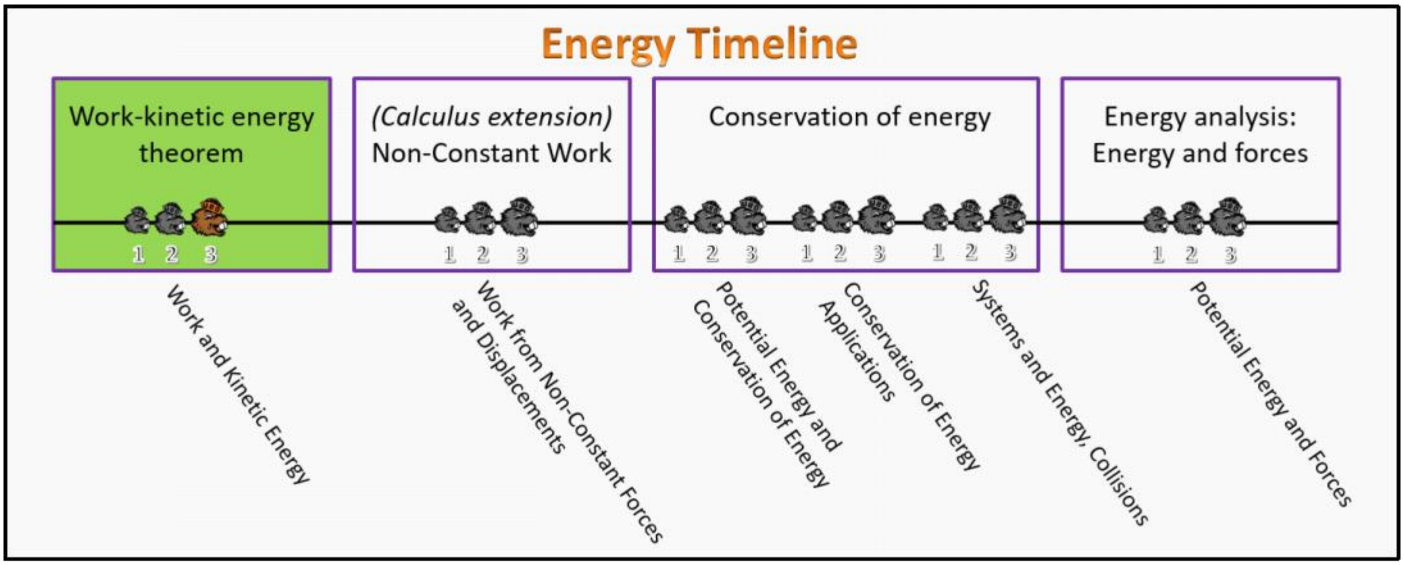
(WE.L1.3-sols) Practice Stage Solutions

Thursday, March 29, 2018 8:34 PM

Work-Kinetic Energy Theorem

Foundation Stage (WE.L1.3)

Post-Lecture 1
Work and Kinetic Energy



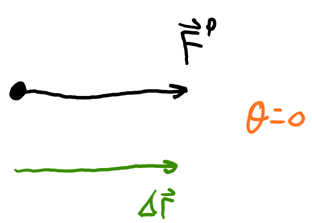
Questions

WE.L1.3-01

Description: Force applied over a distance

Learning Objectives: [x]

Problem Statement: How much work, in joules, does a supermarket checkout attendant do on a can of soup he pushes 0.600 m horizontally with a force of 5.00 N?



$$W^P = \vec{F}^P \cdot \Delta\vec{r}$$

$$= F^P \Delta r \cos\theta$$

$$= (5\,N)(0.6\,m)\cancel{\cos(0)}^{\,1}$$

$$\boxed{W^P = 3\,N}$$

WE.L1.3-02

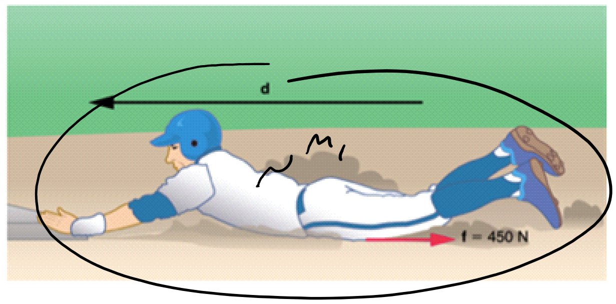
Description: Energy and work done by friction

Learning Objectives: [x]

Problem Statement: Consider the situation shown in the figure, where a baseball player slides to a stop on level ground. Using energy considerations, calculate the distance the 65.0-kg baseball player slides, given that his initial speed is 6.00 m/s and the force of friction against him is a constant 450 N.

NOT A GOOD QUESTION B/C

WE DID'NT TALK ABOUT THERMAL ENERGY YET.



SYSTEM m_1 + GROUND

THERMAL ENERGY FROM FRICTION

$$KE_i + E_i^{Th,f} + \cancel{\Sigma W_{ext}}^{0} = \cancel{KE_f}^{0} + E_f^{Th,f}$$

$$\frac{1}{2} m_1 v_{1i}^2 + E_i^{Th} + 0 = E_f^{Th,f}$$

$$\frac{1}{2} m_1 v_{1i}^2 = \Delta E^{Th,f}$$

$$\frac{1}{2} m v_{1i}^2 = -\vec{F}^{Fk} \cdot \Delta \vec{r}$$

$$\frac{1}{2} m_1 v_{1i}^2 = -F^{Fk} \Delta r \cos\theta$$

$$= -F^{Fk} \Delta r \cancel{\cos(180)}^{-1}$$

$$\frac{1}{2} m_1 v_{1i}^2 = F^{Fk} \Delta r$$

$$\frac{1}{2}(65)(6)^2 = 450 \Delta r$$

$$\boxed{\Delta r = 2.6 \text{ m}}$$

WE.L1.3-03

Description: Multi-part shopping cart work problem

Learning Objectives: [x]

Problem Statement: A shopper pushes a grocery cart 20.0 m at constant speed on level ground, against a 35.0 N frictional force. He pushes in a direction 25 degrees below the horizontal.

(a) What is the change in thermal energy cause by the frictional force acting on the cart and the ground?

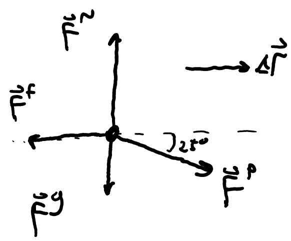
(1) 0 J
(2) 500 J
(3) -500 J
(4) 634 J
(5) -634 J
(6) 700 J
(7) -700 J

$$\Delta E^{Th,f} = -F^f \Delta r \cancel{\cos\theta}^{-1}$$

$$\Delta E^{Th,f} = F^f \Delta r$$

$$= (35)(20)$$

$$\Delta E^{Th,f} = 700\ J$$



(b) What is the work done on the cart by the gravitational force?

(1) 0 J
(2) 500 J
(3) -500 J
(4) 634 J
(5) -634 J
(6) 700 J
(7) -700 J

$\theta_g = 90°$

$\cos(90) = 0$

System

m + Ground

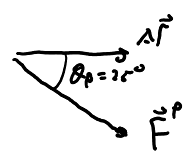
(c) What is the work done on the cart by the normal force from the ground?

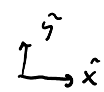
(1) 0 J
(2) 500 J
(3) -500 J
(4) 634 J
(5) -634 J
(6) 700 J
(7) -700 J

$\theta_N = 90°$

(d) What is the work done on the cart by the shopper?

(1) 0 J
(2) 500 J
(3) -500 J
(4) 634 J
(5) -634 J
(6) 700 J
(7) -700 J





$$W^P = \underbrace{F^P \cos\theta_P}\ \Delta r$$

$$= F^P_x \Delta r$$

$$= (35)(20)$$

$$= 700\ J$$

(e) What is the total work done on the cart minus the change in thermal energy?

(1) 0 J
(2) 500 J
(3) -500 J

$$\cancel{KE_i} + E^{Th}_i + \sum W_{ext} = \cancel{KE_f} + E^{Th}_f$$

(1) 0 J
(2) 500 J
(3) -500 J
(4) 634 J
(5) -634 J
(6) 700 J
(7) -700 J

$$\cancel{KE_i} + E_i^{th} + \sum W_{ext} = \cancel{KE_f} + E_f^{th}$$

$$\sum W_{ext} = \Delta E^{th}$$

$$\sum W_{ext} - \Delta E^{th} = 0$$

$$W^P - \Delta E^{th}$$

$$700 - 700 = 0$$

(f) Find the force the shopper exerts, using energy considerations.

(1) 12.2 N
(2) 35.0 N
(3) 38.6 N
(4) 42.7 N
(5) 89.9 N

$$W^P = F^P \Delta r \cos 25$$

$$700 = F^P (20) \cos(25)$$

$$\boxed{F^P \approx 38.6\ N}$$

WE.L1.3-04

Description: Using work in calculations of a car's bumper

Learning Objectives: [x]

Problem Statement: A car's bumper is designed to withstand a 4.0-km/h (1.1 m/s) collision with an immovable object without damage to the body of the car. The bumper cushions the shock by absorbing the force over a distance. Calculate the magnitude of the average force on a bumper that collapses 0.200 m while bringing a 900-kg car to rest from an initial speed of 1.1 m/s.

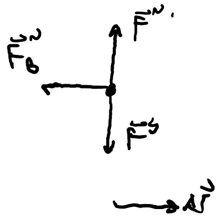
SYSTEM: CAR

(1) $2.7{\times}10^3$ N
(2) $5.6{\times}10^3$ N
(3) $1.1{\times}10^3$ N
(4) $1.3{\times}10^1$ N
(5) $2.3{\times}10^2$ N

$$KE_i + \sum W_{ext} = \cancel{KE_f}^{0}$$

$$KE_i + \cancel{W^g}^{0} + W^N_B + \cancel{W^N}^{0} = 0$$

(3) 1.1×10^{3} N
(4) 1.3×10^{1} N
(5) 2.3×10^{2} N



$$KE_i: \; +\cancel{W^g}^{\,0} + W^N_B + \cancel{W^N}^{\,0} = 0$$

$\theta=90°$ $\quad$ $\theta=90°$

$$\tfrac{1}{2}mv_i^2 + F^N_B \Delta r \cancel{\cos\theta_{NB}}^{\,-1} = 0$$

$$\tfrac{1}{2}mv_i^2 - F^N_B \Delta r = 0$$

$$\tfrac{1}{2}(900)(1.1)^2 - F^N_B(0.2) = 0$$

$$F^N_B = 2722.5\ \text{N}$$

WE.L1.3-05

Description: Work and kinetic energy to determine unknown gravity

Learning Objectives: [x]

Problem Statement: Settlers are trying to determine the value of **g** on a distant planet. They throw a wrench downward from a height of 3 m. If the initial speed of the wrench is 2 m/s, and the speed right before impact is 10 m/s, what is the value of **g**?

(1) 4 m/s^2
(2) 8 m/s^2
(3) 12 m/s^2
(4) 16 m/s^2
(5) 24 m/s^2

$$KE_i + \Sigma W_{ext} = KE_f$$

System: wrench

$$\tfrac{1}{2}mv_i^2 + W^g = \tfrac{1}{2}mv_f^2$$

$$F^g \Delta r \cancel{\cos\theta_g}^{\,1} = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2$$

$$\cancel{m}g\,\Delta r = \tfrac{1}{2}\cancel{m}(v_f^2 - v_i^2)$$

$$g = \frac{v_f^2 - v_i^2}{2\Delta r}$$

$$g = \frac{(10)^2 - (2)^2}{2(3)}$$

$$g = 16\ m/s^2$$

WE.L1.3-06

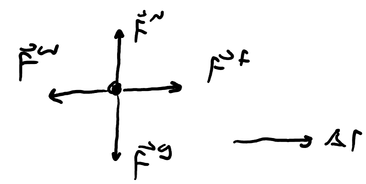
Description: Using work in calculations of sprinter with headwind

Learning Objectives: [x]

Problem Statement: Using energy considerations, calculate the average force a 60.0-kg sprinter exerts backward on the track to accelerate from 2.00 to 8.00 m/s in a distance of 25.0 m, if he encounters a headwind that exerts an average force of 30.0 N against him.

(1) 102 N
(2) 42.0 N
(3) 1,800 N
(4) 174 N
(5) 72 N

System : Person



$$KE_i + \Sigma W_{ext} = KE_f$$

$$W^w + \cancel{W^N} + \cancel{W^g} + W^F = \Delta KE$$

$\theta = 90°$

$$F^w \Delta r \cos(180) + F^F \Delta r \cos(0) = \frac{1}{2} m (v_f^2 - v_i^2)$$

$$-F^w \Delta r + F^F \Delta r = \frac{1}{2} m (v_f^2 - v_i^2)$$

$$F^F = \frac{m(v_f^2 - v_i^2)}{2 \Delta r} + F^w$$

$$= \frac{(60)(8^2 - 2^2)}{2(25)} + 30$$

$$\boxed{F^F = 102\ N}$$

WE.L1.3-07

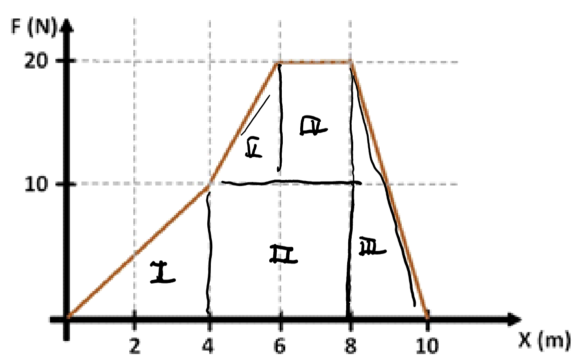
Description: Force vs distance plot and work

Learning Objectives: [x]

Problem Statement: An electromagnetic tagging cannon accelerates ID tags to be fired from a ship at whales. The tags are lodged in the whale to allow scientists to track them. To launch a tag you must load it and fire it by pulling a spring loaded plunger that gives the tag an initial velocity before entering the cannon. The cannon exerts a force as a function of distance, as shown in the figure. If a tag has a mass of 586.7 **g**, and exits the cannon with a speed of 20 m/s, what is approximately the initial velocity of tag when it enters the cannon?

System : TAG

(1) 0 m/s
(2) 2 m/s
(3) 5 m/s
(4) 8 m/s
(5) 12 m/s



$$KE_i + \Sigma W_{ext} = KE_f$$

$$\frac{1}{2} m v_i^2 + W^N = \frac{1}{2} m v_f^2$$

$\vec{F}^N$

$$\frac{1}{2} m v_i^2 + \underline{AREA} = \frac{1}{2} m v_f^2$$

$\Delta \vec{r}$

$$\frac{1}{2} m v_i^2 + 110\ J = \frac{1}{2} m v_f^2$$

$$A_I = \frac{1}{2}(4)(10) = 20\ J$$

$\frac{1}{2}(.5867)V_i^2 + 110 = \frac{1}{2}(.5867)(20)^2$

$\boxed{V_i = 5 \text{ m/s}}$

$A_I = \frac{1}{2}(4)(10) = 20 \text{ J}$

$A_{II} = (8-4)(10) = 40 \text{ J}$

$A_{III} = \frac{1}{2}(16-8)(20) = 20 \text{ J}$

$A_{IV} = (8-6)(20-10) = 20 \text{ J}$

$A_V = \frac{1}{2}(6-4)(20-10) = 10 \text{ J}$

$A_{NET} = 110 \text{ J}$

WE.L1.3-08

Description: Work and kinetic energy proportional reasoning

Learning Objectives: [x]

Problem Statement: Work is applied to your Scooty Puff Jr. spacecraft to accelerate it from rest to some final speed.

(a) If instead it reached a final speed 2.4 times as fast, how many more times work would be required?

(1) 1.20 times
(2) 1.55 times
(3) 2.40 times
(4) 5.76 times
(5) 5.80 times

$\Sigma W_{Ext} = \Delta KE$

$= KE_f - \cancel{KE_i}^{\,0}$

$\Sigma W_{Ext} = \frac{1}{2} m v_f^2$

$m = \text{const}$

$\Sigma W_{Ext} \propto V_f^2$

If $V_f \rightarrow 2.4\, V_f$

then $\Sigma W_{Ext} \rightarrow (2.4)^2$

$\Sigma W_{Ext} \rightarrow 5.76\, \Sigma W_{Ext}$

(b) If instead 2.4 times as much work was done on your spaceship, how many times faster would it be going?

(1) 1.20 times
(2) 1.55 times
(3) 2.40 times
(4) 5.76 times
(5) 5.80 times

$V_f \propto \sqrt{\Sigma W_{Ext}}$

If $\Sigma W_{Ext} \rightarrow 2.4\, \Sigma W_{Ext}$

then $V_f \rightarrow \sqrt{2.4}\, V_f$

4 MEW $\quad v_f \rightarrow \sqrt{2.4}\, v_f$

$v_f \rightarrow 1.55\, v_f$

WE.L1.3-09

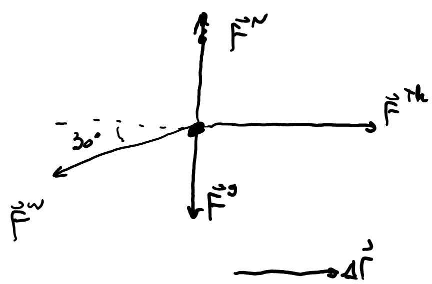
Description: Rocket sled with headwind

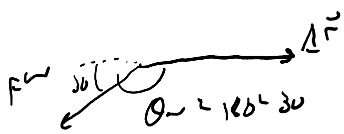
Learning Objectives: [x]

Problem Statement: A rocket sled is attempting to break the land speed record out in a flat dry lakebed. The thrusters apply a 20,000 N force for 1000 m. There is a constant 4,000 N head wind against the motion of the sled and down 30° from the horizontal. If the sled reaches a top speed of 300 m/s, what is its mass? Ignore friction from the ground, assume the wind force is constant, and mass of the sled is a constant (all fairly large assumptions).

(1) 23.7 kg
(2) 112 kg
(3) 367 kg
(4) 500 kg
(5) 1000 kg

SYSTEM : SLED





$\theta_W = 180 - 30$

$\theta_W = 150$

$\cancel{KE_i}^{0} + \sum W_{Ext} = KE_f$

$\cancel{W^N}^{0} + \cancel{W^g}^{0} + W^{Th} + W^W = \frac{1}{2} m v_f^2$

$\theta = 90°$

$F^{Th} \Delta r \cos(0) + F^W \Delta r \cos(150) = \frac{1}{2} m v_f^2$

$F^{Th} \Delta r + F^W \Delta r \cos(150) = \frac{1}{2} m v_f^2$

$(20000)(1000) + (4000)(1000)\cos(150) = \frac{1}{2} m (300)^2$

$20000000 - 3464101.615 = 45000\, m$

$m = 367.46$ kg

$\boxed{m \approx 367 \text{ kg}}$