Week 8 Challenge Homework Solutions

Question 1

Watch the pendulum video found at (https://media.oregonstate.edu/media/t/0_5ebk2zoy) and answer the following questions.

(a) Find an equation for the angle as function of time that describes the motion of the pendulum, include damping.

(b) Use the equation to predict the position (positive or negative) and velocity (positive or negative) at t = 30 s. How well were you able to predict the position? Excluding human measurement error, what could result in your predictions being off?

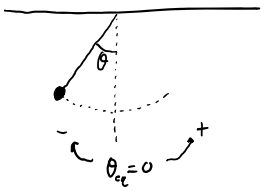
MEASUREMENTS FROM VIDEO

@ $t=0$ $\theta_{max} \approx 20^\circ \longrightarrow \theta_{max}(t=0) = 20^\circ$

10 OSCILLATIONS $\approx$ 18 SEC $\longrightarrow T = 1.8\ s \rightarrow f = \frac{5}{9}$ Hz

@ ≈ 48 sec $\theta \approx 10^\circ \longrightarrow \theta(48\ s) = 10^\circ$

a)



$\theta(t) = -\theta_{max}(t=0)\, e^{-t/\tau} \cos(\omega t)$

BUT WHAT IS τ?

$\theta_{max}(t) = \theta_{max}(t=0)\, e^{-t/\tau}$

$\theta_{max}(t=48) = \theta_{max}(t=0)\, e^{-48/\tau}$

$10 = 20\, e^{-48/\tau}$

$\frac{1}{2} = e^{-48/\tau}$

$\ln(\frac{1}{2}) = -\frac{48}{\tau}$

$\tau = \frac{-48}{\ln(\frac{1}{2})} \approx 69.25$ sec

$\theta(t) = -20^\circ e^{-t/69.25} \cos(2\pi f t)$

$\theta(t) = -20^\circ e^{-1.444\times10^{-2} t} \cos(3.49t)$

OR

$\theta(t) = -0.349\ \text{RAD}\ e^{-1.444\times10^{-2} t} \cos(3.49t)$

b)

$\theta(t=30) \approx 6.7^\circ$

↑ + SO R.H.S. OF θ_{eq}

FOR $\mathcal{R}(t=30)$ LOOK AT $\theta(t)$ W/ t A LITTLE BEFORE & AFTER $t=30$ ... e.g. 29.9 & 30.1

$\theta(t=29.9) \approx 10^\circ$

$\theta(t=30.1) \approx 2.5^\circ$

θ IS ↓ AND PENDULUM IS ON RIGHT HAND SIDE

THUS $\mathcal{R}(t=30)$ MUST BE NEGATIVE