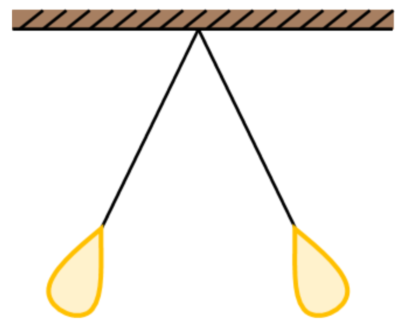
Week 4 Challenge Homework Solutions

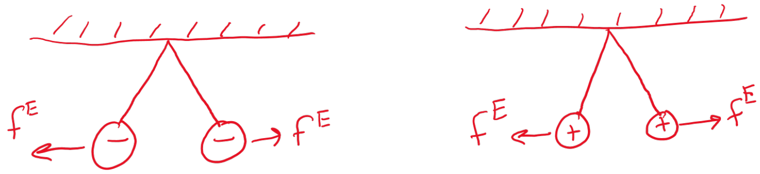
Question 1

Two balloons filled with air are tied to very light 0.5-m-long string. The loose ends of the strings are taped to the same position on a horizontal bar. When both are equally rubbed with the same material they both develop a static charge and repel each other making an angle of 30º with respect to the vertical.



(a) With this experiment alone can you tell what type of charge is on the balloon? Explain.

Cannot tell, both + and − charges will repel each other



$$\vec{F} = k\frac{q^2}{|\Delta\vec{r}|^2}\Delta\hat{r} \qquad q^2 \Rightarrow (-q)(-q) \text{ or } (+q)(+q)$$

give the same sign

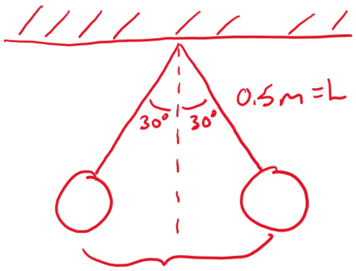
$\Rightarrow$ same direction

(b) Is this a stable or unstable equilibrium?

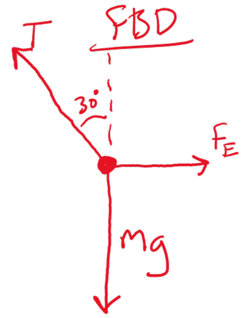
Stable, - increasing $|\Delta\vec{r}|$ between charges will decrease $F_E \Rightarrow \vec{F}_{net}$ pulls charges together

- decreasing $|\Delta\vec{r}|$ between charges will increase $F_E \Rightarrow F_{net}$ pushes charges apart

(c) *Estimate* the number of excess charges on each balloon. Clearly state any assumptions made in your estimation.



$\Delta x = 2L\sin 30° = L$



$F_E = T\sin 30°$

$mg = T\cos 30°$

$T = \dfrac{mg}{\cos 30°} \Rightarrow F_E = mg\tan 30°$

$F_E = k\dfrac{q^2}{(\Delta x)^2}$

$q^2 = \dfrac{\Delta x^2}{k}\, mg\, \dfrac{1}{\sqrt{3}}$

$q^2 = \dfrac{(0.5)^2}{9\times10^9}(0.005)(9.81)\dfrac{1}{\sqrt{3}}$

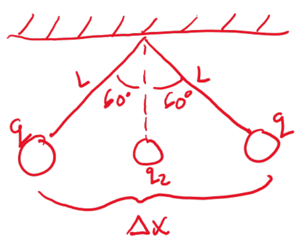
$q = 8.87\times10^{-7}\ C \Rightarrow \#\ \text{charges} = \dfrac{8.87\times10^{-7}}{1.6\times10^{-19}}$

$\boxed{\#\ \text{charges} = 5.54\times10^{12}}$

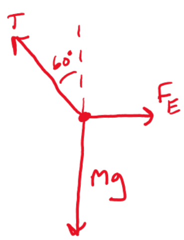
assumptions

- ignore mass of air inside balloon b/c buoyancy will exactly negate gravity
- balloon rubber is 5g
- every excess charge has magnitude e

(d) If a small charged sphere is placed in the middle of the two balloons and the angle between them doubles, *estimate* the number of excess charges on the sphere, again being sure to state any assumptions used in your calculation.



$\Delta x = 2L\sin 60° = \sqrt{3}L = \frac{\sqrt{3}}{2}$ meters



$mg = T\cos 60° = \frac{1}{2}T$

$F_E = T\sin 60° = \frac{\sqrt{3}}{2}T$

$\Rightarrow F_E = \sqrt{3}\, mg$

$F_E = k\dfrac{q^2}{(\Delta x)^2} + k\dfrac{q q_2}{\left(\frac{\Delta x}{2}\right)^2}$

$\sqrt{3}\, mg = k\dfrac{q^2}{(\Delta x)^2} + 4k\dfrac{q q_2}{(\Delta x)^2}$

$\dfrac{\sqrt{3}\, mg\, (\Delta x)^2}{k} = q^2 + 4 q q_2$

$4 q q_2 = \dfrac{\sqrt{3}\, mg\,(\Delta x)^2}{k} - q^2 \Rightarrow q_2 = \dfrac{\sqrt{3}\, mg\,(\Delta x)^2}{4kq} - \dfrac{1}{4}q = \dfrac{\sqrt{3}\,(0.005)(9.81)\left(\frac{\sqrt{3}}{2}\right)^2}{4(9\times10^9)(8.87\times10^{-7})} - \frac{1}{4}(8.87\times10^{-7})$

$q_2 = 2.00\times10^{-6}\ C - 2.22\times10^{-7}\ C$

$q_2 = 1.77\times10^{-6}\ C \Rightarrow \#\ \text{of Charges} = \dfrac{1.77\times10^{-6}}{1.60\times10^{-19}} = \boxed{1.11\times10^{13}\ \text{charges}}$