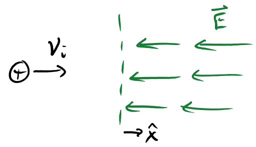
Week 5 Challenge Homework Solutions

Question 1

A proton is fired directly into a uniform electric field. The field is able to momentarily reduce the charge's momentum to zero.

(a) What must be the orientation of the electric field relative to the initial momentum of the proton?
(b) What type of charge configuration would generate a uniform electric field?
(c) If the proton is initially traveling at 5% the speed of light, what field strength will stop the charge within 1.00 cm of entering the field region?
(d) How much work does the electric field do in stopping the proton?

a.)



must be parallel + point in opposite direction

b.) very large plate(s)

c.) $\sum K_i + W = \sum \cancel{K_f}^{\,0}$, $W = \vec{F}^E \cdot \Delta\vec{r} = \underbrace{|\vec{F}^E|}_{eE_0}\underbrace{|\Delta\vec{r}|}_{\Delta x}\cancel{\cos\theta}^{\,-1}$

$\frac{1}{2}mv_i^2 = eE_0\Delta x$

$E_0 = 1.17\times10^{8}\ \frac{N}{C}$

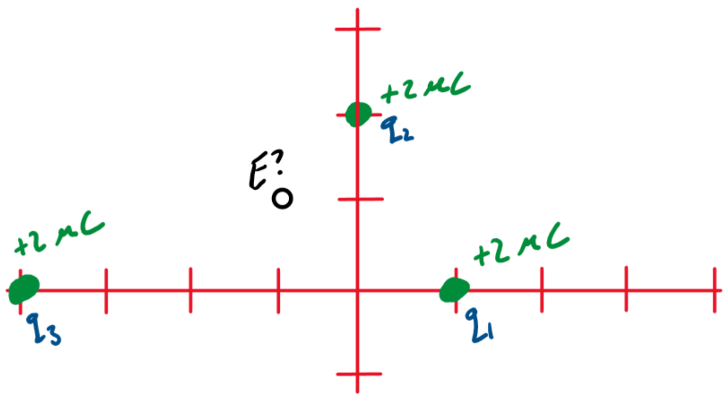
d.) $W = -K_i = -\frac{1}{2}mv_i^2$

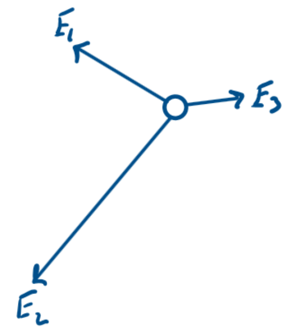
$= -1.88\times10^{-13}\ J$

Question 2

Three +2 μC point charges are fixed in space in the following locations: the first at <1, 0, 0> cm, second at <0, 2, 0> cm, and a third at <-4, 0, 0> cm.

(a) Find the net electric field vector at position <-1, 1, 0> cm.
(b) At this location, what is the force vector on a -3 μC test charge?
(c) Use the *sign* sense-making technique for each dimension (X, Y, Z) to check that your answer to part (b) is consistent with your answer to part (a).





$$\underline{\vec{E}_{tot} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3}$$

$\underline{E_1}$

$$\vec{E}_1 = \frac{kq}{|\Delta\vec{r}_1|^2}\hat{r}_1$$

$$\Delta\vec{r}_1 = \langle -2, 1, 0\rangle \text{ cm}$$

$$\Delta\hat{r}_1 = \frac{\vec{r}_1}{|\vec{r}_1|} = \frac{\langle -2, 1, 0\rangle \text{ cm}}{\sqrt{2^2+1^2}\text{ cm}} = \left\langle -\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}, 0\right\rangle$$

$$\vec{E}_1 = \frac{(9\times10^9)(+2\times10^{-6})}{(0.0005)}\left\langle -\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}, 0\right\rangle \frac{N}{C}$$

$$= 3.6\times10^7\left\langle -\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}, 0\right\rangle \frac{N}{C}$$

$\vec{E}_2$

$$E_2 = \frac{kq}{|\Delta\vec{r}_2|^2}\,\Delta\hat{r}_2$$

$$\Delta\vec{r}_2 = \langle -1, -1, 0 \rangle \text{ cm}$$

$$\Delta\hat{r}_2 = \frac{\langle -1, -1, 0 \rangle \text{ cm}}{\sqrt{2\text{ cm}}} = \left\langle -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0 \right\rangle$$

$$E_2 = 9\times10^7 \left\langle -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0 \right\rangle \frac{N}{C}$$

E_3

$$E_3 = \frac{kq}{|\Delta\vec{r}_3|^2}\,\Delta\hat{r}_3$$

$$\Delta\vec{r}_3 = \langle +3, +1, 0 \rangle \text{ cm}$$

$$\Delta\vec{r}_3 = \left\langle \frac{3}{\sqrt{10}}, \frac{1}{\sqrt{10}}, 0 \right\rangle$$

$$E_3 = 1.8\times10^7 \left\langle \frac{3}{\sqrt{10}}, \frac{1}{\sqrt{10}}, 0 \right\rangle \frac{N}{C}$$

$$E_{tot} = 10^7 \left\langle -\frac{7.2}{\sqrt{5}} - \frac{9}{\sqrt{2}} + \frac{5.4}{\sqrt{10}}, \frac{3.6}{\sqrt{5}} - \frac{9}{\sqrt{2}} + \frac{1.8}{\sqrt{10}}, 0 \right\rangle \frac{N}{C}$$

$$\boxed{E_{tot} = 10^7 \langle -7.88, -4.18, 0 \rangle \frac{N}{C}}$$

$$\sqrt{7.88^2 + 4.18^2} = 8.92$$

$$\boxed{E_{tot} = 8.92\times10^7 \langle -0.883, -0.468, 0 \rangle \frac{N}{C}}$$

(b) $\vec{F} = q\vec{E}$

$$\vec{F} = (-3 \times 10^{-6})(E_{tot})$$

$$= 268 \langle +0.883, +0.468, 0 \rangle \text{ N}$$

(c) I expect the force vector on a negative charge to be opposite direction from the electric field vector. The x, y, and z components of the force vector have opposite signs from the corresponding components of the $\vec{E}$. This matches my expectation!