Week 4 Challenge Homework Solutions

Question 1:

Beer brewing begins with steeping grains in hot water, releasing the sugars inside. The sugar water is then heated to a boil and hops added. The hot temperature of the boil extracts oils from the hops and provides sanitation from unwanted bacteria. The whole mess is cooled down and once it is safe enough for yeast to survive, they are added. The yeast converts the sugars to alcohol, and the oils from the hops provide many things including flavor and antibacterial benefits.

(a) If 16.5 lbs of grain at 67 °F are added to 5 gal of hot water and the equilibrium temperature of the mixture is to be 154 °F, what must the initial temperature (strike temp) of the hot water be? The specific heat of malt grains is about 0.44 times that of water. Also assume no energy is lost during the time the system comes to equilibrium.

(b) How much did it cost to heat the 5 gals of tap water from 110 °F up to the strike temp? Assume the heating takes 47 minutes, electricity costs 15 ¢ per kilowatt-hour, and only 10% energy is lost during heating.

(c) During the one hour steeping stage, where the water and grain mixture started at 154 °F, the mixture only lost 2 °F. What was the average energy per time lost by the mixture.

(d) The steeping was done in a cylindrical vessel (cooler) with outside dimensions 20 inches tall and 15 inches in diameter. The thickness of the walls is 1 inch. Estimate the thermal conductivity of the steeping vessel assuming a room temperature of 67 °F.

a)

SI

16.5 lbs GRAIN = 7.48427 kg

67 °F = 19.444 °C

154 °F = 67.777 °C

5 GAL = $0.0189271\ m^3$

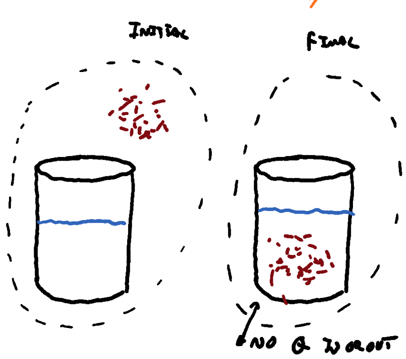
CONSTANTS

$C_w = 4190\ J/kg\,K$

$C_{GRAINS} = 0.44\, C_w = 1843.6\ J/kg\,K$

$\rho_w = \frac{m_w}{V_w} \rightarrow m_w = \rho_w V_w = (1000\ \frac{kg}{m^3})(0.0189271\ m^3)$

$m_w = 18.9271\ kg$



$$\Sigma Q = 0$$

$$Q_w + Q_{GRAINS} = 0$$

$$m_w C_w \Delta T_{w\, ? \rightarrow 67.777} + m_{GRAINS} C_{GRAINS} \Delta T_{GRAIN\, 19.444 \rightarrow 67.777} = 0$$

$$(18.9271)(4190)(67.777 - T_{iw}) + (7.48427)(1843.6)(67.777 - 19.444) = 0$$

$$T_{iw} = 76.187\ °C \approx \boxed{169\ °F}$$

(b)

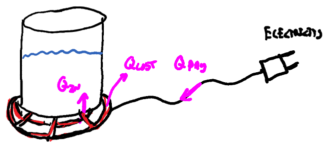
SI

$110°F = 316.483\ K$

$169°F = 349.261\ K$

Constants

Assume $e = 0.90$



$e = \frac{GET}{PAY}$

$e = \frac{Q_{w}}{Q_{PAY}}$

$Q_{w} = M_w C_w \Delta T_{316 \to 349}$

$= (18.9271)(4190)(349.261 - 316.483)$

$Q_w \approx 2599444.507\ J$

$Q_{PAY} = \frac{Q_w}{e}$

$Q_{PAY} = \frac{2599444.507}{0.9}\ J$

$Q_{PAY} = 2888271.674\ J$

NEED THIS MUCH ENERGY FROM POWER COMPANY

$J \to kWHr$

$Q_{PAY} = 2.888271.674\ J \times \frac{1\ kWHr}{3600000\ J}$

$Q_{PAY} \approx 0.8023\ kWHr$

$kWHr \to ¢$

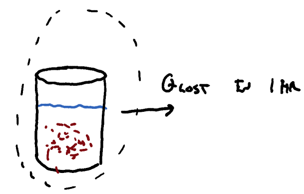
$0.8023\ kWHr \times \frac{15¢}{1\ kWHr} \approx$ **12 ¢**

(c)

SI

$154°F = 67.777\ °C$

$152°F = 66.667\ °C$



$Q_{LOST} = Q_{w_{67 \to 66}} + Q_{GRAINS_{67 \to 66}}$

$= M_w C_w \Delta T_w + M_{GRAINS} C_{GRAINS} \Delta T_{GRAINS}$

$Q_{LOST} = (18.9271)(4190)(66.667 - 67.777) + (7.48427)(1843.6)(66.667 - 67.777)$

$Q_{LOST} \approx -103344\ J$

$Power = \frac{Energy}{Time}$

$= \frac{|Q_{LOST}|}{\Delta t} = \frac{103344\ J}{3600\ s} \approx$ **$29\ \frac{J}{s}$** or **$29\ W$**

(d)

SI

Assume constant T of inside

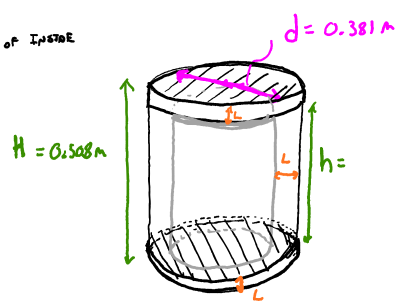
$153°F = 67.222\ °C$

$67°F = 19.444\ °C$

$20\ in = 0.508\ m$

$15\ in = 0.381\ m$

$1\ in = 0.0254\ m$



*Assume negligible energy lost due to radiation so only convection.

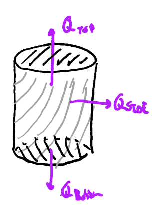
*Estimate cross-sectional area using outside dimensions.

Conduction

$\frac{Q}{\Delta t} = \frac{k A \Delta T}{L}$

$k = \frac{L\,Q}{A \Delta T \Delta t}$

From part (c) $Q_{LOST} = 103344\ J$



$A = A_T + A_B + A_S$

$= \frac{\pi d^2}{4} + \frac{\pi d^2}{4} + \pi d H$

$A = \frac{\pi d^2}{2} + \pi d H \approx 0.836067\ m^2$

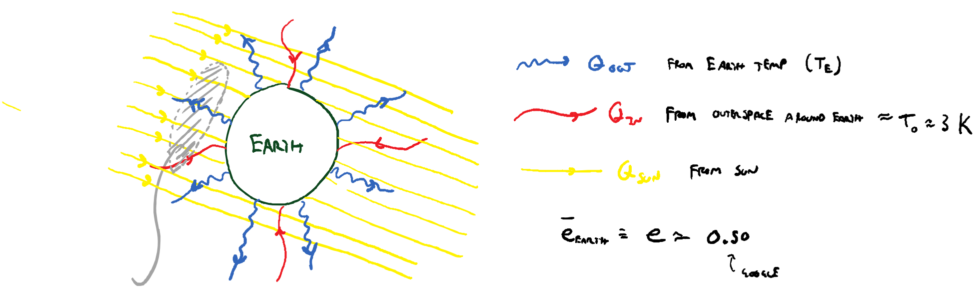
$k \approx \frac{(0.0254)(103344)}{(0.836067)(67.222 - 19.444)(3600\ s)}\ W/mK$

$k \approx 0.018\ W/mK$

Question 2:

At the Earth's distance from the Sun, the intensity of solar radiation is 1370 W/m². The temperature of the Earth is affected by the *greenhouse effect* of the atmosphere. This phenomenon is caused be our atmosphere's absorption of infrared light emitted by the surface, which makes the surface temperature of the Earth higher than if it were airless. For comparison, consider a spherical object of radius *r* with no atmosphere at the same distance from the Sun as the Earth. Assume its emissivity is the same for all kinds of electromagnetic radiation and its temperature is uniform over its surface.

(a) Explain why the projected area over which it absorbs sunlight is πr^2 and the surface area over which it radiates is $4\pi r^2$.
(b) Compute its steady-state temperature.
(c) Use *Known Values* sense-making to determine if this a reasonable value.
(d) Why would an increase in greenhouse gasses such as water vapor, carbon dioxide, and methane be cause for alarm?



$\bar{e}_{EARTH} = e \approx 0.50$ ← GOOGLE

a) CROSS SECTIONAL AREA OF EARTH THAT Q_{SUN} SEES. } AREA OF CIRCLE πR_E^2

EARTH RADIATES SPHERICALLY IN ALL DIRECTIONS } AREA OF SPHERICAL SHELL $4\pi R_E^2$

* INTENSITY → $\frac{W}{m^2}$ → $\frac{POWER}{AREA}$ → $\frac{(ENERGY/TIME)}{AREA}$

$$I_s = \frac{P_s}{A_s} = \frac{(Q_s/\Delta t)}{A_s}$$

b) STEADY STATE...

$$\frac{Q}{\Delta t} = 0$$

$$\frac{Q_s}{\Delta t} + \frac{Q_E}{\Delta t} + \frac{Q_o}{\Delta t} = 0$$

$$I_s A_s + (-e\sigma A_E T_E^4) + e\sigma A_E T_o^4 = 0$$

$$I_s \pi R_E^2 - e\sigma 4\pi R_E^2 T_E^4 + e\sigma 4\pi R_E^2 T_o^4 = 0$$

$$T_E^4 = \frac{I_s \pi R_E^2 + e\sigma 4\pi R_E^2 T_o^4}{e\sigma 4\pi R_E^2}$$

$$T_E = \left(\frac{I_s}{4e\sigma} + T_o^4\right)^{\frac{1}{4}} \qquad \left(\frac{1370}{4(.5)(5.67\times10^{-8})} + 3^4\right)^{\frac{1}{4}}$$

$\approx$ **320 K** $\approx 116\,^\circ F$ ← HOT !

c) GREENHOUSE GASSES ABSORB Q TRYING TO LEAVE THE EARTH; THIS EFFECTIVELY DECREASING THE $\bar{e}$ OF THE EARTH ... NOTE IF $e\downarrow$ THEN $T_E\uparrow$

EX... IF $e \downarrow \frac{1}{2}$ .... $e = \frac{1}{2}(.5) = .25$

$T_E \approx 394\,K \approx 250\,^\circ F$