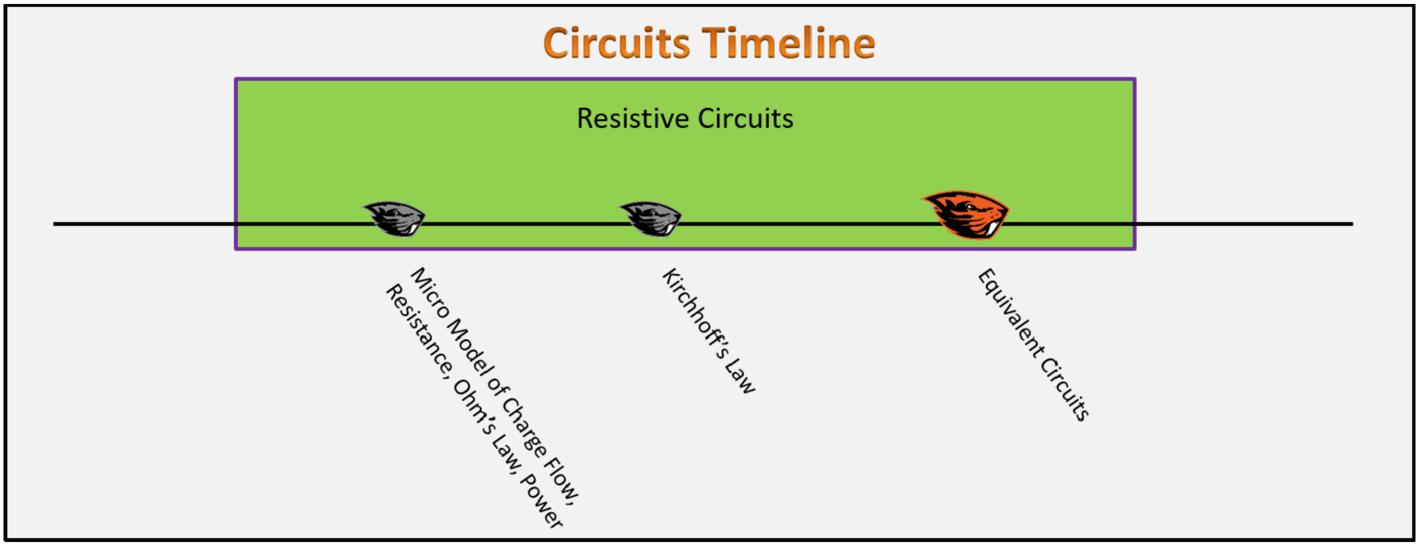
# (RC.L3.2-sols) Foundation Stage Solutions

Thursday, March 29, 2018 8:34 PM

# Circuits

## Foundation Stage (RC.L3.2)

### Lecture 3
### Equivalent Circuits



## Textbook Chapters (* Calculus version)

- **BoxSand** :: KC videos ( Microscopic Charge Flow )
- **Knight** (College Physics : A strategic approach 3rd) ::
- ***Knight** (Physics for Scientists and Engineers 4th) ::
- **Giancoli** (Physics Principles with Applications 7th) ::

## Warm up

**RC.L3.2-01:**

**Description:**

**Learning Objectives:** [?] - Can you identify the objectives from the previous lecture, and this lecture, that this question is relevant to?

**Problem Statement:**

## Selected Learning Objectives

1. Coming soon to a lecture template near you.

## Key Terms

- Electric Force
- Charge
- Coulombs
- Electron
- Proton
- Neutron
- Conductor
- Charge transfer

## Key Equations

| | |
|---|---|
| $\Delta V = IR$ | $I = \frac{\Delta q}{\Delta t}$ |
| $R_{eq} = R_1 + R_2 + \cdots$ | $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots$ |

## Key Concepts

- Coming soon to a lecture template near you.

## Questions

## Act I: Resistance and Equivalent Circuits

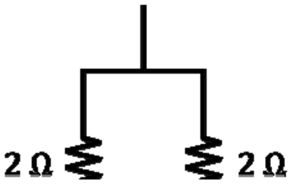
**RC.L3.2-02:**

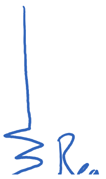
**Description:** [?]

**Learning Objectives:** [?]

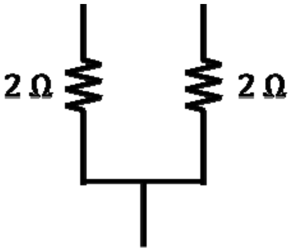
**Problem Statement:** Find the equivalent resistances for each of the following circuits.

**(a)**





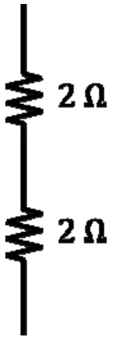
$$\frac{1}{R_{eq}} = \frac{1}{2\Omega} + \frac{1}{2\Omega} = \frac{2}{2\Omega}$$



$R_{eq}$ $2\Omega$ $2\Omega$ $2\Omega$

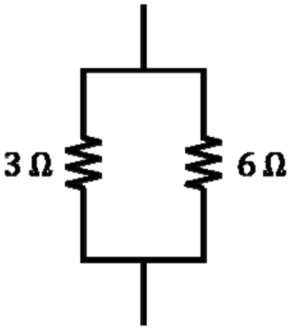
$$\Rightarrow \boxed{R_{eq} = 1\,\Omega}$$

**(b)**



$$R_{eq} = 2\Omega + 2\Omega = \boxed{4\Omega}$$

**(c)**



$$\frac{1}{R_{eq}} = \frac{1}{3\Omega} + \frac{1}{6\Omega} = \frac{3}{6\Omega}$$

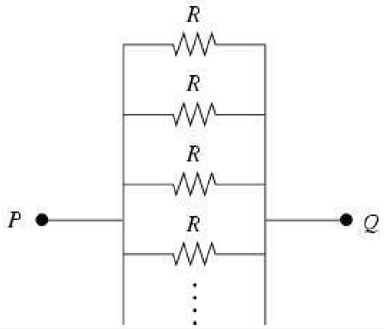
$$\boxed{R_{eq} = 2\,\Omega}$$

**RC.L3.2-03:**

**Description:** [?]

**Learning Objectives:** [?]

**Problem Statement:** As more identical resistors, *R*, are added to the parallel circuit shown here, the total resistance between points *P* and *Q* ...



(1) Increases.
(2) Remains the same.
[✓ (3) Decreases.]

$$R_{eq} = \frac{1}{\frac{1}{R}+\frac{1}{R}} = \frac{1}{\frac{2}{R}} = \frac{R}{2}$$

$$\vdots$$

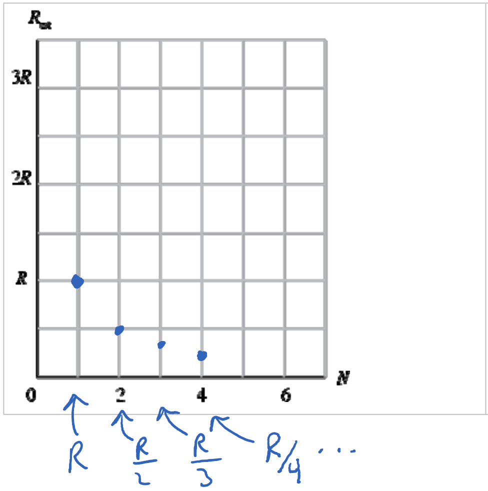
$$\frac{1}{\frac{1}{R}+\frac{1}{R}+\frac{1}{R}+\frac{1}{R}} = \frac{1}{\frac{4}{R}} = \frac{R}{4}$$

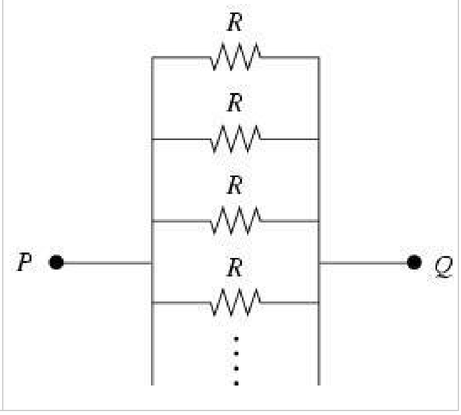
**RC.L3.2-04:**

**Description:** [?]

**Learning Objectives:** [?]

**Problem Statement:** Draw a graph relating the total resistance, $R_{total}$, between points *P* and *Q* versus the total number N of resistors placed between them.



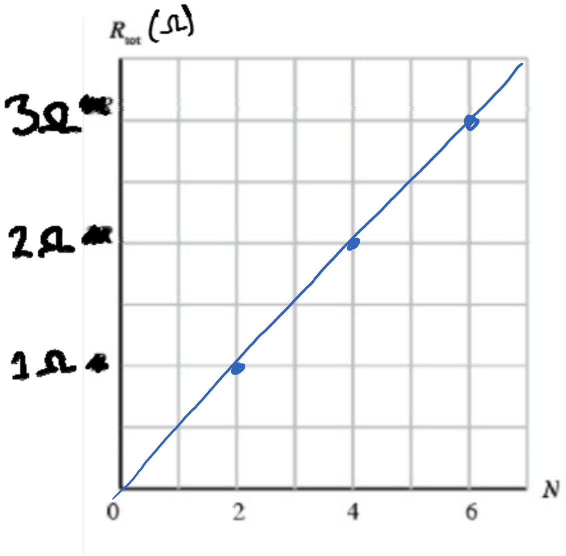


**RC.L3.2-05:**

**Description:** [?]

**Learning Objectives:** [?]

**Problem Statement:** Draw a graph relating the total resistance as a function of N, where N is the number of equivalent 0.5 Ω resistors added in series.



$N=2 \Rightarrow 0.5\Omega + 0.5\Omega = 1\Omega$

$N=3 \Rightarrow 1.5\Omega$

$\vdots$

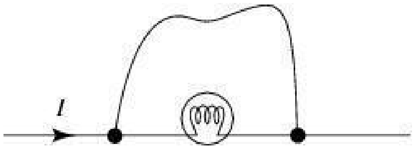
## Act II: Power and Equivalent Circuits

**RC.L3.2-06:**

**Description:** [?]

**Learning Objectives:** [?]

**Problem Statement:** Charge flows through a light bulb. Suppose a wire is connected across the bulb as shown. When the wire is connected,



(1) all the charge continues to flow through the bulb.
(2) half the charge flows through the wire, the other half continues through the bulb.
[✓ (3)] all the charge flows through the wire.
[✓ (4)] none of the above.

the resistance of wire is so small that it can

↑

the resistance of wire is so small that it can be approximated as 0 ⇒ all current chooses the wire

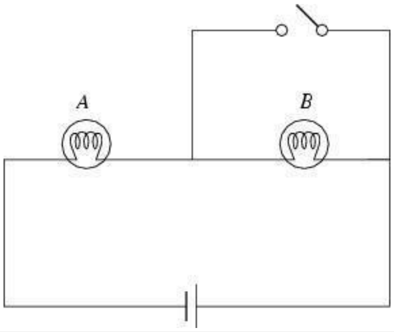
technically $R_{wire} \neq 0$, so some <u>very</u> small amount of current flows through lightbulb

**RC.L3.2-07:**

**Description:** [?]

**Learning Objectives:** [?]

**Problem Statement:** The circuit below consists of two identical light bulbs burning with equal brightness and a single 12 V battery. When the switch is closed, the brightness of bulb *A*



[✓ (1)] increases.
(2) remains unchanged.
(3) decreases.

$$P = I^2 R = \underbrace{\frac{\Delta V^2}{R}}_{} = I \Delta V$$

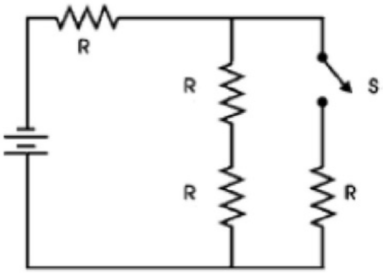
$$\Delta V_A \uparrow, \; R = \text{const}$$

**RC.L3.2-08:**

**Description:** [?]

**Learning Objectives:** [?]

**Problem Statement:** Consider the following circuit.



**(a)** With the switch S closed, what is the equivalent resistance of the circuit?

(1) 4 R
(2) 2 R / 3
(3) 2 R
[✓ (4) 5 R / 3]
(5) 5 R / 6
(6) R
(7) R / 3
(8) R / 2

$$\frac{1}{\frac{1}{2R}+\frac{1}{R}} + R = \frac{1}{\frac{3}{2R}} + R = \frac{2}{3}R + R$$

**(b)** Will more power be dissipated by the circuit when the switch S is open or closed?

(1) Open
[✓ (2) Closed]
(3) No difference

$$R_{eq,open} = 3R > \frac{5}{3}R$$

$$P = \frac{\Delta V^2 \leftarrow \text{same}}{R \leftarrow \text{less when closed}}$$

**RC.L3.2-09:**

**Description:** [?]

**Learning Objectives:** [?]

**Problem Statement:** Use the voltage vs. current curve for the resistors to determine what power is delivered to the circuit by a 12 V battery when the switch is closed?

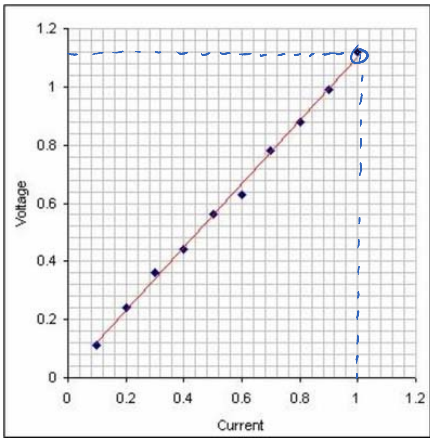
$$\Delta V = I R$$

$$\Rightarrow R = \frac{\Delta V}{I} = \text{slope of} \rightarrow$$

$$R = \frac{1.12\,V}{1\,A} = 1.12\,\Omega$$

$$P = \frac{\Delta V^2}{R} = \frac{144\,V^2}{1.12\,\Omega}$$

$$\boxed{P = 129\,W}$$



## Act III: Solving Circuits

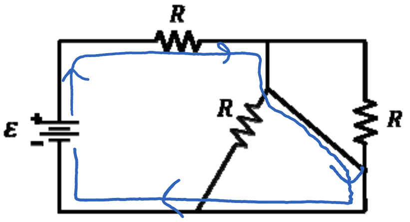
**RC.L3.2-10:**

**Description:** [?]

**Learning Objectives:** [?]

**Problem Statement:** Find the equivalent resistance of the following circuit.

all current will follow blue path



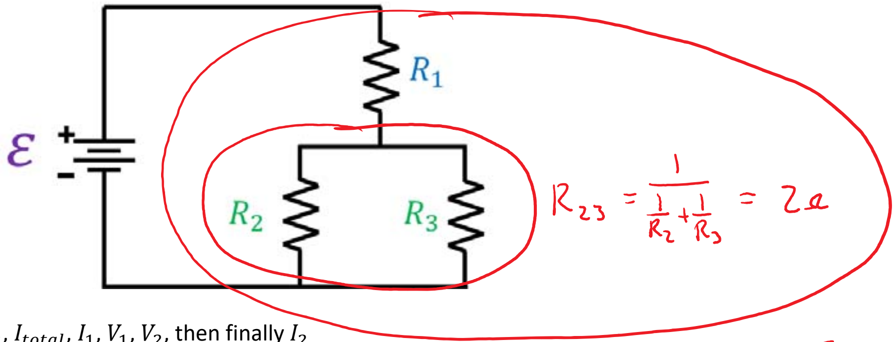
$\Rightarrow R_{eq} = R$

**RC.L3.2-11:**

**Description:** [?]

**Learning Objectives:** [?]

**Problem Statement:** Find the currents $I_1, I_2,$ and $I_3$ in the following circuit. Let $R_1 = 3\ \Omega$, $R_2 = R_3 = 4\ \Omega$, and $\mathcal{E} = 5\ V$



$R_{23} = \frac{1}{\frac{1}{R_2}+\frac{1}{R_3}} = 2\Omega$

Hint: try finding quantities in this order: $R_{eq}, I_{total}, I_1, V_1, V_2$, then finally $I_2$

$R_{123} = R_1 + R_{23} = 5\Omega$

$R_{tot} = R_{123} = 5\Omega \Rightarrow I_{tot} = \frac{\Delta V_{tot}}{R_{tot}} = 1\ A$

$I_{tot} = \boxed{I_1 = 1\ A}$

$\Delta V_1 = I_1 R_1 = (1\ A)(3\Omega) = 3V$

$\Rightarrow \Delta V_2 = \Delta V_3 = \Delta V_{tot} - \Delta V_1 = 2V$

$I_2 = \frac{\Delta V_2}{} = \frac{2V}{} = \boxed{\frac{1}{} A = I_2}$

$$I_2 = \frac{\Delta V_2}{R_2} = \frac{2V}{4\Omega} = \boxed{\frac{1}{2}A = I_2}$$

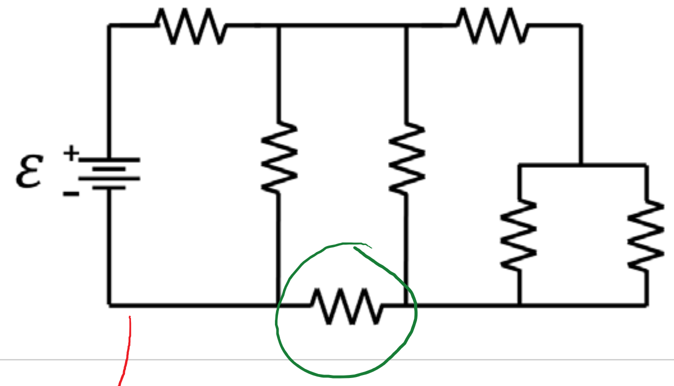
$$I_3 = I_1 - I_2 = \boxed{\frac{1}{2}A = I_3} \rightarrow \text{or } I_3 = \frac{\Delta V_3}{R_3} = \frac{2V}{4\Omega} = \frac{1}{2}A$$

**RC.L3.2-12:**

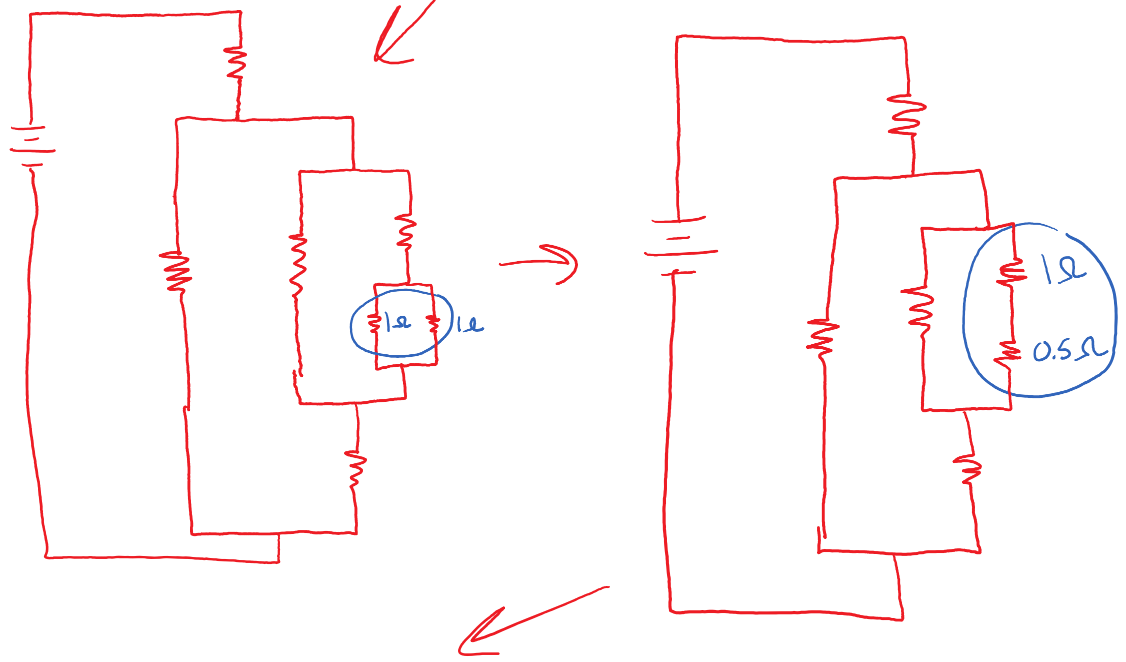
**Description:** [?]

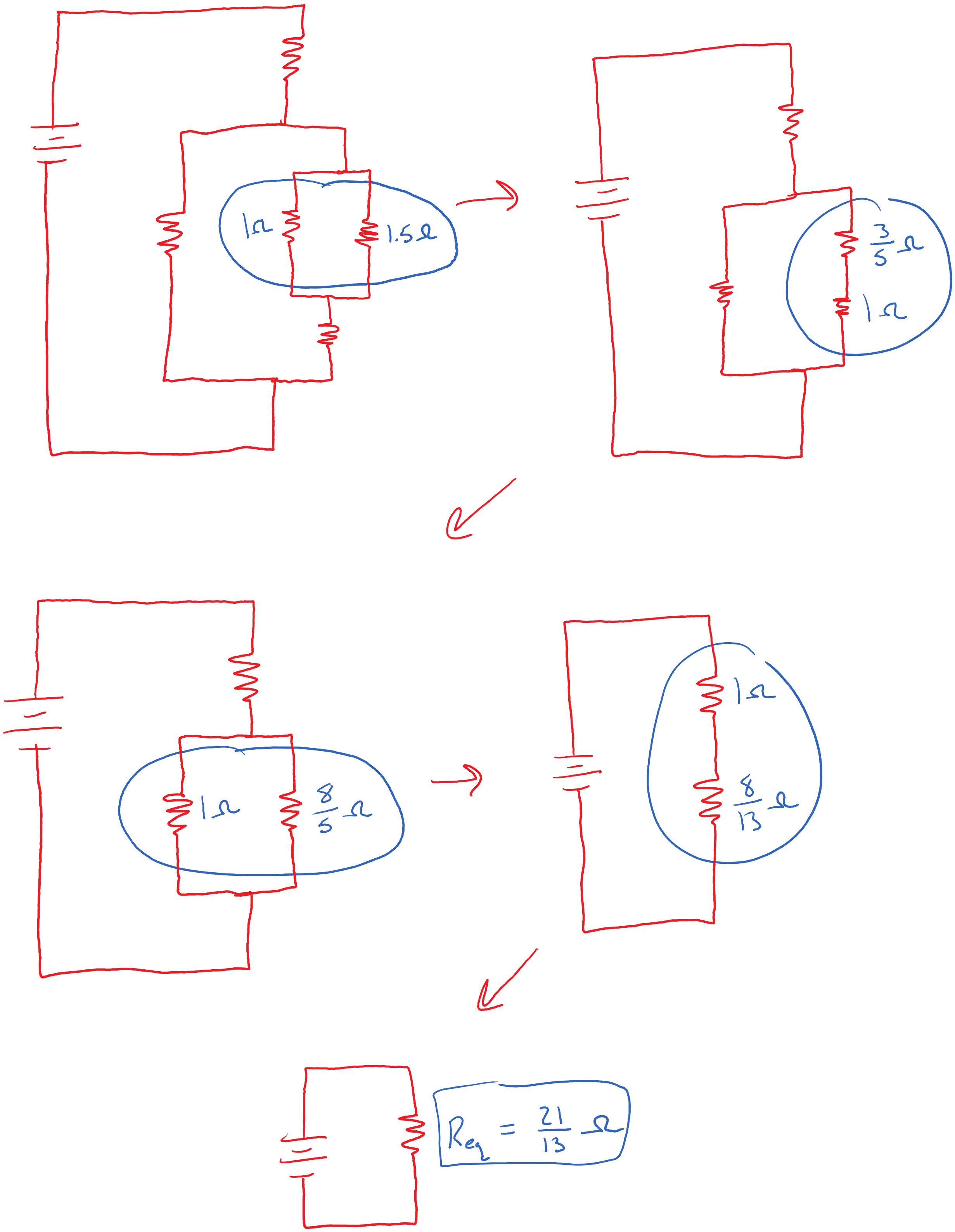
**Learning Objectives:** [?]

**Problem Statement:** Assume each resistor has a resistance of 1 Ω.



**(a)** Find the equivalent resistance of the circuit.





$$\boxed{R_{eq} = \frac{21}{13}\,\Omega}$$

**(b)** How much power would the bottom resistor dissipate if the battery's EMF was 9V?

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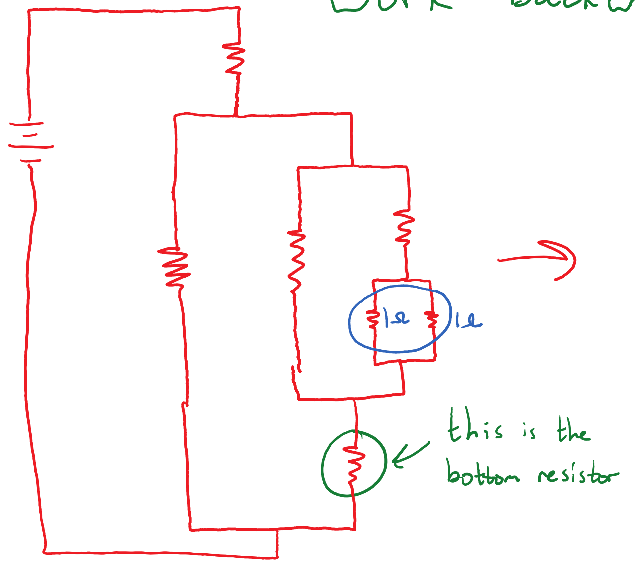
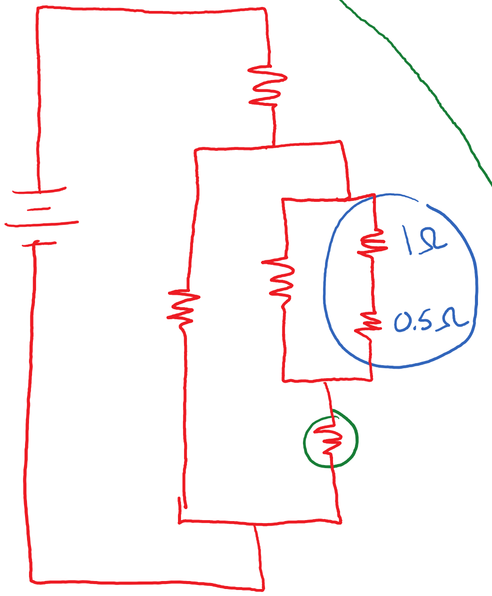
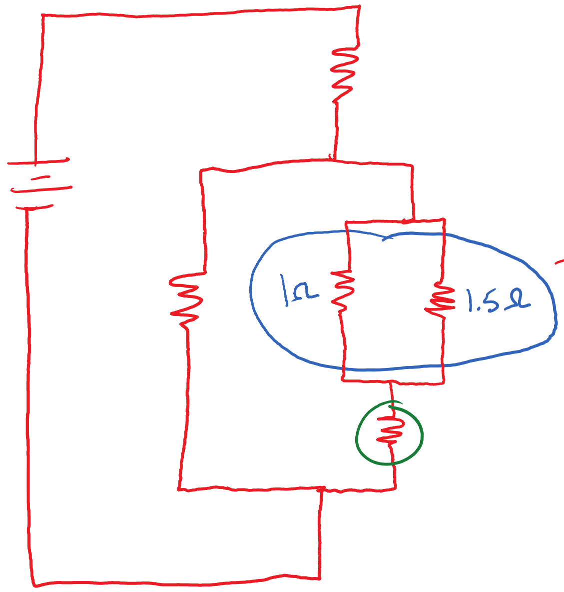
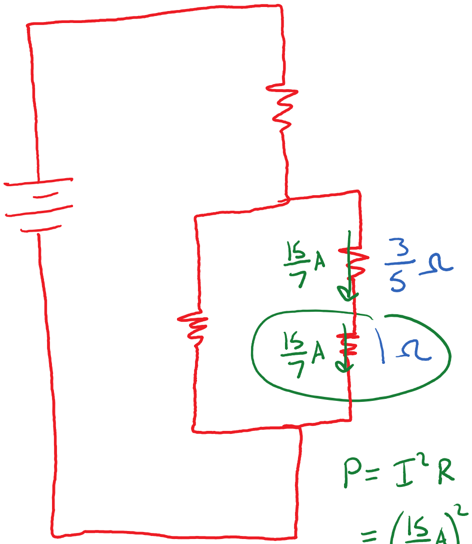
Work backwards from above





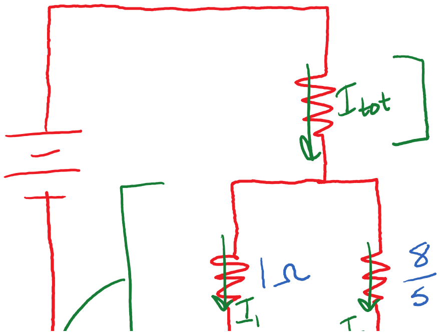




$$P = I^2 R$$

$$= \left(\tfrac{15}{7} A\right)^2 (1\Omega)$$

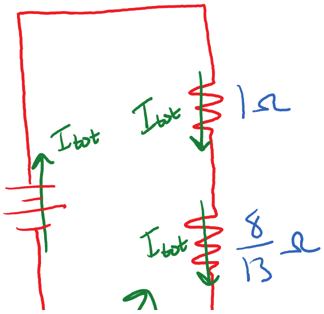
$$\boxed{= \tfrac{225}{49} W \quad = 4.59 W}$$

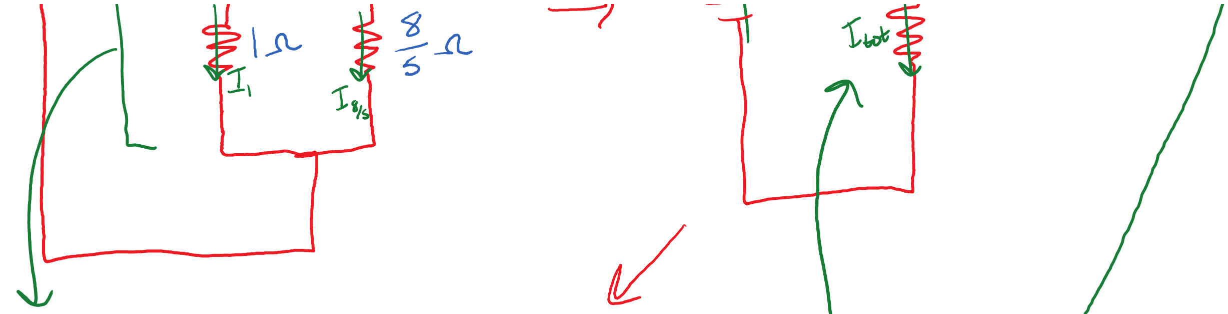


$$\Delta V = IR$$

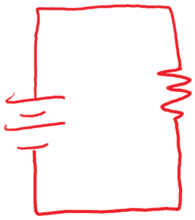
$$= \left(\tfrac{117}{21} A\right)(1\Omega)$$

$$= \tfrac{117}{21} V$$





$$\Delta V = \Delta V_{tot} - \Delta V_{above}$$
$$= 9V - \frac{117}{21}V$$
$$= \frac{72}{21}V$$
$$\Rightarrow I_{8/5} = \frac{\Delta V}{R} = \frac{72/21}{8/5}$$
$$= \frac{360}{168}A = \frac{15}{7}A$$



$$\boxed{R_{eq} = \frac{21}{13}\,\Omega}$$

$$I_{tot} = \frac{9V}{21/13\,\Omega} = \boxed{\frac{117}{21}A = 5.57A}$$

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## Conceptual questions for discussion

1. Coming soon to a lecture template near you.

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## Hints

**RC.L1.2-01**: No hints.

**RC.L1.2-02**: No hints.

**RC.L1.2-03**: No hints.

**RC.L1.2-04**: No hints.

**RC.L1.2-05**: No hints.

**RC.L1.2-06**: No hints.

**RC.L1.2-07**: No hints.

**RC.L1.2-08**: No hints.

**RC.L1.2-09**: No hints.

**RC.L1.2-10**: No hints.

**RC.L1.2-11**: No hints.

**RC.L1.2-12**: No hints.

**RC.L1.2-13**: No hints.

**RC.L1.2-14**:  No hints.

**RC.L1.2-15**:  No hints.

**RC.L1.2-16**:  No hints.

**RC.L1.2-17**:  No hints.