(N3.L2.2-sols) Foundation Stage Solutions

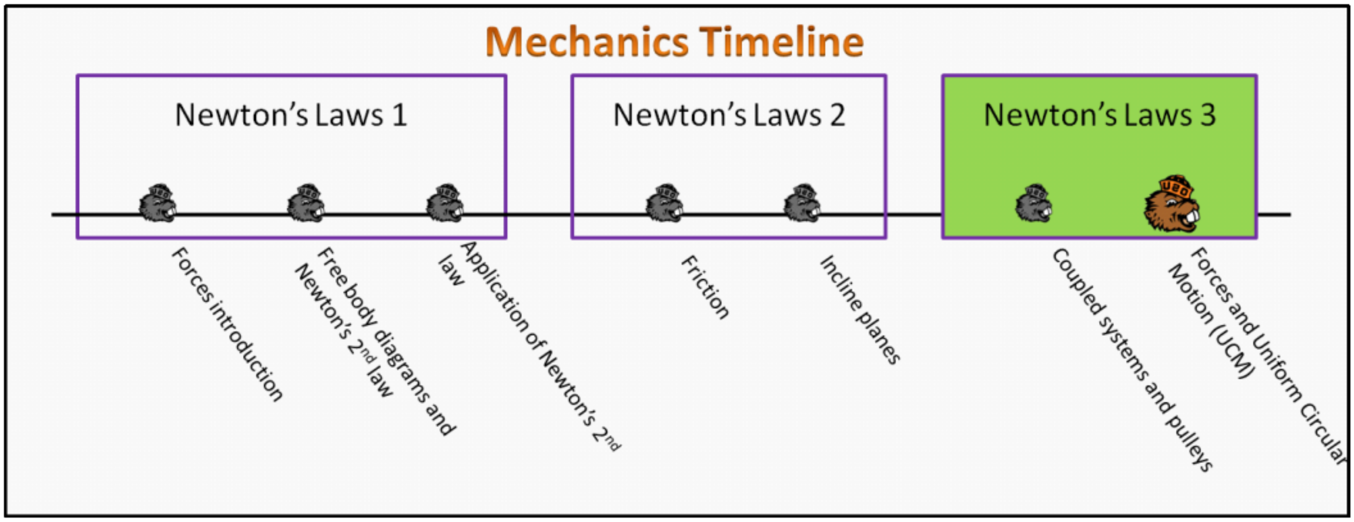
Monday, January 22, 2018 5:44 PM

Newton's Laws 3

Foundation Stage (N3.2.L2)

lecture 2

Forces and Uniform Circular Motion (UCM)



Textbook Chapters

- **BoxSand** :: KC videos (Uniform Circular Motion)
- **Giancoli** (Physics Principles with Applications 7th) :: 5-1 ; 5-2 ; 5-3
- **Knight** (College Physics : A strategic approach 3rd) :: 6.1 ; 6.2 ; 6.3
- **Knight** (Physics for Scientists and Engineers 4th) :: 8.1 ; 8.2

Warm up

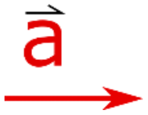
N3.2.L2-1:

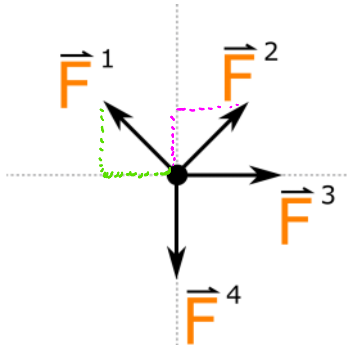
Description: Determine how the magnitude of acceleration for two objects connected via a pulley are related to each other.

Learning Objectives: [?] - Can you identify the objectives from the previous lecture, and this lecture, that this question is relevant to?

Problem Statement: The acceleration of an object is observed to be horizontally to the right as shown below. A FBD for this object is also shown below. Which force(s) have a component along the same line of the acceleration?

[✓ (1)] Force 1
[✓ (2)] Force 2
[✓ (3)] Force 3
(4) Force 4





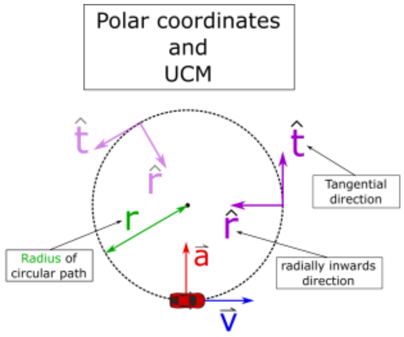
Selected Learning Objectives

1. Analyze situations involving multiple objects and choose appropriate systems.
2. Differentiate between external and internal forces.
3. Use 3rd law force pairs to analyze coupled systems.
4. Use constraints to analyze coupled systems.
5. Choose coordinate systems relative to each other that simplify the analysis.
6. Choose different systems (e.g. the system 1 and 2, or the combined 1 2 system) for the same physical situation and use the analysis from both to arrive at a solution.
7. Identify the features of an ideal pulley.
8. Demonstrate the ability to use a FBD and Newton's 2nd law to determine the mechanical advantage of a pulley system.
9. Identify systems that exhibit uniform circular motion.
10. Draw physical representations from multiple directions and choose the perspective that helps the analysis the most.
11. Draw a FBD that only includes real forces.
12. Identify which forces are responsible for keeping the object traveling in a circle.
13. Define centripetal force.
14. Show that the direction of the net force, and thus acceleration, are perpendicular to the direction of velocity during UCM.
15. Show that the direction of the net force and acceleration point towards the center of the circle during UCM.
16. Identify the radially inward direction and align the coordinate system with it.
17. Identify the tangential direction.
18. Realize that UCM is a situation where the speed of the object affects the magnitude of the acceleration (v^2/r) and thus net force.
19. (UPMF) Explain the nature of fictitious forces that arise in UCM and inertia's role in the effect.
20. Show how period, frequency, speed, and distance are related.

Key Terms

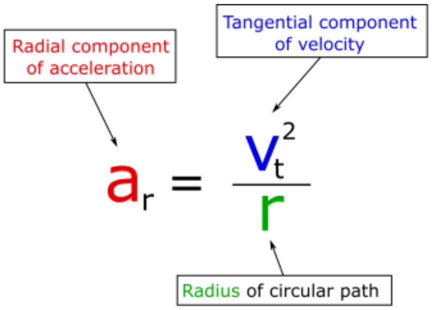
- Frequency
- Period
- Uniform circular motion
- Polar coordinates
- Radial component
- Tangential component
- Radial component of acceleration
- Tangential component of velocity
- Centripetal force
- Centrifugal force

Key Equations



$$\vec{v} = < v_r , v_t > \xrightarrow{\text{In UCM}} \vec{v} = < 0 , v_t >$$

$$\vec{a} = < a_r , a_t > \xrightarrow{\text{In UCM}} \vec{a} = < a_r , 0 >$$



$$a_r = \frac{v_t^2}{r}$$

In words: The radial component of acceleration is equal to the tangential component of velocity squared divided by the radius of the circular path the object is traveling.

Key Concepts

- The force analysis process (defining system, drawing FBD, choosing a coordinate system, applying Newton's 2nd law to construct equations) is the same for an object undergoing UCM. However there are two main differences: First, the coordinate system is now polar not Cartesian, so you must identify the radial direction; Second, the acceleration in the radial direction is a function of the tangential component of velocity and radius, so you can substitute $\mathbf{v}_t^2/\mathbf{r}$ for the radial component of acceleration.
- The net force, and acceleration always points radially inwards for an object in UCM.
- The velocity is always tangent to the circular path and constant in magnitude when in UCM.
- Frequency is the number of repeated events per one unit of time (e.g. 1000 revolutions per minute, 5 cycles per second, 70 beats per second, 1020 hand claps per minute, etc..)
- Period is the amount of time per one repeated event (e.g. 10 seconds per 1 revolution, 24 hours per 1 revolution, 5 seconds per 1 clap, 42 seconds per one click noise).
- Centripetal force is not a real force, it is just another way to say "net force in the radial direction".
- Centrifugal force is a pseudo force that is needed when trying to apply Newton's 2nd law in a rotating (i.e. non-inertial) reference frame. A non-inertial reference frame is a frame where Newton's 1st law does not hold (e.g. inside a car when it is undergoing UCM).

Act I: UCM

Questions

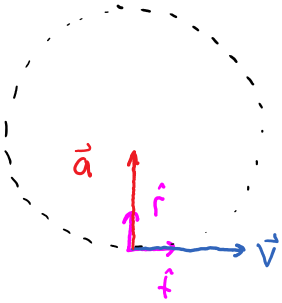
N3.2.L2-2:

Description: Conceptual question exploring the characteristics of physical quantities when an object undergoes UCM. (3 minutes)

Learning Objectives: [14, 15,17]

Problem Statement: What is true about an object in UCM?

T [✓ (1)] net force points towards the center
T [✓ (2)] net force points radially inwards
F (3) net force is tangent to path of circle
F (4) net force points radially outwards
T [✓ (5)] velocity is tangent to circular path
F (6) velocity is radially inwards
T [✓ (7)] acceleration points radially inwards
T [✓ (8)] acceleration points towards the center



$$\underbrace{\Sigma \vec{F}}_{n} = m\vec{a}$$

SAME DIRECTION

N3.2.L2-3:

Description: Differentiate between real, fake, and pseudo forces. (2 minutes)

Learning Objectives: [13]

Problem Statement: Which of the following forces are real forces?

F (1) Centripetal force — DEFINED AS ΣF_r
F (2) Centrifugal force — COMPLICATED... BUT NOT A REAL FORCE
T [✓ (3)] Normal force
T [✓ (4)] Buoyant force
T [✓ (5)] Gravitational force
? (6) Jedi mind tricks

$$\Sigma F_r = F_r^N + F_r^g + F^f$$

? (6) Jedi mind tricks

$$\Sigma F_r = F_r^n + F_r^g + F_f^f \dots$$

REAL FORCES

N3.2.L2-4:

Description: Conceptual question exploring the characteristics of physical quantities when an object undergoes UCM. (3 minutes)

Learning Objectives: [14, 15,17, 18]

Problem Statement: An object moves in uniform circular motion; what can be said?

CONSTANT SPEED

CONSTANT RADIUS

F (1) The velocity is constant
T [✓ (2) The speed is constant]
F (3) The acceleration is constant
F (4) The net force is constant
F (5) The magnitude of the displacement is constant FROM WHERE TO WHERE?!

$\vec{V}, \vec{a}, \Sigma\vec{F}$ ALL VECTORS w/ THEIR DIRECTION CHANGING

N3.2.L2-5:

Description: Given speed and circular distance, find frequency. (5 minutes)

Learning Objectives: [20]

Problem Statement: You are at a half-mile long circular race track and the cars are screaming by with a constant speed of 60 mph. How frequently do you see a car pass your location?

$$\text{REPEATED EVENT} = \underbrace{\text{CAR PASSES YOUR LOCATION}}_{\frac{1\ \text{PASS}}{0.5\ \text{MILE}}}$$

(1) 60 passes/hour
(2) 120 passes/hour
(3) 0.0167 passes/hour
(4) 1 passes/minute
(5) 2 passes/minute
(6) 0.0167 passes/second
(7) 0.0333 passes/second

$$\frac{1\ \text{PASS}}{0.5\ \text{MILE}} \times \frac{60\ \text{MI}}{\text{HR}} = \frac{120\ \text{PASSES}}{1\ \text{HR.}}$$

FREQUENCY $\frac{\#\ \text{REPEATED EVENTS}}{1\ \text{UNIT TIME}}$

N3.2.L2-6:

Description: Given radius, and period, determine speed of an object in UCM. (6 minutes)

Learning Objectives: [20]

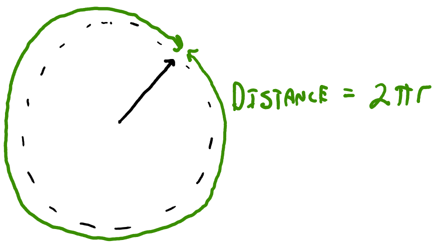
Problem Statement: What is the speed of a point on the tip of a 3.00-cm-long second-hand of a clock?

(1) 1.57 m/s
(2) 1.57 x 10^{-3} m/s
(3) 6.28 x 10^{-4} m/s
(4) 3.14 x 10^{-5} m/s
[✓ (5)] 3.14 x 10^{-3} m/s

FOR UCM SPEED IS CONSTANT

$$\therefore \quad \text{SPEED} = \frac{\text{DISTANCE}}{\text{TIME}} = \frac{2\pi r}{T} = \frac{2\pi(3/100\,\text{m})}{60\ \text{s}}$$

TIME FOR 1 FULL REVOLUTION



$$\text{PERIOD} = \frac{\text{AMOUNT OF TIME}}{1 \text{ REPEATED EVENT}}$$

HERE 1 REPEATED EVENT = 1 REVOLUTION

N3.2.L2-7:

Description: Identify the relationship between frequency and period. (2 minutes)

Learning Objectives: [20]

Problem Statement: Which of the following is the correct relationship between frequency (**f**) and period (**T**)?

(1) **f** = **T**
[✓ (2)] **f** = 1/**T**
[✓ (3)] **T** = 1/**f**

N3.2.L2-8:

Description: Find the frequency and period of an object in UCM. (3 minutes)

Learning Objectives: [20]

Problem Statement: Consider the second-hand of an analog clock.

(a) What is the period of the second hand on a clock?

(b) What is the frequency of the second hand on a clock?

(a) What is the period of the second hand on a clock?

$$\boxed{T = 60\ s}$$

(b) What is the frequency of the second hand on a clock?

$$F = \frac{1}{T} = \frac{1}{60\ s} = \boxed{0.0167\ \frac{1}{s}}$$

$$\frac{1}{s} = Hz$$

$$\boxed{0.0167\ Hz}$$

N3.2.L2-9:

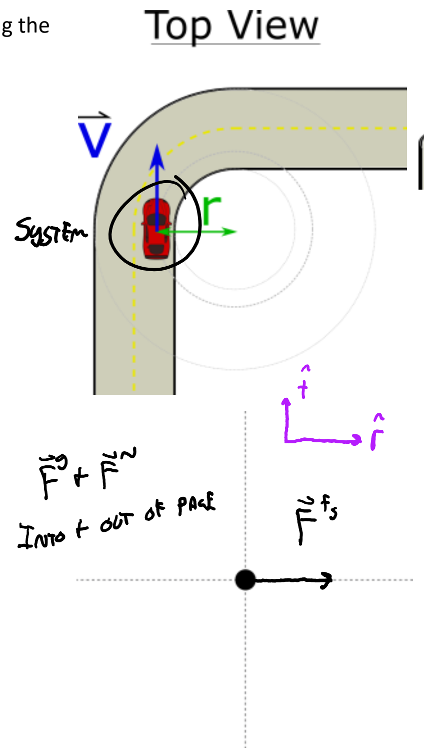
Description: Identify the forces responsible for UCM and apply a force analysis to find maximum speed a car can go around an unbanked turn when given mass, radius, and coefficient of static friction. (2 minutes + 5 minutes + 6 minutes)

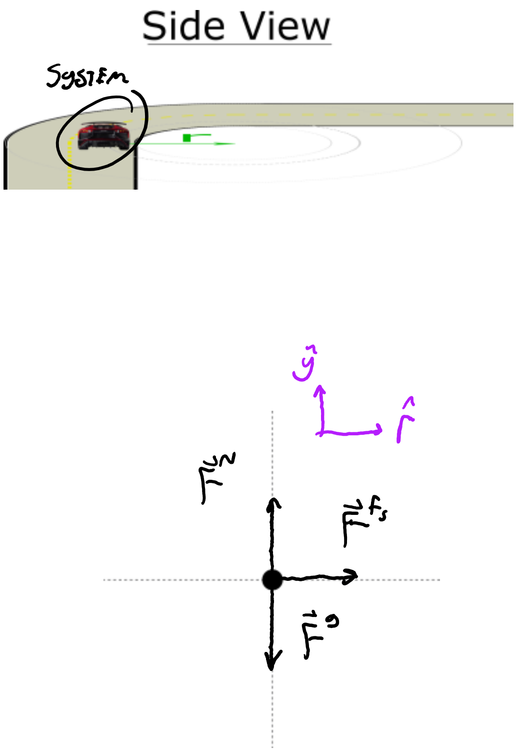
Learning Objectives: [9, 10, 11, 12, 16]

Problem Statement: A car is traveling around a circular corner of radius **r** at a constant speed. The corner is not banked.

(a) Which force is responsible for keeping the car on the road?

F (1) Normal force
X (2) Centripetal force
T [✓ (3)] Friction
F (4) Gravity
X (5) Force of acceleration





(b) Which of the following 2nd law equations correctly describes this situation?

(1)

(2)

$$\Sigma F_y = m\cancel{a_y}^{\,0} \qquad \Sigma F_r = ma_r$$

$$|\vec{F}^N| - |\vec{F}^g| = 0 \qquad |\vec{F}^{f_s}| = ma_r$$

(b) Which of the following 2nd law equations correctly describes this situation?

$\sum F_g = m\cancel{a}_y$ $\sum F_r = m a_r$

$|\vec{F}^N| - |\vec{F}^g| = 0$ $|\vec{F}^{f_s}| = m a_r$

(1)

$$\left|\vec{F}^{centripetal}\right| = m\,a_r$$

$$\left|\vec{F}^{N}\right| - \left|\vec{F}^{g}\right| = m\,a_y$$

(2)

$$\left|\vec{F}^{acceleration}\right| = m\,a_r$$

$$\left|\vec{F}^{N}\right| - \left|\vec{F}^{g}\right| = m\,a_y$$

[✓ (3)]

$$\left|\vec{F}^{f,s}\right| = m\,a_r$$

$$\left|\vec{F}^{N}\right| - \left|\vec{F}^{g}\right| = m\,a_y$$

(4)

$$\left|\vec{F}^{f,s}\right| - \left|\vec{F}^{centrifugal}\right| = m\,a_r$$

$$\left|\vec{F}^{N}\right| - \left|\vec{F}^{g}\right| = m\,a_y$$

(c) A popular car magazine uses what is known as a "skid-pad" to determine the "lateral-g's" a car can withstand before skidding off the track. The skid-pad has a radius of about 103 ft (31.4 m). A 2016 Dodge Viper ACR has a mass of about 1530 kg and the coefficient of static friction between the ground and the tires is about 1.11. What is the speed that the Viper can go around this trick before starting to skid?

Max speed

∴ Max friction

$\sum F_y = m\cancel{a_y}^{0}$ $\sum F_r = m a_r$

$|\vec{F}^N| - mg = 0$ $|\vec{F}^{f_{s,max}}| = m\frac{V_t^2}{r}$

$|\vec{F}^N| = mg$ $\mu_s|\vec{F}^N| = m\frac{v^2}{r}$

$\vec{V} = \langle \cancel{V_r}^{0}, V_t \rangle$

constant

$|\vec{V}| = V_t \equiv V$

$\mu_s \cancel{m} g = \cancel{m}\frac{v^2}{r}$

$\boxed{V = \sqrt{\mu_s g r}} \approx \boxed{18.5\ m/s} \approx 40$ mph

N3.2.L2-10:

Description: Given radius as a variable, and magnitude of normal force, find speed of ball in UCM inside a hula-hoop. (8 minutes)

Learning Objectives: [9, 10, 11, 12, 16]

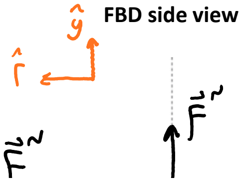
Problem Statement: A ball of mass **m** is rolling on a horizontal table on the inside of a hula-hoop of radius **r**. At what speed must the ball be moving so that the normal force from the hoop is equal to twice the normal force from the floor?

X (1) $v = 2\,g\,r$ $\frac{m^2}{s^2}$

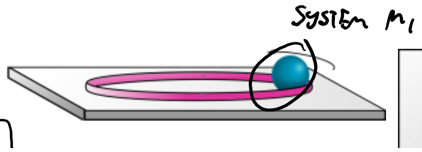
[✓ (2)] $v = \sqrt{2\,g\,r}$

X (3) $v = 2\sqrt{\frac{g}{r}}$ $\frac{1}{s}$

FBD side view



Side view



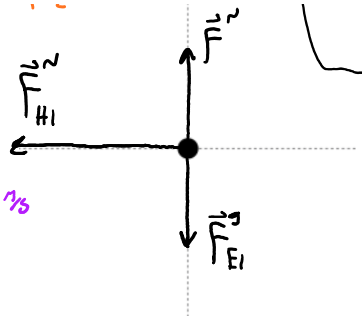
Top view

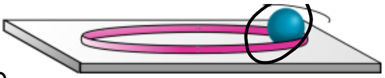
X (3) $v = 2\sqrt{\frac{g}{r}}$ $\frac{1}{s}$

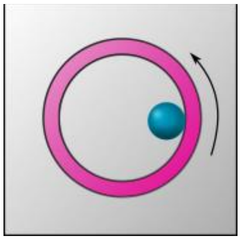
X (4) $v = 2\,m\,g\,r$

X (5) $v = 2\,m\sqrt{g\,r}$ $kg\,m/s$

X (6) $v = 2\,m\sqrt{\frac{g}{r}}$ $\frac{kg}{s}$







$$|\vec{F}^N_{HI}| = 2|\vec{F}^N|$$

$$\Sigma F_r = m a_r \qquad \Sigma F_y = m \cancel{a_y}^{0}$$

$$|\vec{F}^N_{HI}| = m\frac{v^2}{r} \qquad |\vec{F}^N| - mg = 0$$

$$|\vec{F}^N| = mg$$

$$2mg = m\frac{v^2}{r}$$

$$\boxed{v = \sqrt{2gr}}$$

* SPEED SI UNITS m/s

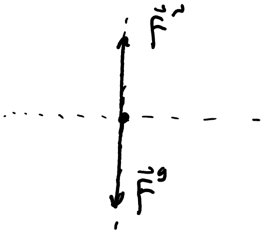
N3.2.L2-11:

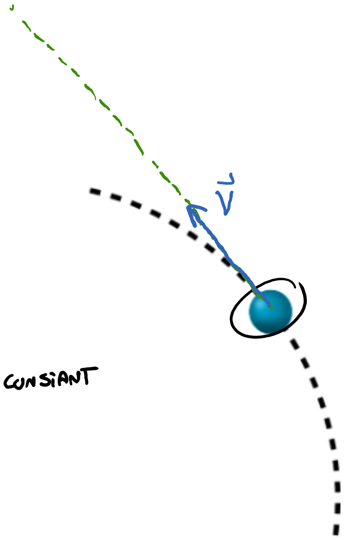
Description: Sketch a trajectory of an object after removing the only force responsible for the object undergoing UCM. (2 minutes)

Learning Objectives: []

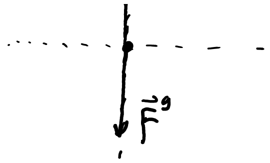
Problem Statement: A ball is rolling counter-clockwise inside a hula-hoop and is located at the dot in the figure below. The dashed line represents the outline of the hula-hoop. When at the dot, the hoop is removed. Sketch a trajectory of the ball when the hoop is removed.

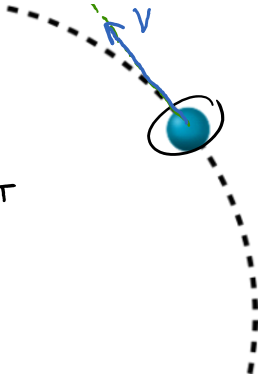
SIDE VIEW WHEN HOOP IS REMOVED





$\Sigma\vec{F} = \vec{0}$ so $\vec{v}$ = CONSTANT





$\sum \vec{F} = \vec{0}$ so $\vec{v}$ = CONSIANT

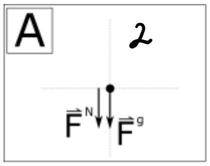
N3.2.L2-12:

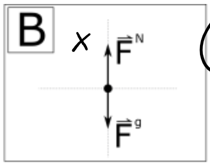
Description: Match FBDs with snapshots of a roller coaster at various locations along a loop-the-loop. Apply a force analysis to find speed of cart when it is at the top of the loop and weightless. (3 minutes + 6 minutes)

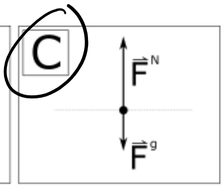
Learning Objectives: [9, 11, 12, 14, 15, 16]

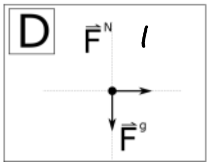
Problem Statement: A roller coaster cart goes through a loop-the-loop as shown in the figure below. A group of physics students drew four free body diagrams but didn't label which snapshot each FBD was drawn at. Which set of free body diagrams are matched with the proper snapshot of the cart?

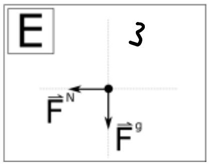
(a) Which FBD correctly represents the forces on the cart when it is at location 0?

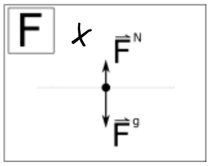


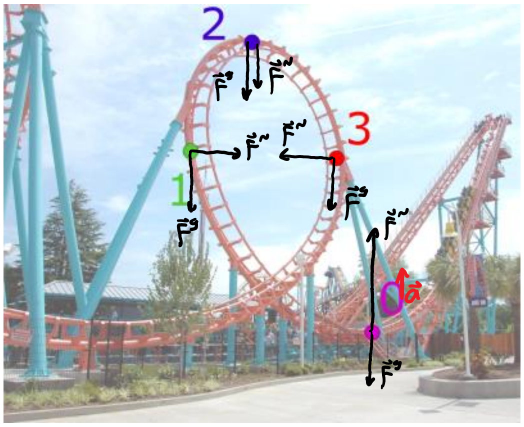












(b) At location 2, what must the speed of the cart be to feel momentarily weightless?

FEEL NO NORMAL FORCE FROM SEAT

(1) $v = g\,r$

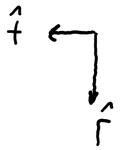
[✓ (2)] $v = \sqrt{g\,r}$

(3) $v = \sqrt{\frac{g}{r}}$

(4) $v = m\,g\,r$

(5) $v = m\sqrt{g\,r}$

(6) $v = m\sqrt{\frac{g}{r}}$



$$\sum F_r = m a_r$$

$$\overset{0}{\cancel{|\vec{F}^N|}} + mg = \frac{mv^2}{r}$$

$$mg = \frac{mv^2}{r}$$

$$\boxed{v = \sqrt{gr}}$$

N3.2.L2-13:

Description: Conceptual question exploring the relative scale of forces involved in UCM. (5 minutes)

Learning Objectives: [9, 11, 12, 14, 15, 17]

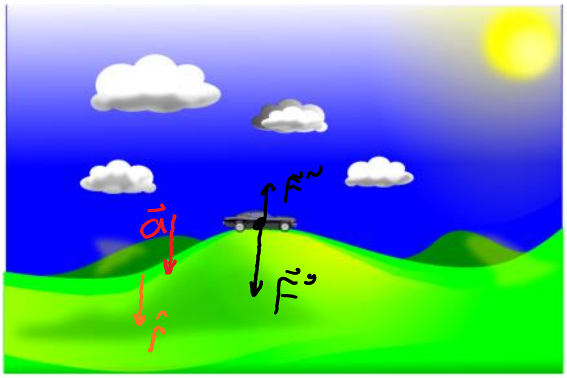
Problem Statement: A car is traveling over the top of a circular hill with a constant speed v. What can be said at the instant the car is on the top of the hill?

F (1) The normal force from the ground on the car is greater than the weight of the car

T [✓ (2)] The normal force from the ground on the car is less than the weight of the car

F (3) The normal force from the ground on the car is equal to the weight of the car

F (4) The relative magnitude of the normal force and the weight of the car can't be determined without knowing the speed of the car.



N3.2.L2-14:

Description: Identify forces that have a radial component for an object on a banked frictionless curve. Draw a FBD for an object on a banked frictionless curve. (4 minutes)

Learning Objectives: [9, 11, 12, 14, 15, 16]

Problem Statement: A car is travels around a frictionless banked circular corner of radius r at a constant speed. Which force(s) has a radial component?

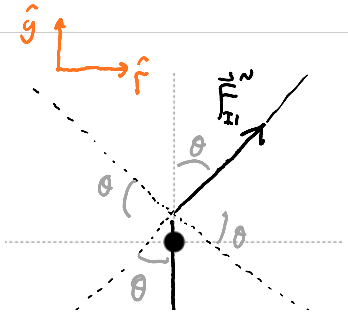
X [✓ (1)] Normal force

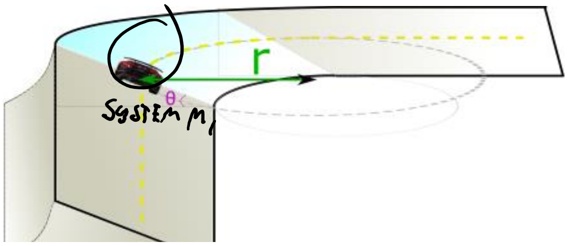
F (2) Centripetal force

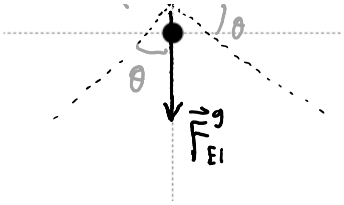
F (3) Friction

X (4) Gravity

(5) Force of acceleration







N3.2.L2-15:

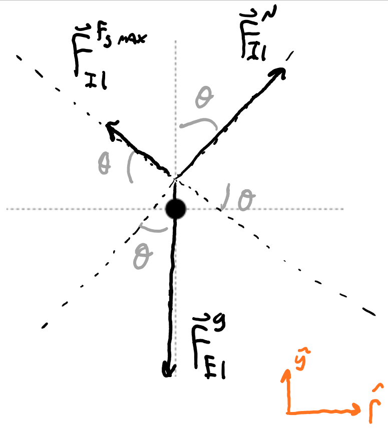
Description: Identify the direction of friction on a banked curve for maximum and minimum speeds. Draw a FBD for an object on a banked curve with friction. (4 minutes)

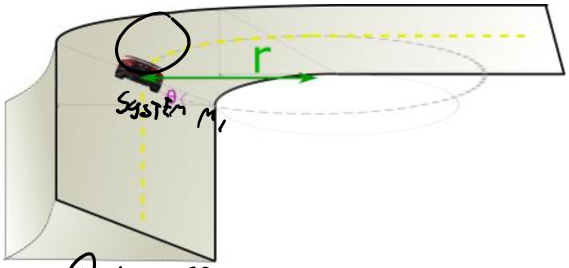
Learning Objectives: [9, 11, 12, 14, 15, 16]

Problem Statement: A car is travels around a banked circular corner of radius r at a constant speed. Which direction does the force of static friction point if the car is going around the corner at the minimum speed possible before slipping?

O

(1) Up the incline
(2) Down the incline
(3) Forwards
(4) Backwards
(5) Horizontally to the right





@ MIN SPEED, WANTS TO SLIDE DOWN THE INCLINE

N3.2.L2-16:

Description: Identify the direction of friction on a banked curve for maximum and minimum speeds. Draw a FBD for an object on a banked curve with friction. (4 minutes)

Learning Objectives: [9, 11, 12, 14, 15, 16]

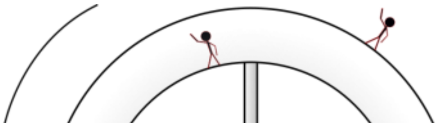
Problem Statement: Three students are discussing the possibility for a two level rotating space station, far from any gravitational bodies. One of them finds this drawing on the internet and the following conversation occurs. Which student do you agree with the least? Ignore oxygen issues.

T

(1) "A rotating space station uses a normal force acting on the people to simulate gravity yo."

T

(2) "Yeah, but this station is drawn wrong, those on the inside level would float



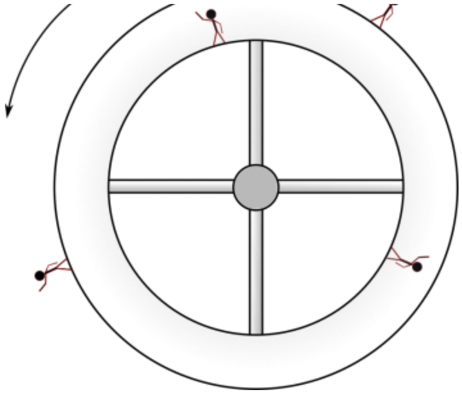
(1) "A rotating space station uses a normal force acting on the people to simulate gravity yo."

T

(2) "Yeah, but this station is drawn wrong, those on the inside level would float towards the ceiling and those on the outside would float away from the station."

F

(3) "Nah, the station is drawn correctly. Just think about what you look like standing on Earth, it's just like this."



Conceptual questions for discussion

1. If you are driving along a straight road at a constant speed, is it possible to accelerate without adjusting the brake or gas pedal?
2. When sitting in a car that is turning, why do you feel like you are being tossed to the outside of the turn? Explain this from the perspective of an outside observer on the ground.
3. Consider whirling a bucket full of water around in a vertical circle. When at the top of the circle the bucket is upside-down. If you are whirling the bucket around fast enough, the water does not spill out when at the top of the circle. Use Newton's laws to explain why the water doesn't fall out. Explain this from the perspective of you as you swing the bucket around.

Hints

N3.2.L2-1: No hints.

N3.2.L2-2: No hints.

N3.2.L2-3: No hints.

N3.2.L2-4: Velocity, acceleration, and net force are all vectors.

N3.2.L2-5: How many miles do the cars travel in one hour, and how many miles is the circular track?

N3.2.L2-6: With zero acceleration; speed = distance / time. What distance does the tip of a second hand go through in one minute?

N3.2.L2-7: No hints.

N3.2.L2-8: No hints.

N3.2.L2-9: No hints.

N3.2.L2-10: No hints.

N3.2.L2-11: After the hoop is removed, what is the net force on the ball equal to?

N3.2.L2-12: No hints.

N3.2.L2-13: What is the direction of acceleration? How does direction of acceleration relate to direction of net force?

N3.2.L2-14: No hints.

N3.2.L2-15: No hints.

N3.2.L2-16: No hints.