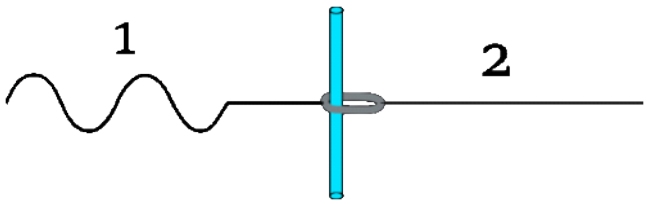
Week 10 quiz

Thursday, March 11, 2021 12:42 PM

An interesting material property of strings is known as impedance (**Z**). The impedance is equal to the square root of the tension in the string multiplied by the square root of the linear mass density of the string. Consider two different strings connected to a very light hoop that can slide vertically on a pole with negligible friction between the pole and the hoop. The pole allows the tension in each string to be different. A traveling wave is created on the left side of **string 1** and begins to move towards **string 2**. If the impedance of **string 1** ($\mathbf{Z}_1$) is equal to the impedance of **string 2** ($\mathbf{Z}_2$), then 100% of the traveling wave in **string 1** gets transmitted to **string 2** (i.e. none of the wave is reflected at the pole boundary). The linear mass density of **string 2** is 1/3 of the linear mass density of **string 1**.



$$Z = \sqrt{F^T \mu}$$

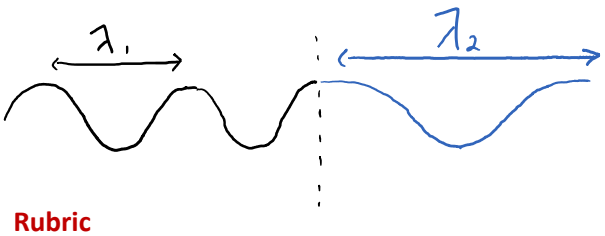
$$v = \sqrt{\frac{F^T}{\mu}} = f\lambda$$

(a) By what factor must the tension in **string 2** be compared to **string 1** if the impedance of both strings are equal?

$$Z_1 = Z_2 \Rightarrow \sqrt{F_1^T \mu_1} = \sqrt{F_2^T \mu_2}, \quad F_2^T = \frac{\mu_1}{\mu_2} F_1^T, \quad \text{w/ } \mu_2 = \frac{\mu_1}{3}$$

$$F_2^T = 3 F_1^T$$

(b) Sketch a physical representation showing the traveling wave in both sections of the string. Label the wavelengths in each string and scale them relative to each other. Use words, diagrams, equations, etc… to explain the relationship between the frequencies and wavelengths in each string.



$f_1 = f_2$ for continuity

So, $\frac{v_1}{\lambda_1} = \frac{v_2}{\lambda_2} \Rightarrow \sqrt{\frac{F_1^T}{\mu_1}} \frac{1}{\lambda_1} = \sqrt{\frac{F_2^T}{\mu_2}} \frac{1}{\lambda_2}$

$$\ldots \ldots = \sqrt{\frac{(3)3F_1^T}{\mu_1}} \frac{1}{\lambda_2}$$

$$\sqrt{\frac{F_1^T}{\mu_1}} \frac{1}{\lambda_1} = 3\sqrt{\frac{F_1^T}{\mu_1}} \frac{1}{\lambda_2}$$

$$\lambda_2 = 3\lambda_1$$

Rubric

Part (a)	Part (b)
1 pt - setting $Z_1 = Z_2$	1 pt - $f_1 = f_2$
1 pt - Z = sqrt(F^T*mu) equation	1 pt - v = sqrt(F^T/mu) equation
1 pt - $mu_2 = mu_1/3$ equation	1 pt - v = f*lambda equation
0.5 pt - algebra	1 pt - combining equations
0.5 pt - answer	0.5 pt - algebra
	1 pt - wavelength relationship
	0.5 pt - sketch