Week 9 Quiz

Tuesday, March 2, 2021 1:30 PM

On a beautiful spring Corvallis day, ever the adorable couple, Benny and Bernice swing gently on a porch swing pendulum. Bernice initially notices that as they swing back and forth, for the first few swings their motion can be described by the following equation (before any damping can be noticed):

$$\theta(t) = 0.2\cos(2.4\,t)\ radians$$

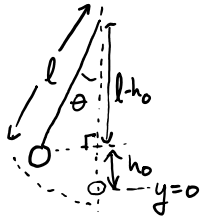
Benny, Bernice, and the swing together have a mass of 62 kg.

(a) Find the length of the pendulum.

$\omega = \sqrt{\frac{g}{l}} = 2.4\ \frac{rads}{s} \Rightarrow l = \frac{g}{\omega^2} = 1.7\,m$

(b) Find the initial total energy of the oscillation (before any energy leaves the system through damping).

$\theta_{max}(t=0) = 0.2$ rads @ max height $E^\circ_{tot} = K^\circ + U_{max}$, $U_{max} = mgh_0$



$\cos\theta^\circ_{max} = \frac{l-h_0}{l} \Rightarrow h_0 = l(1-\cos\theta^\circ_{max})$

So, $E^\circ_{tot} = mgl(1-\cos(\theta^\circ_{max})) = 20.6\,J$

(c) Bernice notices that their maximum kinetic energy has decreased to 47% of its initial value after 73 seconds. Find the time constant, τ, of their damped simple harmonic motion.

Need to find $\theta^{(t=73s)}_{max}$

$E_{tot} = K_{max} = U_{max}$

1 refers to @ $t = 73s$

$E'_{tot} = 0.47\,E^\circ_{tot} = 9.685\,J$

$U'_{max} = E'_{tot} = mgl(1-\cos(\theta'_{max}))$

$\theta'_{max} = 0.137$ rads

Now apply damping

$\theta_{max}(t) = \theta^{(t=0)}_{max} e^{-t/\tau}$

$0.137\,rads = 0.2\,rads\ e^{-73s/\tau}$

$0.685 = e^{-73s/\tau}$

$\tau = \frac{-73s}{\ln(0.685)} = 193s$

Rubric

~~ Part (a) - 2 pts ~~
1 pt - omega equation
0.5 pts - omega = 2.4 rad/s
0.5 pts - answer and units

~~ Part (b) - 4 pts ~~
1 pt - E_tot = U_max
1 pt - U = mgh
1.5 pts - geometry
0.5 pts - answer and units

~~ Part (c) - 4 pts ~~
1 pt - Finding new theta max
1 pt - damping equation
1.5 pts - application to find tau
0.5 pts - answer and units