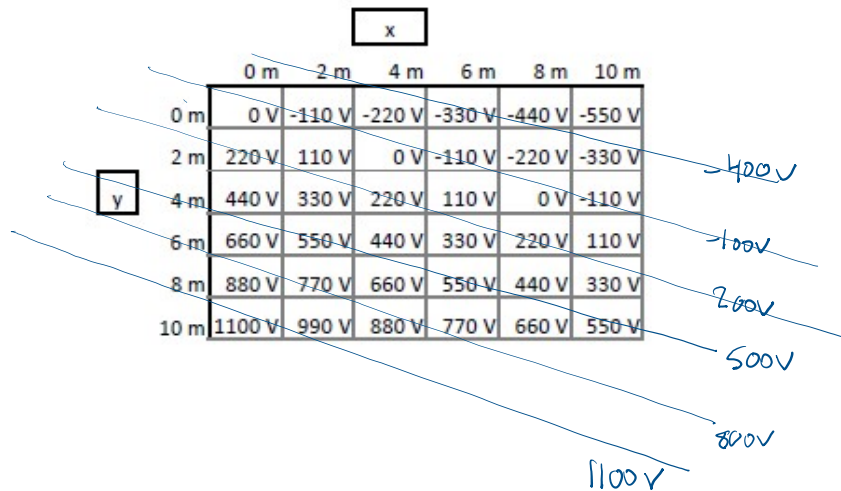


# Week 7 Challenge Homework Solutions

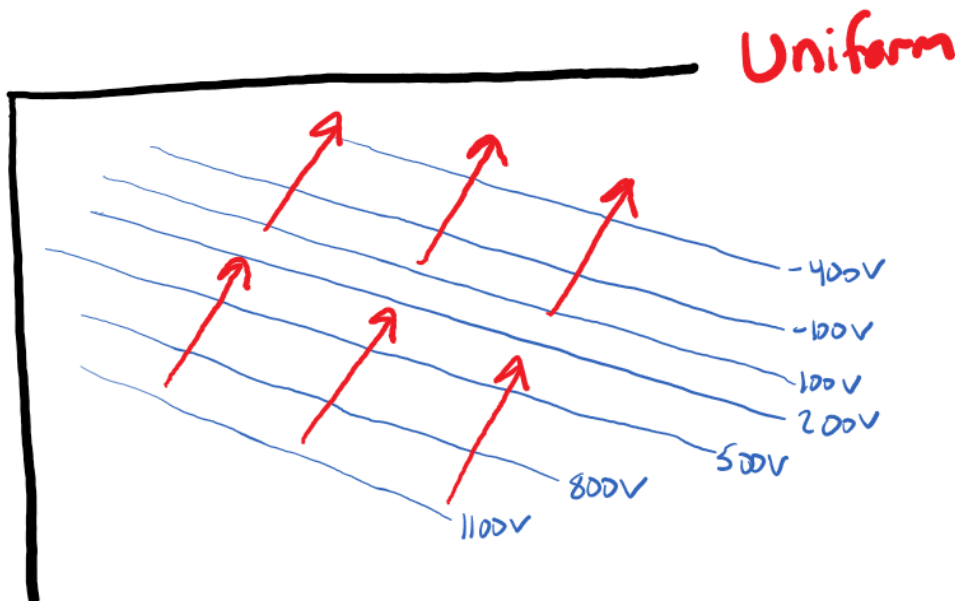
## Question 1

The voltage is mapped out everywhere in a 2D space for a certain distribution of charge. The voltage data is presented in the table to the right and each value is in volts.

- (a) Map out these voltage values by sketching a set of equipotential surfaces for  $V = -400\text{V}$ ,  $-100\text{V}$ ,  $200\text{V}$ ,  $500\text{V}$ ,  $800\text{V}$  and  $1100\text{V}$ .



- (b) Sketch the electric vector field.



(c) What is an equation for the electric potential as a function of x and y?

X	V
0	0V
2	-110V
4	-220V
6	-330V
8	-440V
10	-550V

 $\Rightarrow V_x = -55x$ 

Y	V
0	0V
2	220V
4	440V
6	660V
8	880V
10	1100V

$V_y = 110y \leftarrow$

$V(x,y) = -55x + 110y$

(d) What is the electric field as a function of x and y?

$$E_x = - \frac{\Delta V}{\Delta x} = - \text{slope of } V = 55 \text{ N/C in } x \text{ direction}$$

$$E_y = - \frac{\Delta V}{\Delta y} = -110 \text{ N/C in } y \text{ direction}$$

$$\vec{E}(x,y) = \langle 55, -110 \rangle \frac{\text{N}}{\text{C}}$$

$$|\vec{E}| = \sqrt{55^2 + 110^2} = 123.0$$

$$\vec{E}(x,y) = 123 \frac{\text{N}}{\text{C}} \left\langle \frac{55}{123}, -\frac{110}{123} \right\rangle$$

Note: the +y axis has been chosen to point down the page to match the voltage map given.

Sense-making: the electric field found algebraically ( $\leftarrow$  there) points in the +x, -y direction. This matches the drawn electric field from part b! Yay!

## Question 2

Certain fish, such as the Elephant Fish (Mormyridae), concentrate charges in their head and tail, thereby producing an electric field in the water around them. This field creates a potential difference of a few volts between the head and tail, which in turn causes current to flow in the conducting seawater. As the fish swims, it passes near objects that have a resistivity different from that of



seawater, which in turn causes the current to vary. Cells in the skin of the fish are sensitive to this current and can detect changes in it. The changes in the current allow the fish to navigate. (In the next few chapters, we shall investigate how the fish might detect this current.) Since the electric field is weak far from the fish, we shall consider only the field in the vicinity of the fish. If the fish is 20 cm long (certain members of the family can reach 1.5 m), we can model the seawater through which that field passes as a conducting tube around the fish of area  $4.0 \text{ cm}^2$  which has a potential difference of 3.0 V between its ends. The resistivity of seawater is  $0.13 \text{ } \Omega \cdot \text{m}$ . For this model let's assume that the fish has negligible width so that it does not impact the conductivity of the tube.

- (a) How large is the current through the tube of the seawater?

$$R = \rho \frac{L}{A} = (0.13) \frac{(0.2)}{4 \times 10^{-4}} = 65 \Omega$$

$$I = \frac{\Delta V}{R} = \frac{3V}{65 \Omega} = 46.2 \text{ mA}$$

- (b) Suppose the fish swims along the side of a vertical rock face such that one quarter of the tube is now taken up by the rock. The rock has a metal content that gives it twice the conductivity of seawater. What is the difference in current that the fish detects? (Hint: How are the rock and the remaining water in the tube connected, in series or in parallel?)

$$R_1 = \rho_1 \frac{L}{A_1} = (0.13) \frac{(0.2)}{3 \times 10^{-4}} = 86.7 \Omega$$

$$R_2 = \rho_2 \frac{L}{A_2} = \frac{\rho_1}{2} \frac{L}{A/4} = \frac{2}{2} R_1 = 130 \Omega$$

$\sigma_2 = 2\sigma_1$   
 $\Rightarrow \rho_2 = \frac{\rho_1}{2}$

$$I_{\text{new}} = \frac{3V}{52 \Omega} = 57.7 \text{ mA}$$

$$\Delta I = 57.7 \text{ mA} - 46.2 \text{ mA} = 11.5 \text{ mA}$$