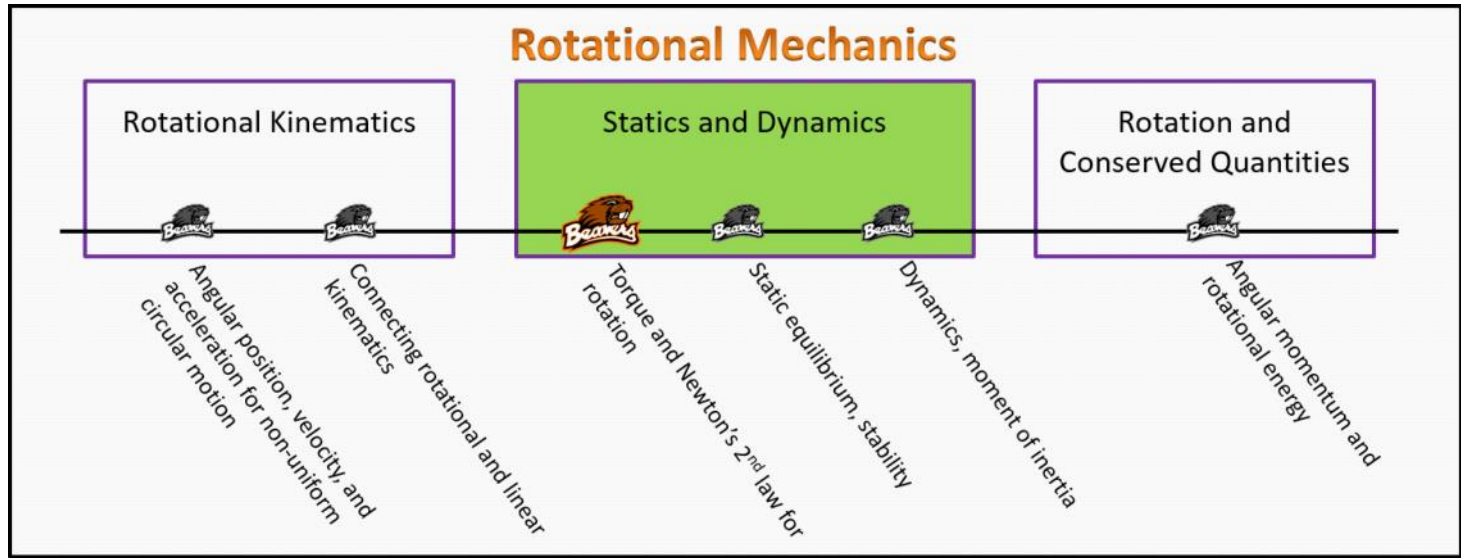


Statics and Dynamics Familiarize Stage (SD.L1.1)

Lecture 1 Torque and Newton's 2nd Law for Rotation



SD.L1.1-01

Description: Infographic quiz torque - label matching

Learning Objectives: [x,xx,...] Put the learning objective numbers here

Problem Statement: Match each term in the equation with the correct description from the following list. (1) Smallest angle between position vector and force vector, (2) Force vector, (3) Position vector, (4) Torque

$$|\vec{\tau}| = |\vec{r}| |\vec{F}| \sin\theta$$

(a) _____

(b) _____

(c) _____

(d) _____

SD.L1.1-02

Description: Ways to change the torque on a wrench

Learning Objectives: [x,xx,...] Put the learning objective numbers here

Problem Statement: A force is being applied at an angle of 45° with respect to a wrench, halfway between the bolt and the end of the wrench. Which of the following are ways you can increase the torque on the bolt?

- | |
|---|
| (1) Increase the force applied |
| (2) Apply the force closer to the end of the wrench |
| (3) Apply the force parallel to the wrench |
| (4) Apply the force perpendicular to the wrench |
| (5) Use a heavier wrench |

SD.L1.1-03

Description: increasing lever arm to increase torque with a wrench

Learning Objectives: [x,xx,...] Put the learning objective numbers here

Problem Statement: Mechanics sometimes put a length of pipe over the handle of a wrench when trying to remove a very tight bolt.

(a) How does this make it easier for the mechanic to remove the bolt?

- | |
|---|
| (1) It increases the torque the mechanic can apply, without the need to exert a greater force. |
| (2) It increases the force exerted by the mechanic, without the need to apply a greater torque. |
| (3) It allows the mechanic to apply the same torque, while decreasing the force. |
| (4) It allows the mechanic to exert the same force, while decreasing the torque. |

(b) How does this increase the torque?

- | |
|---|
| (1) It increases the lever arm. |
| (2) It increases the force you can apply. |
| (3) It changes the angle you can apply the force relative to the lever arm. |

SD.L1.1-04

Description: Infographic quiz Newton's second law - label matching

Learning Objectives: [x,xx,...] Put the learning objective numbers here

Problem Statement: Match each term in the equation with the correct description from the following list. (1)

The diagram shows the equation $\sum \vec{\tau}_{ext,o} = I_o \vec{\alpha}_o$. Label (a) points to the summation symbol \sum . Label (b) points to the vector $\vec{\tau}_{ext,o}$. Label (c) points to the moment of inertia I_o . Label (e) points to the angular acceleration vector $\vec{\alpha}_o$.

SD.L1.1-05

Description: Physical representations in a torque analysis

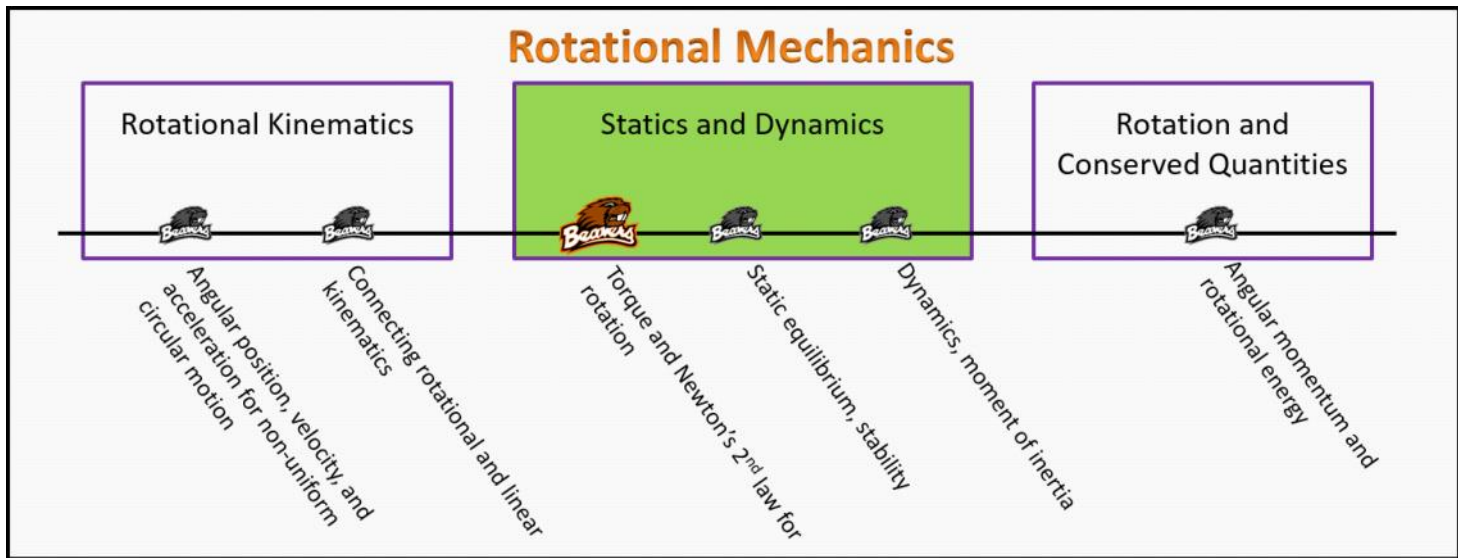
Learning Objectives: [x,xx,...] Put the learning objective numbers here

Problem Statement: Which of the following physical representations are relevant when analyzing rigid bodies?

- | |
|--|
| (1) Free-body diagram (FBD) |
| (2) Extended free-body diagram (eFBD) |
| (3) Vector operation for the dot product between a force and lever arm |
| (4) Vector operation for the cross-product of a force and a lever arm |
| (5) Vector operation for subtraction of torque vectors |

Statics and Dynamics Foundation Stage (SD.L1.2)

Lecture 1 Torque and Newton's 2nd Law for Rotation



Textbook Chapters (* Calculus version)

- **BoxSand** :: KC videos ([statics & dynamics](#))
- **Knight** (College Physics : A strategic approach 3rd) :: 7.3
- ***Knight** (Physics for Scientists and Engineers 4th) :: 12.5
- **Giancoli** (Physics Principles with Applications 7th) :: 8-4

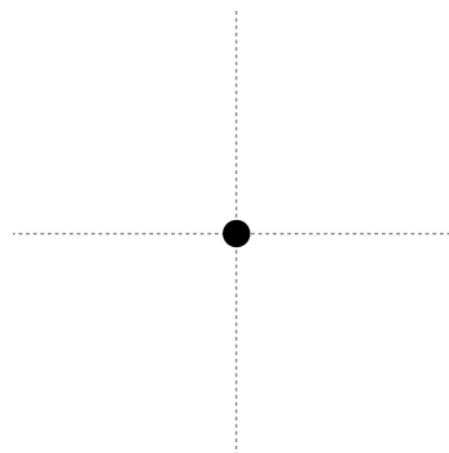
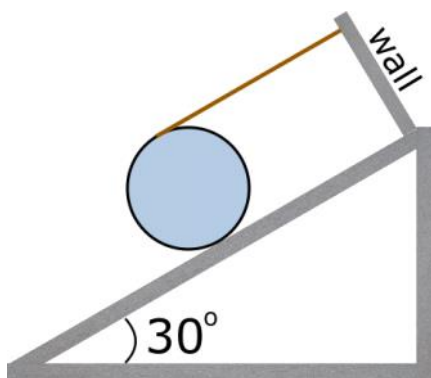
Warm up

SD.L1.2-01:

Description: Draw a FBD.

Learning Objectives: [?] - Can you identify the objectives from the previous lecture, and this lecture, that this question is relevant to?

Problem Statement: A disk with uniform mass distribution sits in static equilibrium on an incline with the aid of a rope as shown in the figure below. Draw a FBD for the disk.



Selected Learning Objectives

- Coming soon to a lecture template near you.

Key Terms

- Reference axis
- Cross product
- Torque
- Extended Free Body Diagram (e-FBD)

Key Equations

$$\vec{\tau}_O^A = \vec{r}_A \times \vec{F}^A$$

In words: The torque from the applied force about reference axis "o" is equal to the cross product between the position vector that points from "o" to the applied force with the applied force.

$$|\vec{\tau}_O^A| = |\vec{r}_A| |\vec{F}^A| \sin(\theta)$$

In words: The magnitude of torque from the applied force about reference axis "o" is equal to the magnitude of the position vector that points from "o" to the applied force times the magnitude of the applied force times the sin of the smallest angle between the two vectors when placed tail to tail.

$$\tau_O^A = \pm |\vec{\tau}_O^A|$$

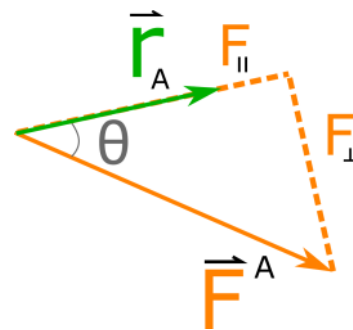
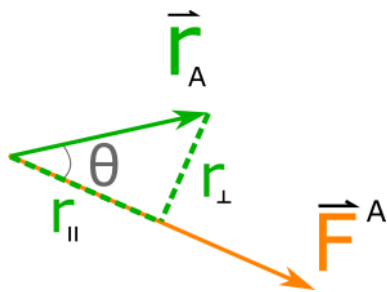
In words: The component of torque from the applied force about reference axis "o" is equal to plus or minus the magnitude of the torque. The plus or minus is determined by the desired rotation about "o": ccw(+), cw(-).

$$\tau_O^A = \pm r_{\perp} |\vec{F}^A|$$

In words: The component of torque from the applied force about reference axis "o" is equal to plus or minus the perpendicular component of the position vector with respect to the applied force times the magnitude of the applied force.

$$\tau_O^A = \pm |\vec{r}_A| F_{\perp}$$

In words: The component of torque from the applied force about reference axis "o" is equal to plus or minus the magnitude of the position vector that points from "o" to the applied force times the perpendicular component of the applied force with respect to the position vector.



Key Concepts

- A reference axis is analogous to an origin of a coordinate system. You can place the reference axis anywhere you like. A reference axis differs from an origin because it is an axis, while the origin is just a point. Imagine a thin cylindrical rod as the reference axis, thus this rod (reference axis) can be oriented in different directions. Remember that the reference axis does not need to be at the location where an object is rotating about, you can choose your reference axis location and orientation anywhere you'd like.
- The cross product is a mathematical operation that takes in two vectors, and returns a vector. Conceptually, you can think of the cross product as asking, "how perpendicular two vectors are to each other". The more perpendicular the two vectors are, the larger the magnitude of the cross product, reaching its maximum magnitude when the two vectors are perpendicular, and is zero when the two vectors are parallel.
- Torque is a vector quantity found by taking the cross product between the position vector from a reference axis and the force applied to an object. The magnitude of torque depends on the magnitude of the applied force, the magnitude of the position vector, and the angle between the two.
- The vector nature of torque is not covered in this template. We will only be working with a component of torque about a reference axis. Recall that component of vectors can be positive or negative. To determine if the component of torque is positive or negative use the following convention: if the force is trying to rotate the object ccw about the reference axis, then the torque from this force is positive ; if the force is trying to rotate the object cw about the reference axis, then the torque from this force is negative.
- An extended free body diagram is a tool to help organize information when a torque analysis is required. The e-FBD should include an outline of the system, the reference axis, all of the forces acting on the system with their tails at the locations they act, and the position vectors from the reference axis to all of the forces.

Questions

Act I: Torque

SD.L1.2-02

Description: Determine dimensions of torque. Determine which physical quantity has same dimensions as torque. Compare quantities with same dimensions. (2 minute + 2 minutes + 1 minute)

Problem Statement: Recall that the magnitude of torque is: $|\vec{\tau}_o| = |\vec{r}_o| |\vec{F}| \sin(\theta)$.

(a) Which of the following are the correct dimensions for torque?

(1) N m

(2) $\frac{kg \ m}{s}$

(3) $\frac{kg \ m^2}{s^2}$

(b) Which of the following physical quantities have the same dimensions as torque?

(1) Momentum

(2) Energy

(3) Work

(4) Impulse

(5) Force

(6) Displacement

(7) Velocity

(8) Acceleration

(4) $\frac{[M]^2 [L]}{[T]^2}$

(5) $\frac{[M]^2 [L]^2}{[T]}$

(6) $\frac{[M] [L]}{[T]^2}$

(7) $\frac{[M] [L]}{[T]}$

(8) $\frac{[M] [L]^2}{[T]^2}$

(9) Heat

(c) What is the difference between torque and your answer(s) to part b?

(1) Torque is related to rotational motion and part b answer(s) are only related to translational motion.

(2) Torque is a scalar and part b answer(s) are vectors.

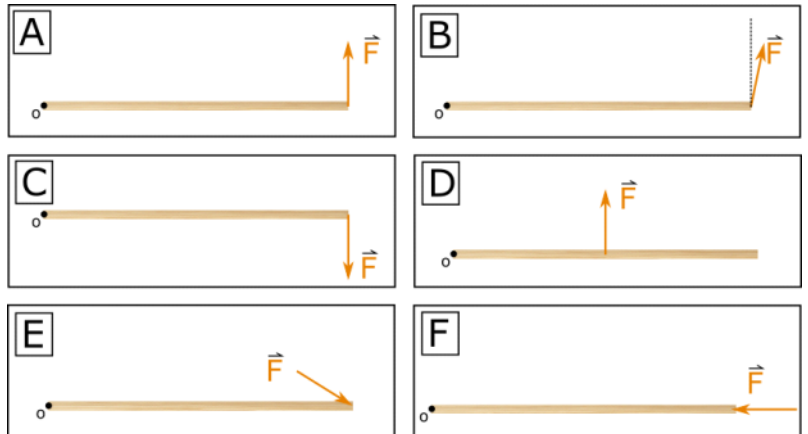
(3) Torque is a vector and part b answer(s) are scalars.

SD.L1.2-03:

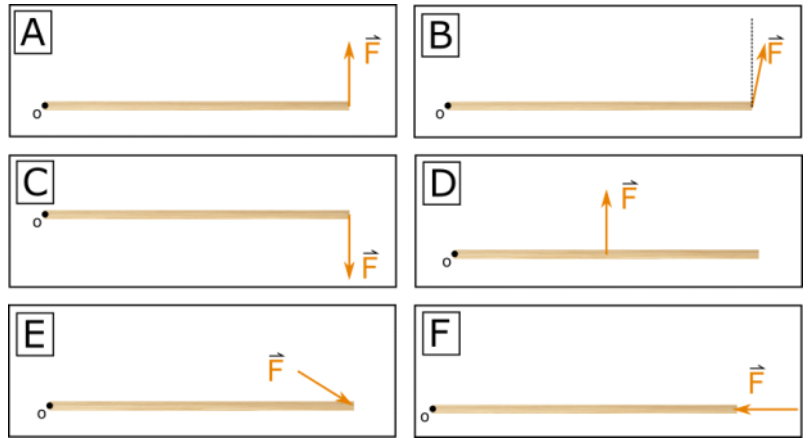
Description: Ranking torque. (3 minutes + 3 minutes)

Problem Statement: Consider a block of wood being pushed/pulled on by a force as shown in the images below. All forces have the same magnitude and the images are drawn to scale.

(a) In which situation is the torque produced by the force about axis *o* the most positive?



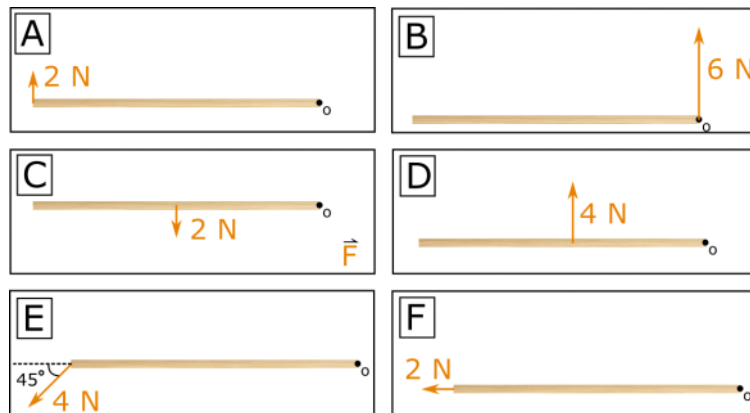
(b) Rank the torques in each situation from most negative to most positive.



SD.L1.2-04:

Description: Ranking torque. (5 minutes)

Problem Statement: Consider a block of wood being pushed/pulled on by a force as shown in the images below. The images are drawn to scale. Rank the torques in each situation from most negative to most positive.



SD.L1.2-05:

Description: Find torque about given reference axis. (1 minute + 4 minutes)

Problem Statement: Benny and Bernice the beavers are building a space dam in outer space far away from other massive objects. While moving a 24000 kg log into the correct place, Benny accidentally nudges the constant 1990 N thruster off the log's axis as shown in the image below.

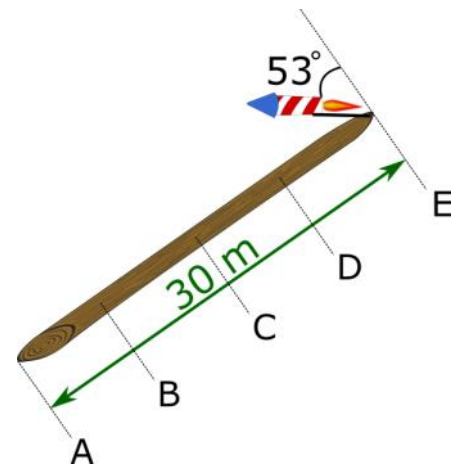
Part I: Spinning out of control.

(a) At what location is the center of mass of the log assuming uniform mass distribution?

- (1) A
- (2) B
- (3) C
- (4) D
- (5) E

(b) What torque does the constant 1990 N rocket thruster provide to the log about the center of mass of the log?

- (1) 18000 N·m
- (2) 23800 N·m
- (3) 29900 N·m
- (4) 36000 N·m
- (5) 47700 N·m
- (6) 59700 N·m



Part II: Bernice to the rescue.

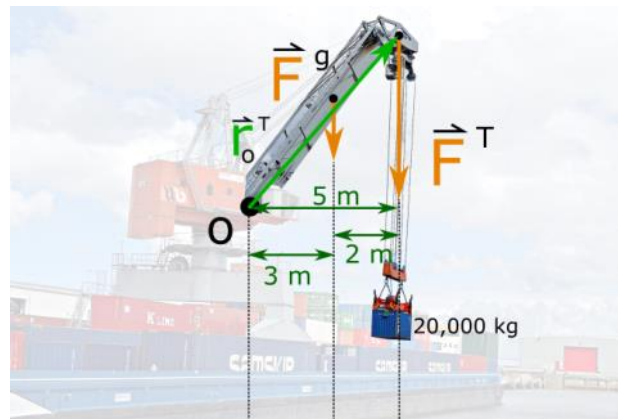
(c) , (d) , (e) , (f) : See problem SD.2.L1-8

SD.L1.2-06:

Description: Find torque about a given reference axis. (4 minutes)

Problem Statement: A 3,000-kg crane is supporting a 20,000-kg shipping container as seen in the image below. Find the torque due to the 20,000 kg container on the crane about the reference axis o . Recall $g = 9.8 \text{ m/s}^2$.

- (1) -100,000 N·m
- (2) 392,000 N·m
- (3) -588,000 N·m
- (4) -980,000 N·m
- (5) 1,068,000 N·m



Act II: Newton's 2nd law for rigid rotators

SD.L1.2-07:

Description: Label forces and their locations of action on an object. (3 minutes + 1 minute + 3 minutes + 2 minutes)

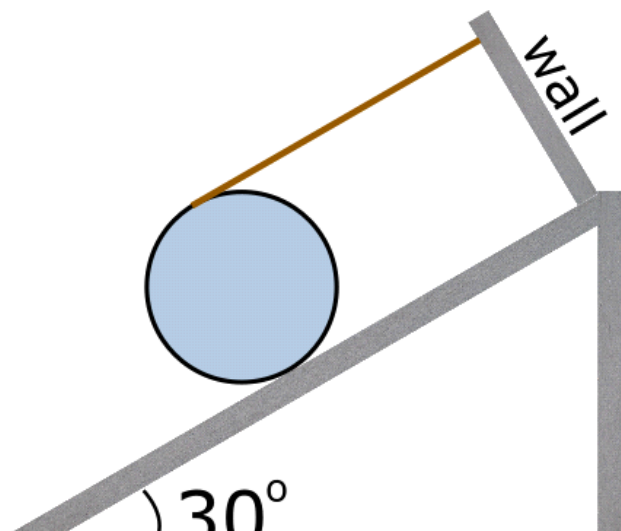
Problem Statement: A disk with uniform mass distribution sits in static equilibrium on an incline with the aid of a rope as shown in the figure below.

(a) There are 4 forces acting on the disk: tension, normal, static friction, and gravity. On top of the image, draw tension, normal, and static friction by placing the tail of each force vector at the location which they act on the disk.

(b) With your neighbors, take 1 minute to discuss where you think the force of gravity acts on this disk. Then draw the force of gravity vector on the image by placing the tail of the vector at the location that it is acting on.

(c) If we wished to solve for the tension, what location would you choose to place the reference axis?

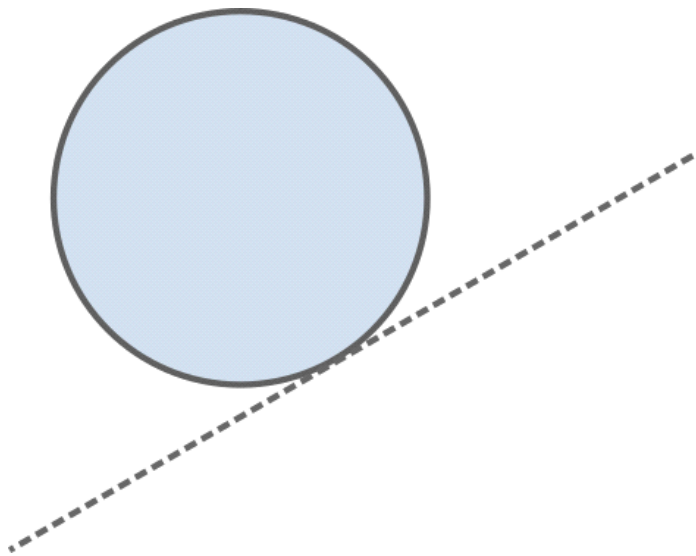
- (1) At location of contact between disk and incline.
- (2) At location of contact of rope and disk.



- (3) At center of mass of the disk.
- (4) At contact point of rope and wall.



(d) Create an e-FBD to organize all of the information that is needed to solve for the tension in the rope.



SD.L1.2-08:

Description: Calculate net torque. (3 minutes + 1 minute + 3 minutes + 2 minutes)

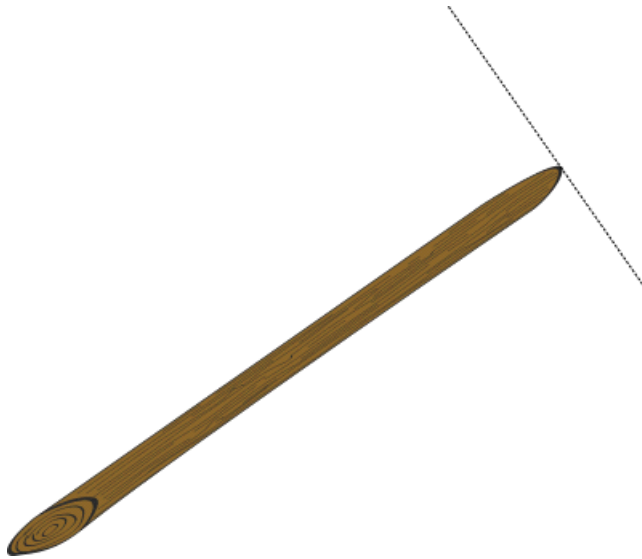
Problem Statement: Benny and Bernice the beavers are building a space dam in outer space far away from other massive objects. While moving a 24000 kg log into the correct place, Benny accidentally nudges the 1990 N thruster off the log's axis as shown in the image below.

Part I: Spinning out of control.

(a) and (b): See problem SD.2.L1-5

Part II: Bernice to the rescue.

(c) With the 24000 kg log now spinning out of control, Bernice quickly springs to action by placing a 2nd rocket of unknown force on the log such that the rockets uses the minimum amount of constant thrust to cancel out the torque. Draw an e-FBD for the log with both Benny's and Bernice's rockets acting on the log.



(d) If we wish to determine the minimum force the rocket needs to apply, where should we place the reference axis?

(e) Calculate the minimum amount of force of the rocket the Bernice attached to cancel out the torque on the 30 m long log.

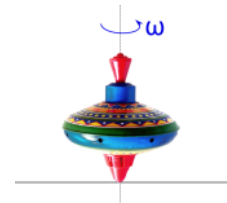
(f) What happens to the log after Bernice places the rocket on the log?

- (1) The log stops spinning
- (2) The log continues spinning but slows down and eventually reverses direction.
- (3) The log continues to spin at the same rate as it was the moment the 2nd rocket was added.

Description: Conceptual question regarding Newton's laws of motion in rotational form. (4 minutes)

Problem Statement: A few students are discussing a top that is spinning vertically in one place, but the rotational speed is slowing down. Which student(s) do you agree with?

- (1) If you look at a FBD for the top, there is no net external force, thus the velocity of the center of mass is constant.
- (2) The angular velocity of the top is changing, so there must be a non-zero net external force acting on the top.
- (3) I agree that there is no net external force, but since the angular velocity is changing there must be a net external torque on the top.
- (4) The top spins for a long time because the angular impulse is small, thus the top loses angular momentum slowly.
- (5) The top spins for a long time because the friction between the top and table is small, thus the rotational kinetic energy of the top slowly converts to thermal energy of the top+table system.



SD.L1.2-10:

Description: Conceptual question regarding equilibrium. (4 minutes)

Problem Statement: An axe is thrown into the air near the surface of the earth. A red LED is attached to the center of mass of the axe and a yellow LED is attached to one end of the axe so that a picture taken at night can be used to create a trajectory of each part of the axe as seen in the image below. Which statement(s) is(are) true? Ignore air resistance and consider only the time that the axe has left your hand and before it hits the ground.

- (1) The axe has no net external force acting on it.
- (2) The axe has no net external torque acting on it.
- (3) The energy of the axe is constant.
- (4) The axes' momentum is constant.



Conceptual questions for discussion

1. Is a torque a force?
 2. A friend of yours claims that it is easiest to open a door by pushing at a location furthest away from the hinges. Do you agree? Would you add anything to this statement if you don't necessarily agree?
 3. Do you agree with the following statement: A small force can never apply a torque larger in magnitude than a larger force on the same object.
 4. How might you experimentally find the center of mass of an object near the surface of the earth?
 5. Is a torque a force?
 6. If you wish to stop a door from closing using a door stop that wedges between the floor and the bottom of the door, where should you place the door stop and why?
 7. Do you agree with the following statement: If an object is not rotating, then it has no torques acting on it.
 8. Do you agree with the following statement: If an object is rotating, then there must be a net external torque on it.
-

Hints

SD.L1.2-01: No hints.

SD.L1.2-02: No hints.

SD.L1.2-03: Determine sign of torques first, then focus on magnitudes.

SD.L1.2-04: Determine sign of torques first, then focus on magnitudes.

SD.L1.2-05: Draw a vector operation to determine the angle that goes into the sin function in the definition of torque.

SD.L1.2-06: Recall that a cross product asks, "how perpendicular two vectors are".

SD.L1.2-07: No hints.

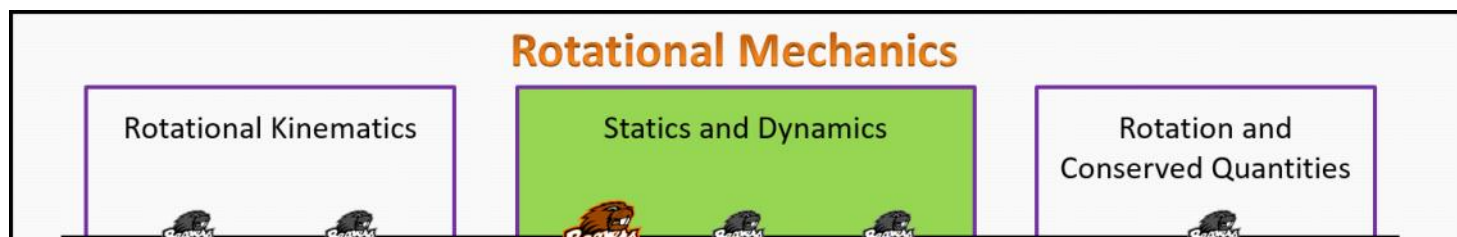
SD.L1.2-08: Talk to your neighbors about what angle relative to the log would give the largest torque due to a force.

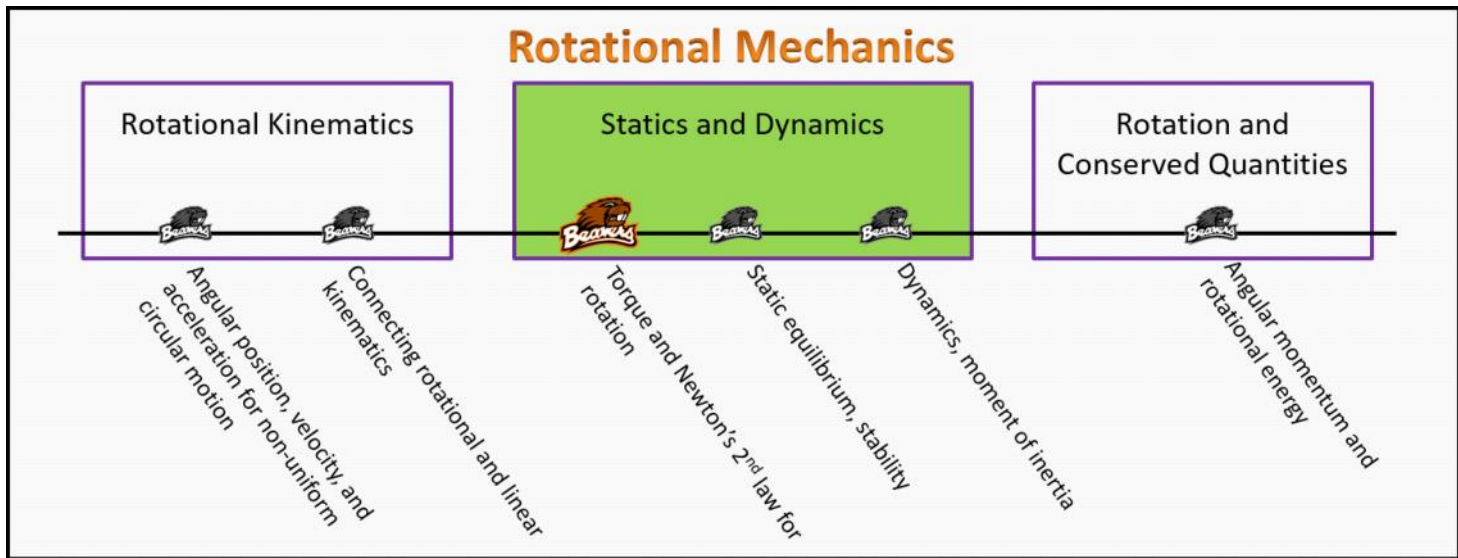
SD.L1.2-09: No hints.

SD.L1.2-10: No hints.

Statics and Dynamics Practice Stage (SD.L1.3)

Lecture 1 Torque and Newton's 2nd Law for Rotation





SD.L1.3-01

Description: Proportional reasoning for a force applied to a pivot point

Learning Objectives: [x]

Problem Statement: A force is applied 1 m away from the pivot point on a lever arm and it produces torque. How much force would have to be applied to produce the same amount of torque if it were applied 4 m from the pivot point? Assume that both forces are applied perpendicularly to the lever arm.

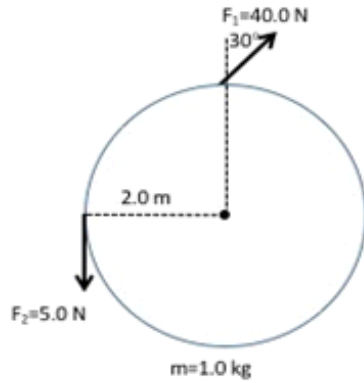
- | |
|--|
| (1) Four times the initial force |
| (2) Sixteen times the initial force |
| (3) One fourth of the initial force |
| (4) One sixteenth of the initial force |
| (5) same as the initial force |

SD.L1.3-02

Description: Net torque of a rotating disk

Learning Objectives: [x]

Problem Statement: Consider this figure of a disk rotating about an axis through the middle. The mass if the disk is 1.0kg.



(a) What is the net torque acting on the disk?

- | |
|-------------|
| (1) 50 N·m |
| (2) -50 N·m |
| (3) +40 N·m |
| (4) -40 N·m |
| (5) +30 N·m |
| (6) -30 N·m |

(b) What is the angular acceleration of the disk?

- | |
|-----------------------------|
| (1) 6 rad/s ² |
| (2) -6 rad/s ² |
| (3) 30 rad/s ² |
| (4) -30 rad/s ² |
| (5) 150 rad/s ² |
| (6) -150 rad/s ² |

SD.L1.3-03

Description: Lever arm and fulcrum mechanical advantage

Learning Objectives: [x]

Problem Statement: A small branch is wedged under a 200 kg rock and rests on a smaller object. The small object that creates a pivot point is called the fulcrum. The smaller object is 2.0 m from the large rock and the branch is 12.0 m long.



(a) If the mass of the branch is negligible, what force must be exerted on the free end to just barely lift the rock?

- | |
|------------|
| (1) 16.7 N |
| (2) 100 N |
| (3) 158 N |
| (4) 224 N |
| (5) 311 N |
| (6) 392 N |

(b) What is the mechanical advantage of this fulcrum and lever system? Mechanical advantage is defined as the force required to do a task with a simple machine (fulcrum in this case) divided by the force to do the task without it.

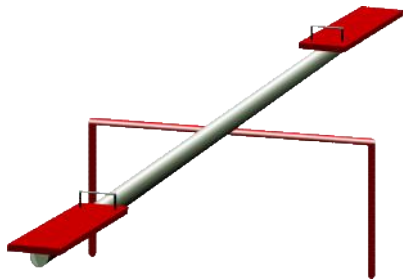
- | |
|-------|
| (1) 1 |
| (2) 2 |
| (3) 3 |
| (4) 4 |
| (5) 5 |
| (6) 6 |

SD.L1.3-04

Description: How two children balance on a seesaw

Learning Objectives: [x,xx,...] Put the learning objective numbers here

Problem Statement: Two children of different weights are riding a seesaw.

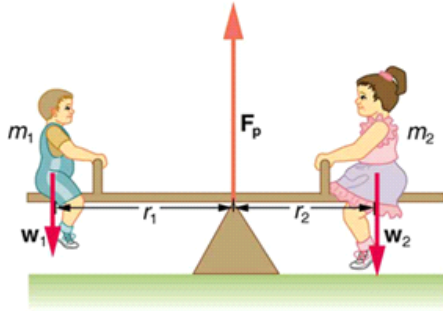


(a) How do they position themselves with respect to the pivot point (the fulcrum) so that they are balanced?

- | |
|---|
| (1) The heavier child sits closer to the fulcrum. |
|---|

- | |
|---|
| (2) The heavier child sits farther from the fulcrum. |
| (3) Both children sit at equal distance from the fulcrum. |
| (4) Since both have different weights, they will never be in balance. |

(b) Suppose two children are using a uniform seesaw that is 3.00 m long and has its center of mass over the pivot. The first child has a mass of 30.0 kg and sits 1.40 m from the pivot. Calculate where the second 18.0 kg child would have to sit to balance the seesaw.



- | |
|----------------------------|
| (1) $x_2 = 2.84$ m |
| (2) $x_2 = 2.67$ m |
| (3) $x_2 = 2.33$ m |
| (4) Not enough information |

(c) What is unreasonable about the result in the previous question?

- | |
|--|
| (1) The seesaw is 3.0 m long, and hence, there is only 1.50 m of board on the other side of the pivot. The second child is off the board. |
| (2) The seesaw is 3.0 m long, and hence, there is only 1.50 m of board on the other side of the pivot. The second child is on the wrong side of the pivot. |
| (3) Nothing is unreasonable. |