m/s}^2)$

$= -250\ \text{N}$

$V_x(t)$ SLOPE $a_x(t)$



N1.2-4:

Description: Given a plot of net force as a function of time, determine the velocity at a certain time. (5 minutes)

Learning Objectives: [21]

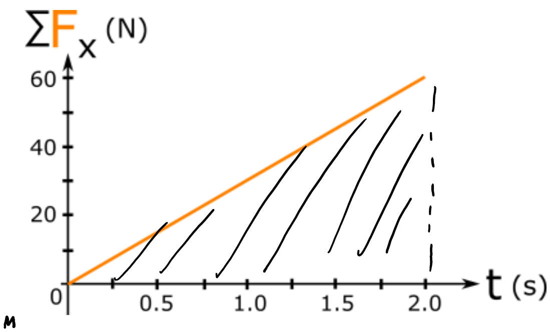
Problem Statement: Westyln the wolf attaches a rocket on to her back and fires it at **t** = 0 s while standing on ice initially at rest. Westyln is 80 kg and the rocket's mass is negligible. A plot of the net force on Westyln as a function of time is given below. What is Westyln's velocity after 2 seconds has elapsed?

(1) 0.375 m/s $\hat{x}$
(2) 0.75 m/s $\hat{x}$
(3) 30.0 m/s $\hat{x}$
(4) 60 m/s $\hat{x}$
(5) 2400 m/s $\hat{x}$
(6) 4800 m/s $\hat{x}$

$\Sigma F_x = m a_x$

$a_x = \frac{\Sigma F_x}{m}$ AREA $v_x(t)$ $a_x(t)$

... OR ... $\frac{AREA}{m} = a_x$



$AREA = \frac{1}{2}(2\ s)(60\ N) = 60\ \frac{kg\,m}{s}$

$\frac{AREA}{m} = 60\ \frac{kg\,m}{s} \cdot \frac{1}{80\ kg} = \boxed{0.75\ \frac{m}{s}}$

N1.2-5:

Description: Given a plot of a force as a function of time, determine the acceleration. (2 minutes)

Learning Objectives: [17, 21]

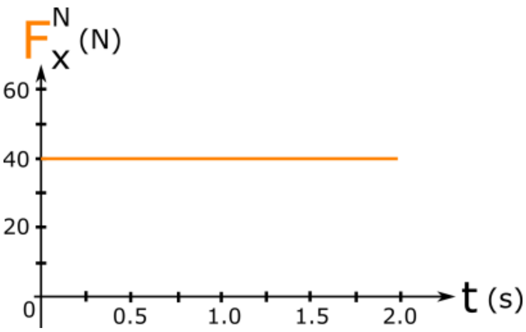
Problem Statement: A normal force on Jackie the jackal is plotted as a function of time below. Jackie's mass is 10 kg. What is Jackie's acceleration between 0 and 2.0 seconds?

(1) 0 m/s² $\hat{x}$
(2) 4 m/s² $\hat{x}$
(3) 40 m/s² $\hat{x}$
(4) Unable to determine.

$\Sigma F_x = m a_x$

↑

OTHER FORCES ACTING ON JACKIE?



Act II: Connecting forces and kinematics

N1.2-6:

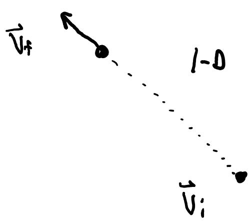
Description: Given net force and mass, determine how far an object travels in a given amount of time. (1.5 minutes + 1.5 minutes + 5 minutes)

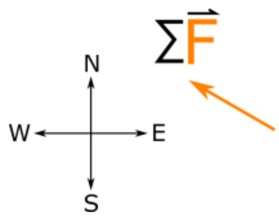
Learning Objectives: [14, 18, 21, 22]

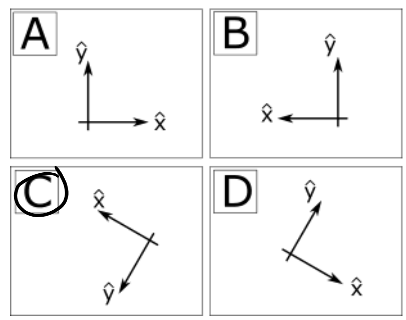
Problem Statement: Your 50-kg Scooty Puff Jr. spaceship provides a constant thrust of 500 N 25 degrees north of west as shown below. The thrust is the only force acting on the spaceship. We wish to find out how far you've travelled in the first 2 minutes assuming you start from rest.

(a) To help us get started, choose which coordinate system will make the task of finding the distance most easy.

(1) A
(2) B
(3) C
(4) D







(b) What physics quantities connect a force analysis to a kinematics analysis?

(1) displacement
(2) velocity
(3) acceleration
(4) time
(5) mass

(c) Your 50-kg Scooty Puff Jr. spaceship provides a constant thrust of 500 N 25 degrees north of west as shown above. The thrust is the only force acting on the spaceship. How far did you travel in the first 2 minutes assuming you started from rest.

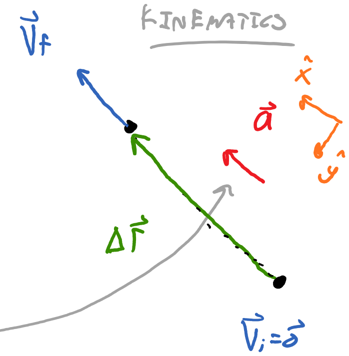
FORCE ANALYSIS

$\Sigma F_x = m a_x$

$500\ N = 50\ kg\ a_x$

$a_x = 10\ m/s^2$

KINEMATICS



K	UK
$V_{ix} = 0$	Δx
$a_x = 10\ m/s^2$	V_{fx}
$\Delta t = 120\ s$	

$\Delta x = \cancel{V_{ix}}^{0} \Delta t + \frac{1}{2} a_x \Delta t^2$

$\Delta x = \frac{1}{2}(10)(120)^2$

$\boxed{\Delta x = 72000\ m}$

$\Delta x = \frac{1}{2}(10)(120)^2$

$\boxed{\Delta x = 72000\ m}$

N1.2-7:

Description: Given forces acting on an object, mass, and distance information, determine final velocity. (2 minutes + 4 minutes + 4 minutes)

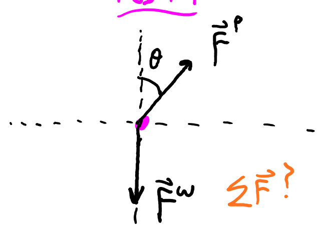
Learning Objectives: [8, 9, 12, 14, 16, 18, 21, 22]

Problem Statement: Tabitha the tadpole is 5 grams and trying to swim to stagnant water. There is a constant 0.001 N force ($\vec{F}^w$) from the water current in the stream in the southern direction. Tabitha's paddling provides an additional constant force ($\vec{F}^p$) of 0.0015 N in the direction towards the pool (30 degrees east of north). We wish to find Tabitha's velocity vector when she reaches the po ol.

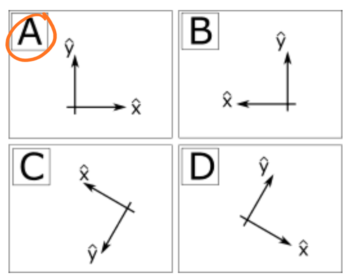
(a) Which coordinate system would make this problem the easiest in the mathematical representation?

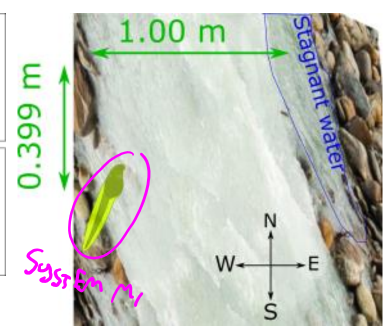
(1) A
(2) B
(3) C
(4) D

FBD m_1



$\theta = 30°$





(b) Which set of equations are a correct application of Newton's 2nd law applied to this scenario?

(1) A
(2) B
(3) C
(4) D

[A]

$\times$ $|\vec{F}^p| = m\,a_x$

$|\vec{F}^p| - |\vec{F}^w| = m\,a_y$

[C]

$\times$ $|\vec{F}^p| = m\,a_x$

$|\vec{F}^p| + |\vec{F}^w| = m\,a_y$

[B]

$\checkmark$ $|\vec{F}^p|\sin(30°) = m\,a_x$

$\checkmark$ $|\vec{F}^p|\cos(30°) - |\vec{F}^w| = m\,a_y$

[D]

$\times$ $|\vec{F}^p|\cos(30°) = m\,a_x$

$|\vec{F}^p|\sin(30°) - |\vec{F}^w| = m\,a_y$

(c) Tabitha the tadpole is 5 grams and trying to swim to stagnant water. There is a constant 0.001 N force ($\vec{F}^w$) from the water current in the stream in the southern direction. Tabitha's paddling provides an additional constant force ($\vec{F}^p$) of 0.0015 N in the direction towards the pool (30 degrees east of north). **Find Tabitha's velocity vector when she reaches the pool**.

FORCE ANALYSIS

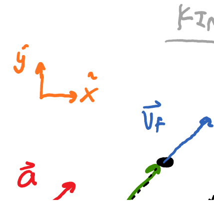
$|\vec{F}^p|\sin(30°) = m a_x$

$0.0015\ \sin(30) = \frac{5}{1000} a_x$

$|\vec{F}^p|\cos(30) - |\vec{F}^w| = m a_y$

$0.0015\cos(30) - 0.001 = \frac{5}{1000} a_y$

KINEMATICS



x	
k	uk

y	
k	uk

$0.0015\sin(30) = \frac{5}{1000} a_x$ $\quad$ $0.0015\cos(30) - 0.001 = \frac{5}{1000} a_y$

$a_x \approx 0.15\ m/s^2$ $\quad$ $a_y \approx 0.0598\ m/s^2$

$\vec{v}_f$ $\quad$ $\vec{a}$ $\quad$ $\Delta\vec{r}$ $\quad$ $\vec{v}_i$

k	uk
$\Delta x = 1m$	v_{fx}
$v_{ix} = 0$	Δt
$a_x = .15\ m/s^2$	

k	uk
$\Delta y = .349$	v_{fy}
$v_{iy} = 0$	Δt
$a_y = 0.0598$	

$v_{fx}^2 = \cancel{v_{ix}^2} + 2a_x\Delta x$ $\quad$ $v_{fy}^2 = \cancel{v_{iy}^2} + 2a_y\Delta y$

$v_{fx}^2 = 2(.15)(1)$ $\quad$ $v_{fy}^2 = 2(0.0598)(.349)$

$v_{fx} = 0.3\ m/s$ $\quad$ $v_{fy} \approx 0.0477\ m/s$

$$\boxed{\vec{v}_f = \langle 0.300,\ 0.0477 \rangle\ m/s}$$

N1.2-8:

Description: Given forces acting on object, mass, and initial velocity, determine how much time to come to a rest. (6 minutes)

Learning Objectives: [8, 9, 12, 14, 18, 21, 22, 23]

Problem Statement: After landing, some airplanes use reverse thruster mechanisms to help slow down. The Fokker 100 used a target-type reverse thruster as shown below. Let's assume that both reverse thrusters on the Fokker 100 are the only devices that slow down the plane after landing, and the total combined reverse thrust is a constant 50 kN. If the 30,000 kg plane lands with a velocity of 55 m/s due west, how much time does it take to come to a complete stop?

FORCE ANALYSIS

FBD M_1

$\hat{y}$ $\hat{x}$

$\vec{F}^N$ $\quad$ $\vec{F}^T$ $\quad$ $\vec{F}^g_{E1}$

W $\quad$ N $\quad$ E $\quad$ S

$\Sigma F_x = m_1 a_{1x}$

$|\vec{F}^T| = m_1 a_{1x}$

$50000\ N = 30000\ kg\ a_x$

$a_x = \frac{5}{3}\ m/s^2$

KINEMATICS

$\hat{y}$ $\hat{x}$ $\quad$ $\vec{a}$

$\vec{v}_f$ $\quad$ $\Delta\vec{r}$ $\quad$ $\vec{v}_i$

k	uk
$v_{ix} = -55\ m/s$	Δx
$v_{fx} = 0$	Δt
$a_x = \frac{5}{3}\ m/s^2$	

$\cancel{v_{fx}} = v_{ix} + a_x\Delta t$

$0 = -55\ m/s + \frac{5}{3}\ m/s^2\ \Delta t$

$$0 = -55\,m/s + \tfrac{5}{3}\,m/s^2\,\Delta t$$

$$\boxed{\Delta t \approx 33\ \text{sec}}$$

Act III: Forces and equilibrium

N1.2-9:

Description: Find magnitude of normal forces for an object in equilibrium. (1.5 minutes + 1.5 minutes + 4 minutes + infinite minutes)

Learning Objectives: [8, 9, 11, 12, 16, 18, 19, 22]

Problem Statement: A 10 kg marble is pushed up and to the right at an angle of 45 degrees relative to the vertical by Hanna's hand, against a frictionless wall tilted 15 degrees relative to the vertical as shown in the image below. The system is in equilibrium. We wish to find the magnitude of both normal forces from Hanna's hand and the wall.

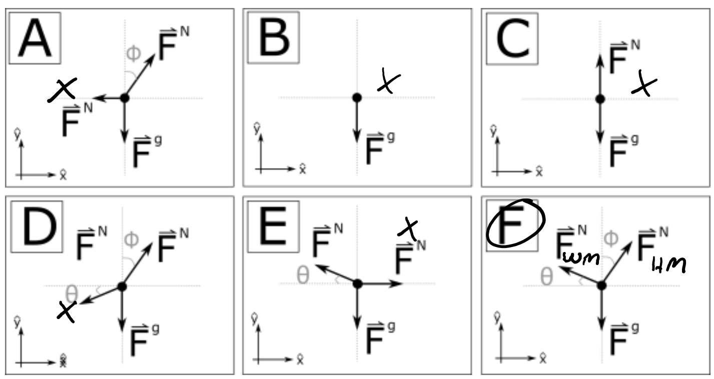
(a) Is the marble moving?

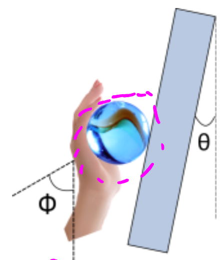
$\sum \vec{F} = \vec{0}$ $\quad \vec{V} = \overrightarrow{\text{CONSTANT}}$

(1) Yes
(2) No
(3) Not enough information.

(b) Pick which FBD correctly describes this scenario.

(1) A
(2) B
(3) C
(4) D
(5) E
(6) F





System m

(c) Which of the following sets of Newton's 2nd law equations correctly describes this scenario using a standard coordinate system.

(1) A
(2) B
(3) C
(4) D

[A]

$$\left|\vec{F}^N_{Hm}\right| - \left|\vec{F}^N_{wm}\right| = m\,a_x$$

$$\left|\vec{F}^N_{Hm}\right| + \left|\vec{F}^N_{wm}\right| - \left|\vec{F}^g_{Em}\right| = m\,a_y$$

[C]

$$\left|\vec{F}^N_{Hm}\right|\sin(\varphi) - \left|\vec{F}^N_{wm}\right|\cos(\theta) = m\,a_x$$

$$\left|\vec{F}^N_{Hm}\right|\cos(\varphi) + \left|\vec{F}^N_{wm}\right|\sin(\theta) - \left|\vec{F}^g_{Em}\right| = m\,a_y$$

[B]

$$\left|\vec{F}^N_{Hm}\right|\cos(\varphi) + \left|\vec{F}^N_{wm}\right|\sin(\theta) = m\,a_x$$

$$\left|\vec{F}^N_{Hm}\right|\sin(\varphi) + \left|\vec{F}^N_{wm}\right|\cos(\theta) + \left|\vec{F}^g_{Em}\right| = m\,a_y$$

[D]

$$\left|\vec{F}^N_{Hm}\right|\cos(\varphi) - \left|\vec{F}^N_{Hm}\right|\sin(\theta) = m\,a_x$$

$$\left|\vec{F}^N_{Hm}\right|\sin(\varphi) + \left|\vec{F}^N_{wm}\right|\cos(\theta) - \left|\vec{F}^g_{Em}\right| = m\,a_y$$

(d) A 75 g marble is pushed up and to the right by Hanna's hand, against a frictionless wall at an angle of 30 degrees up from the horizontal as shown in the image below. The system is in equilibrium. ***Find the magnitude of both normal forces from Hanna's hand and the wall***.

horizontal as shown in the image below. The system is in equilibrium. ***Find the magnitude of both normal forces from Hanna's hand and the wall***.

$$F^N_{HM}\sin\phi - F^N_{WM}\cos\theta = M\cancel{a_x}^{\,0}$$

$$F^N_{HM}\cos\phi + F^N_{WM}\sin\theta - F^g_{EM} = M\cancel{a_y}^{\,0}$$

$$F^N_{HM}\sin\phi - F^N_{WM}\cos\theta = 0$$

1 EQN
2 UNKNOWNS

$$F^N_{HM}\cos\phi + F^N_{WM}\sin\theta - Mg = 0$$

1 EQN
2 UNKNOWNS

2 EQN
2 UNKNOWNS

$$F^N_{HM}\sin\phi = F^N_{WM}\cos\theta$$

$$F^N_{HM} = \frac{\cos\theta}{\sin\phi}F^N_{WM}$$

$$\frac{\cos\theta}{\sin\phi}F^N_{WM}\cos\phi + F^N_{WM}\sin\theta - Mg = 0$$

$$\frac{\cos\theta\cos\phi}{\sin\phi}F^N_{WM} + F^N_{WM}\sin\theta = Mg$$

$$F^N_{WM}\left(\cos\theta\cot\phi + \sin\theta\right) = Mg$$

$$F^N_{WM} = \frac{Mg}{(\cos\theta\cot\phi + \sin\theta)} = \frac{(10)(9.8)}{(\cos(15)\cot(15) + \sin(15))}$$

$$\boxed{\left|\vec{F}^N_{WM}\right| \approx 80.0\ N}$$

$$F^N_{HM} = \frac{\cos(15)}{\sin(15)}(\ \ )$$

$$\boxed{\vec{F}^N_{HM} \approx 109\ N}$$

Conceptual questions for discussion

1. Do you agree with the following statement? *An object with a quadratic positon as a function of time has an acceleration that changes as a function of time.*
2. Do you agree with the following statement? *An object with a quadratic velocity as a function of time has an acceleration that changes as a function of time*.
3. If the net force as a function of time on an object in 1-D is constant, what shape is the velocity vs time for the object?

4. If the net force as a function of time on an object in 1-D is linear, what shape is the velocity vs time for the object?

Hints

N1.2-1: No hints.

N1.2-2: Write Newton's 2nd law down and compare the axes to the mathematical form of the 2nd law.

N1.2-3: How does net force relate to kinematic variables? How to get from velocity to position and from velocity to acceleration in the graphical representation?

N1.2-4: What is the net force divided my mass equal too?

N1.2-5: No hints.

N1.2-6: Is this a 1-D or 2-D problem? Try a force analysis and see if you can connect it to a kinematics analysis.

N1.2-7: Draw a FBD first before trying to identify the correct Newton's 2nd law application.

N1.2-8: No hints.

N1.2-9: No hints.