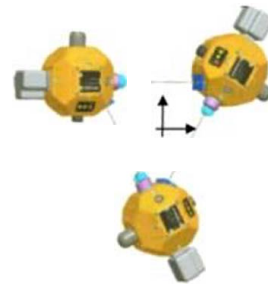


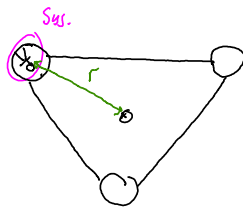
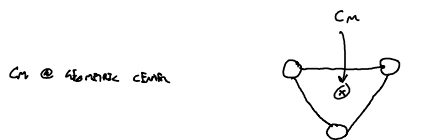
Week 3 Challenge Homework Solutions

Question 1:

A space station consists of three modules, connected to form an equilateral triangle of side length 82.0 m. Suppose 100 people, with an average mass of 75.0 kg each, live in each capsule and the mass of the modules is negligible compared to the mass of the people. At the current rotational rate, the effective acceleration of gravity is $g/2$.



- What is the angular momentum of the system?
- If the modules are pulled closer what should happen to the rotational rate? Explain.
- Determine the distance between the modules which would give an effective acceleration of gravity equal to g .
- Use the Proportionality sensemaking technique to evaluate whether the effective acceleration of gravity changes as expected with a change in the distance between modules. Hint: compare the problem statement and your answer to part (c) with your answer to part (b).



$$\sum F_r = m a_r$$

$$F^c = m \omega^2 r$$

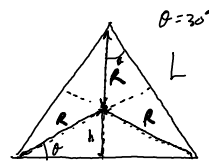
$$\frac{1}{2} m g = m \omega^2 r$$

$$\omega = \sqrt{\frac{g}{2r}}$$

$$\begin{aligned} \textcircled{a} \quad \sum L_{cm} &= L_{1cm} + L_{2cm} + L_{3cm} \\ &= 3 L_{1cm} \\ &= 3 I_{cm} \omega \\ &= 3 m R^2 \omega \end{aligned}$$

$$= 3 m R^2 \sqrt{\frac{g}{2r}}$$

$$\sum L_{cm} = 1.62 \times 10^7 \frac{\text{kg m}^2}{\text{s}}$$



$$\sin \theta = \frac{h}{R}$$

$$L \cos \theta = R + h$$

$$L \cos \theta = R + R \sin \theta$$

$$R = \frac{L \cos \theta}{(1 + \sin \theta)} = 47.34 \text{ m}$$

$$\textcircled{b} \quad \omega / \sum F_{ext} = 0$$

$$L = \text{const}$$

$$\text{so } \cdot \quad \underbrace{L}_{\text{constant}} = I \omega$$

$$\omega \propto \frac{1}{I}$$

$$\boxed{I \downarrow \text{ so } \omega \uparrow}$$

$$\textcircled{c} \quad F^c = m \omega^2 R$$

$$m g = m \omega^2 R$$

$$R = \frac{g}{\omega^2}$$

$$R = 94.68 \text{ m}$$

$$\boxed{L = 164 \text{ m}}$$

$$\textcircled{d} \quad \text{no. } \omega / \sum F_{ext} = 0 \quad \Delta L = 0$$

Question 2:

A helium atom falls in a vacuum. The mass of a helium atom is 6.64×10^{-27} kg.

- (a) From what height must it fall so that its translational kinetic energy at the bottom equals the average translational kinetic energy of a helium molecule at 300 K?
- (b) At what temperature would the average speed of helium atoms equal the escape speed from the Earth, 1.12×10^4 m/s.
- (c) Do **any** of the helium atoms in the Earth's atmosphere have this speed? Explain your answer and any consequences that may arise from it.

a) $\overline{K_e} = \frac{3}{2} k_B T$

$$E_i = E_f = K_e @ 300K$$

$$U^2 = mgy$$

$$\Rightarrow mgy = \frac{1}{2} m v^2 = \frac{3}{2} k_B T$$

$$K_{E_T} = \frac{1}{2} m v^2$$

$$y = \frac{3k_B T}{2mg} = 95,400 \text{ m}$$

$y = 95.4 \text{ km}$

b) $\frac{3}{2} k_B T = \frac{1}{2} m v_{esc}^2$

$T = 20,118 \text{ K}$

- c) yes, likely. There is a distribution of speeds \Rightarrow not all particles have same speed. Some at a given moment will have $v > v_{esc}$, but will collide with other particles and lose v . Also, some particles will escape!