Week 10 Challenge Homework Solutions

Question 1

You're convinced the opera is almost over because of the large person singing a loud vibrato.

(a) If the sound intensity level (dB) increases from one note to the next by 1 dB, what is the percent increase in the intensity?
(b) The singer comes over to your side of the stage, only 10 m away, and your phone applet tells you the sound intensity level is 85 dB. What is the power coming from their voice?
(c) All of the sudden 4 more identically loud people come out of nowhere and join the first person. You think to yourself, will it ever end. Three of them are standing next to the first person and the fourth is on the other side of the stage, twice as far from you as the group of singers. What is the sound intensity level now?

a) $\Delta\beta = 1$ dB

$\Delta\beta = 10\left(\log_{10}\left(\frac{I_f}{I_0}\right) - \log\left(\frac{I_i}{I_0}\right)\right)$

$\Delta\beta = 10\log_{10}\left(\frac{I_f}{I_i}\right)$

$1 = 10\log_{10}\left(\frac{I_f}{I_i}\right)$

$\frac{1}{10} = \log_{10}\left(\frac{I_f}{I_i}\right)$

$10^{1/10} = \frac{I_f}{I_i}$

$I_f = 10^{1/10} I_i$

$\%\ \text{INCREASE} = \frac{\text{FINAL} - \text{INITIAL}}{\text{INITIAL}} \times 100$

$\%\ \text{INCREASE} = \frac{10^{1/10} I_i - I_i}{I_i} \times 100$

$\%\ \text{INCREASE} \approx 25.9\%$

b) $r = 10$ m

$\beta = 85$ dB

$\beta = 10\log_{10}\left(\frac{I}{I_0}\right)$

$\beta = 10\log_{10}\left(\frac{P}{A I_0}\right)$

$\frac{\beta}{10} = \log_{10}\left(\frac{P}{4\pi r^2 I_0}\right)$

$10^{\beta/10} = \frac{P}{4\pi r^2 I_0}$

$P = 4\pi r^2 I_0 10^{\beta/10} \approx 0.397\ \text{W}$

c) $r = 10$ m

$r_5 = 2r$

$r_1 = r_2 = r_3 = r_4 \equiv r$

$P_1 = P_2 = P_3 = P_4 = P_5 \equiv P$

$\Sigma I = \frac{P}{4\pi r^2} + \frac{P}{4\pi (2r)^2}$

$\Sigma I = \frac{17P}{16\pi r^2}$

$\beta = 10\log_{10}\left(\frac{17P}{16\pi r^2 I_0}\right) \approx 91.3$ dB

$\Sigma I = I_1 + I_2 + I_3 + I_4 + I_5$

$\Sigma I = \frac{P_1}{A_1} + \frac{P_2}{A_2} + \frac{P_3}{A_3} + \frac{P_4}{A_4} + \frac{P_5}{A_5}$

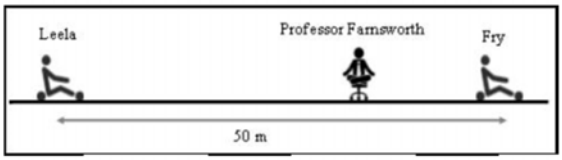
$\Sigma I = \frac{4P}{4\pi r^2} + \frac{P}{4\pi r_5^2}$

SEE *

Question 2

A stationary professor Farnsworth is trying to determine who will win the intergalactic go-kart race. The only two contenders left are Leela and Fry, with Leela trailing Fry by 50 m and Fry 50 m from the finish line. Both engines produce identical 600 Hz tones. Fry thought if pushing one of the pedals down made him go fast, then pushing them both down would make him go faster. As a consequence he is pressing down on both the brake and the gas and is traveling slower than Leela. Fry maintains a steady pace of 10 m/s traveling away from the professor, as shown in the figure. The professor uses his "frequency-O-meter" to determine that there is a 54 Hz sound at his location in addition to the sound from the engines. This additional sound is coming from the beat frequency generated by hearing both Leela and Fry's Doppler shifted sounds at the same time. Whenever two different frequency sounds are present at the same time there will be a third frequency generated called the beat frequency. This frequency is equal to absolute value of the difference in the two other frequencies.

(a) How fast is Leela traveling at that time?
(b) If both racers maintain a constant speed, who will win the race? Show your work.



a)

$\vec{v}_L$ $\vec{v}_{Pr} = \vec{0}$ $\vec{v}_F$

f_L f_B f_F

Professor hears a beat frequency from the Doppler shifted frequencies from Leela and Fry

$f_{L/P}$ and $f_{F/P}$

Knowns

$f_L = f_F = 600$ Hz

$|\vec{v}_f| = v_f = 10$ m/s

$f_{Pr} = 54$ Hz

Doppler Shift

$f_o = f_s\left(\frac{v \pm v_o}{v \mp v_s}\right)$

Leela $\Delta\vec{r}_L \downarrow$ Fry $\Delta\vec{r}_{FP} \uparrow$

$f_{L/P} = f_L\left(\frac{v+0}{v-v_L}\right)$ $f_{F/P} = f_F\left(\frac{v-0}{v+v_f}\right)$

Beat Freq

$f_B = |f_1 - f_2|$

$f_B = f_L\left(\frac{v}{v-v_L}\right) - f_f\left(\frac{v}{v+v_f}\right)$

Alg.. solve for v_L

$v_L = 19.9$ m/s

b)

$\hat{y}$ $\hat{x}$ $\vec{a}_L = \vec{0}$ $\vec{a}_F = \vec{0}$ Finish

$\vec{v}_{oL}$ $\vec{v}_{oF}$ $\vec{v}_{iF}$

50 m 50 m

When Fry finishes the race, where will Leela be? ... x_{iL}?

$x_{oF} = 50$ m

$x_{iF} = 100$ m

$v_{ofx} = 10$ m/s

$v_{ifx} = 10$ m/s

$a_{fx} = 0$

$\Delta t_f = ?$

$x_{iF} = x_{oF} + v_{ofx}\Delta t_f + \frac{1}{2}a_{fx}\Delta t_f^2$

$\Delta x_f = v_{ofx}\Delta t_f$

$\Delta t_f = 5$ sec

$x_{oL} = 0$

$x_{iL} = ?$

$v_{oLx} = 19.9245$ m/s

$v_{iLx} = 19.9245$ m/s

$a_{Lx} = 0$

$\Delta t_L = 5$ sec

$x_{iL} = x_{oL} + v_{oLx}\Delta t_L + \frac{1}{2}a_{Lx}\Delta t^2$

$x_{iL} = v_{oLx}\Delta t_L$

$x_{iL} = 99.6$ m

99.6 m < 100 m

So Fry wins!