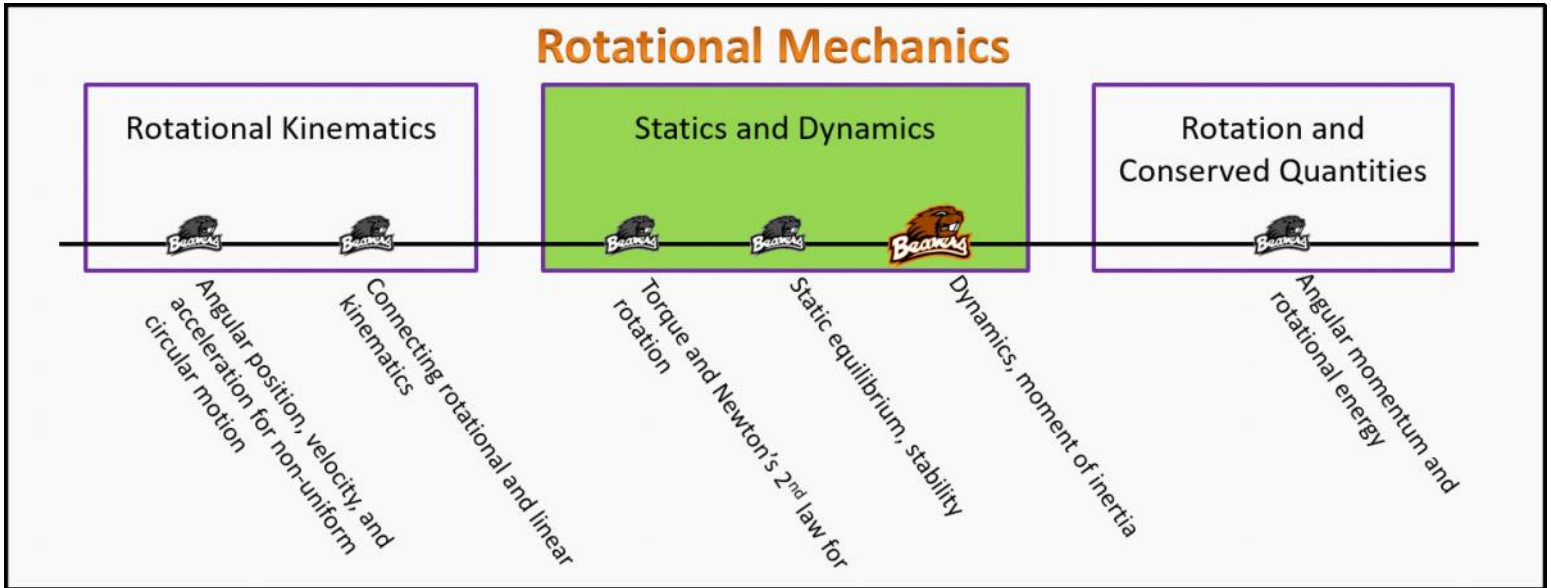


Statics and Dynamics Familiarize Stage (SD.L3.1)

Lecture 3 Dynamics, Momentum of Inertia



SD.L3.1-01

Description: Features of equilibrium

Learning Objectives: [x,xx,...] Put the learning objective numbers here

Problem Statement: Which of the following statements are necessarily true about equilibrium?

- (1) An object in equilibrium does not move
- (2) An object in equilibrium can be moving
- (3) An object in equilibrium has zero acceleration
- (4) An object in equilibrium has a velocity that does not change

SD.L3.1-02

Description: Infographic quiz moment of inertia for a point particle - label matching

Learning Objectives: [x,xx,...] Put the learning objective numbers here

Problem Statement: The equation below is for a point particle. Match each term in the equation with the correct description from the following list. (1) Moment of inertia, (2) Mass, (3) Distance to the rotation axis

(a) _____ (b) _____ (c) _____

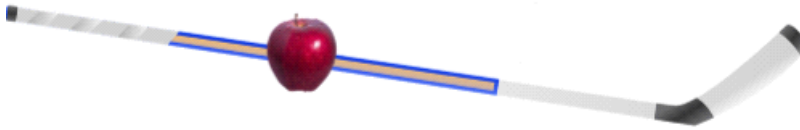
$$I_o = mr_o^2$$

SD.L3.1-03

Description: Proportional reasoning about moment of inertia of point particles

Learning Objectives: [x,xx,...] Put the learning objective numbers here

Problem Statement: Consider an apple on a hockey stick.



(a) If the apple was to move twice as far away from the handle, by what factor would its contribution to the moment of inertia about the handle change?

- (1) 1/4
- (2) 1/2
- (3) 1
- (4) 2
- (5) 4

(b) If the apple was to move twice as far away from the handle and the same torque was applied to the handle, by what factor would the angular acceleration of the stick change? Ignore the mass of the hockey stick.

- (1) 1/4
- (2) 1/2
- (3) 1
- (4) 2
- (5) 4

SD.L3.1-04

Description: Infographic quiz moment of inertia - label matching

Learning Objectives: [x,xx,...] Put the learning objective numbers here

Problem Statement: The equation is for a bunch of point particles - a distribution of mass. Match each term in the equation with the correct description from the following list. (1) Mass, (2) Distance to rotational axis, (3) Summation, (4) Moment of inertia

The diagram shows the equation for the moment of inertia of a system of point particles: $I_o = \sum_i m_i r_{i,o}^2$. Four labels with arrows point to specific parts of the equation: (a) points to I_o , (b) points to the summation symbol \sum , (c) points to the distance term $r_{i,o}^2$, and (d) points to the mass term m_i .

SD.L3.1-05

Description: Comparing moment of inertia of solid disk and hollow ring

Learning Objectives: [x,xx,...] Put the learning objective numbers here

Problem Statement: Why is the moment of inertia about the center axis of a solid disk less than a hollow ring if both have the same mass and radius?

- (1) The disk has less mass closer to the center axis than the disk
- (2) The disk has more mass closer to the center axis than the disk
- (3) There is no difference, both have the same moment of inertia if their mass and radius is identical

SD.L3.1-06

Description: Variable(s) found in both kinematics and 2nd law for rotation

Learning Objectives: [x,xx,...] Put the learning objective numbers here

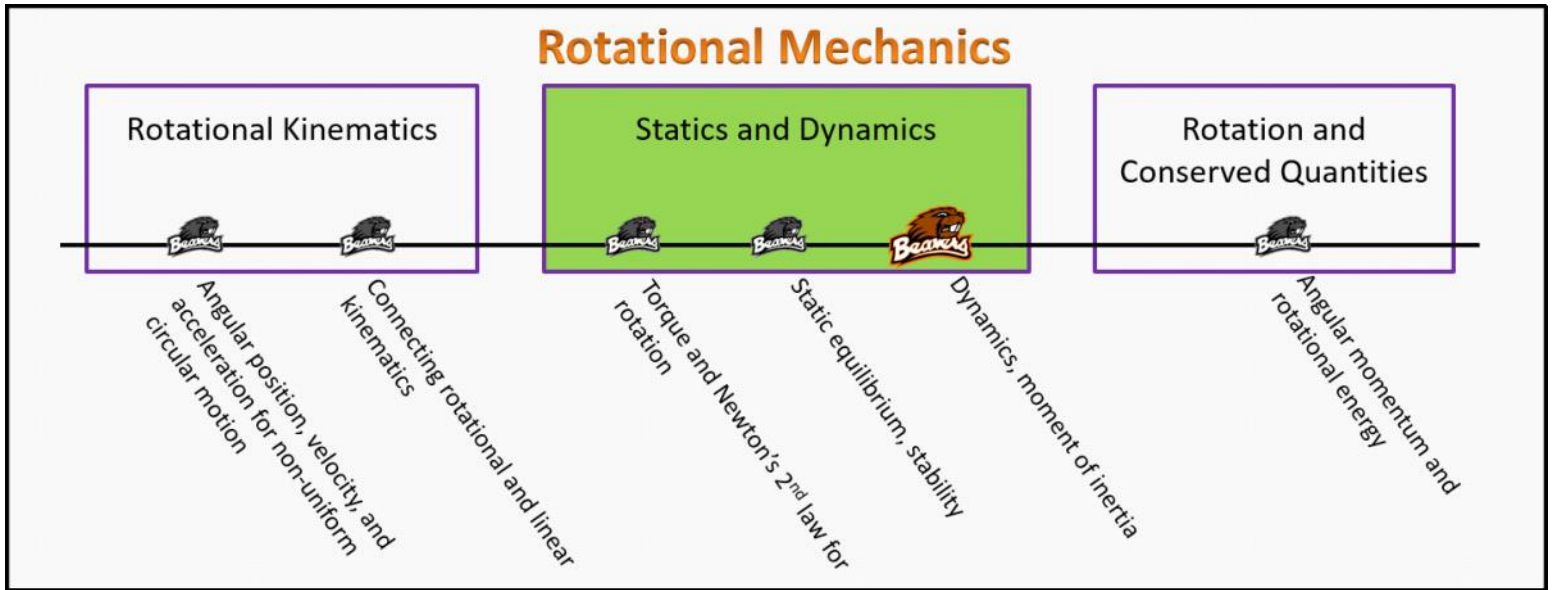
Problem Statement: Which of the following physical quantities show up both in a torque analysis as well as a

(1) Force
(2) Torque
(3) Moment of inertia
(4) Time
(5) Angular displacement
(6) Angular velocity
(7) Angular acceleration

Statics and Dynamics Foundation Stage (SD.L3.2)

Lecture 3 Dynamics, Momentum of Inertia

Rotational Mechanics



Textbook Chapters (* Calculus version)

- **BoxSand** :: KC videos ([statics & dynamics](#))
- **Knight** (College Physics : A strategic approach 3rd) :: 7.5 ; 7.6
- ***Knight** (Physics for Scientists and Engineers 4th) :: 12.6 ; 12.7
- **Giancoli** (Physics Principles with Applications 7th) :: 8-5 ; 8-6

Warm up

SD.L3.2-01:

Description: Apply Newton's 2nd law for rotation given torques and moment of inertia to solve for angular acceleration.

Learning Objectives: [?] - Can you identify the objectives from the previous lecture, and this lecture, that this question is relevant to?

Problem Statement: Consider a system with a moment of inertia of $4 \text{ kg}\cdot\text{m}^2$ with one counter-clockwise torque about reference axis \mathbf{o} of magnitude $12 \text{ N}\cdot\text{m}$, and two clockwise torques about reference axis \mathbf{o} of magnitudes $8 \text{ N}\cdot\text{m}$ and $2 \text{ N}\cdot\text{m}$. What is the angular acceleration of this system?

Selected Learning Objectives

- Coming soon to a lecture template near you.

Key Terms

- Moment of inertia

Key Equations

$$\sum \vec{\tau}_{\text{ext},o} = I_{\text{sys},o} \vec{\alpha}_{\text{sys},o}$$

Labels for the equation above:

- τ : Torque
- I : Moment of inertia
- α : Angular acceleration
- \sum : Net (i.e. add up all of the ___)
- ext,o : External to system, about reference axis o
- sys,o : System, about reference axis o

In words: The net torque external to the system about reference axis o is equal to the moment of inertia of the system about reference axis o multiplied by the angular acceleration of the system about reference axis o.

$$I_{\text{point particle},o} = m r_o^2$$

Labels for the equation above:

- I : Moment of inertia
- m : mass of point particle
- r_o : perpendicular distance of point particle from reference axis o
- point particle,o: Point particle, about reference axis o

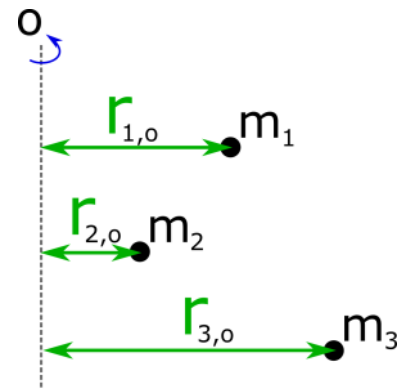
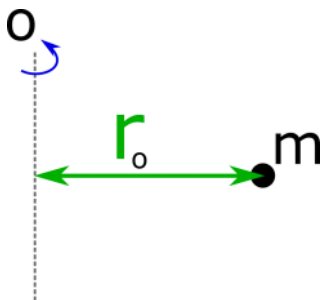
$$I_{\text{sys},o} = \sum m_i r_{i,o}^2$$

Labels for the equation above:

- I : Moment of inertia
- m_i : mass of point particle
- $r_{i,o}$: perpendicular distance of point particle from reference axis o
- sys,o: System, about reference axis o
- \sum : Net (i.e. add up all of the ___)

In words: The moment of inertia for a point particle about axis o is equal to the mass of the point particle times the perpendicular distance of the point particle from o squared.

In words: The moment of inertia for a system of point particles about axis o is equal to the sum of the products between the mass of each point particle and its perpendicular distance from o squared.



Key Concepts

- Recall that the net force acting on a system caused the center of mass of the system to accelerate. For a given net force, the magnitude of the acceleration was scaled by the mass of the system. Similarly, Newton's 2nd law for rotation tells us that a net torque acting on a system about a reference axis will cause an angular acceleration. For a given net torque, the magnitude of the angular acceleration is scaled by a new quantity known as the moment of inertia. Thus the moment of inertia is analogous to mass in the sense that both scale the magnitude of acceleration/angular-acceleration given a net force/net torque.
- The moment of inertia depends on the reference axis and on the distribution of the mass about that reference axis. For example, the moment of inertia about a reference axis oriented vertically going through your head to the floor is larger if you stretch your arms out (more mass further away from axis) compared to if you kept your arms at your side.
- For objects that are not point-like particles about a reference axis, you can break the object up into multiple point particles to approximate the moment of inertia of the object.
- A force analysis and a torque analysis can be used together to analyze a scenario, just remember that the net force relates to the acceleration of the center of mass and the net torque relates to the angular acceleration about a reference axis.

Questions

Act I: Identifying equilibrium in systems

SD.L3.2-02:

Description: Identify equilibrium status of given systems. (2 minutes + 2 minutes)

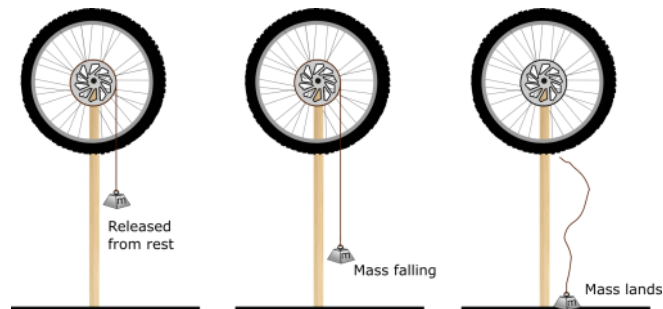
Problem Statement: A bicycle wheel is attached to a post as shown in the images below. A rope attached to a mass is wrapped around the inner radius of the wheel and released from rest.

(a) While the weight is falling, the wheel is in

- (1) translational static equilibrium and rotational static equilibrium
- (2) translational dynamic equilibrium only
- (3) translational dynamic and rotational static equilibrium
- (4) translational static equilibrium only
- (5) rotational dynamic equilibrium only

(b) When the rope and the mass are no longer attached to the wheel, the wheel is in

- (1) translational static equilibrium and rotational dynamic equilibrium
- (2) translational dynamic equilibrium only
- (3) translational dynamics and rotational static equilibrium
- (4) translational static equilibrium only
- (5) rotational dynamic equilibrium only



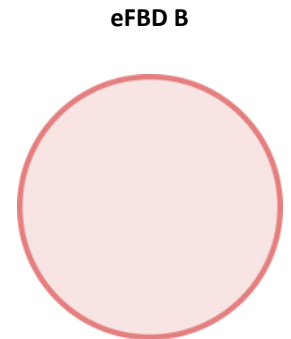
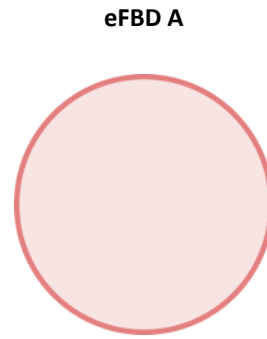
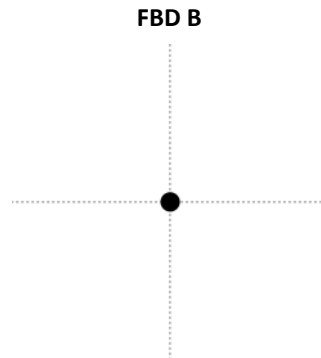
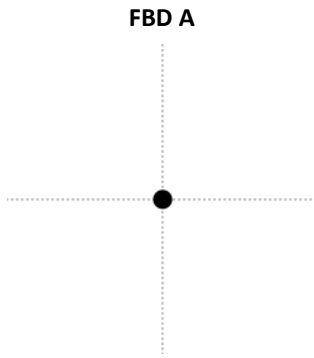
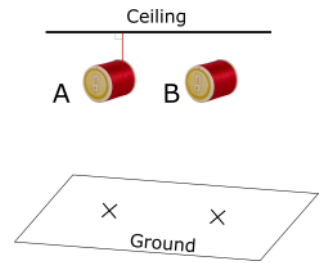
SD.L3.2-03:

Description: Conceptual application of torque analysis and force analysis to determine motion of object. (5 minutes + 3 minutes + 2 minutes)

Problem Statement: Two identical spools of thread are released from rest. Spool A has its thread attached to a ceiling about it, and spool B is not attached to the ceiling.

(a) Which spool reaches the floor first?

- (1) A
- (2) B
- (3) Same time



(b) Where does spool A land on the floor?

- (1) To the left of the X
- (2) To the right of the X
- (3) On the X

(c) Match which state each spool is in.

- (1) Translational dynamic equilibrium and rotational dynamics
- (2) Translational dynamic equilibrium and rotational dynamic equilibrium
- (3) Translational dynamics and rotational static equilibrium
- (4) Translational dynamics and rotational dynamics

Act II: Moment of inertia

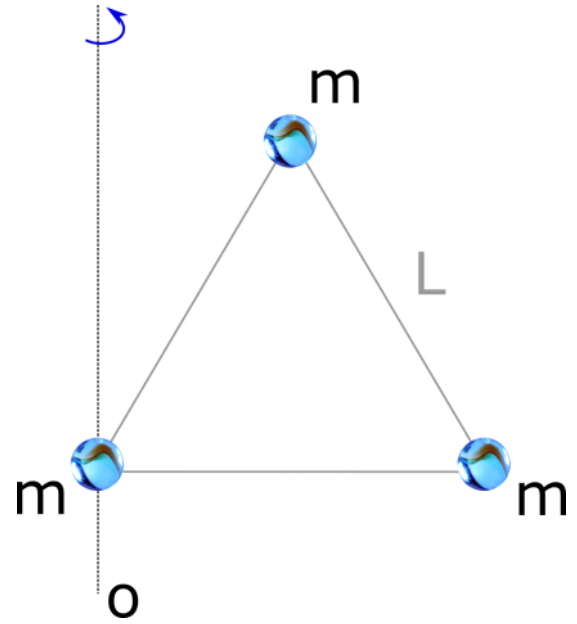
SD.L3.2-04:

Description: Calculate moment of inertia for system of point particles. (4 minutes)

Problem Statement: Consider 3 point particles each of mass m arranged in the configuration shown below (equilateral triangle of side

lengths L). Find the moment of inertia about the axis labeled "o".

- (1) $m L^2$
- (2) $1/2 m L^2$
- (3) $3/4 m L^2$
- (4) $5/4 m L^2$



SD.L3.2-05:

Description: Rank moment of inertias. (3 minutes + 3 minute)

Problem Statement: Consider a point particle, a rod with uniform mass distribution and length R , a hoop with uniform mass distribution and radius R , a solid disk with uniform mass distribution and radius R , and a sphere with uniform mass distribution and radius R . All objects have the same mass m .

(a) Rank the moment of inertia about the center of mass of each object as shown below.

Point particle	Uniform disk	Uniform hoop	Uniform sphere	Uniform rod
I_{cm}^P	I_{cm}^D	I_{cm}^H	I_{cm}^S	I_{cm}^R

(b) Below shows mathematical representations of moments of inertia. Match each representation with each object about the center of mass as show in the image below.

(1) 0

(2) $\frac{1}{12} m R^2$

(3) $\frac{2}{5} m R^2$

(4) $\frac{1}{2} m R^2$

(5) $m R^2$

Point particle



I_{cm}^P

Uniform disk



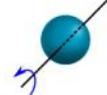
I_{cm}^D

Uniform hoop



I_{cm}^H

Uniform sphere



I_{cm}^S

Uniform rod

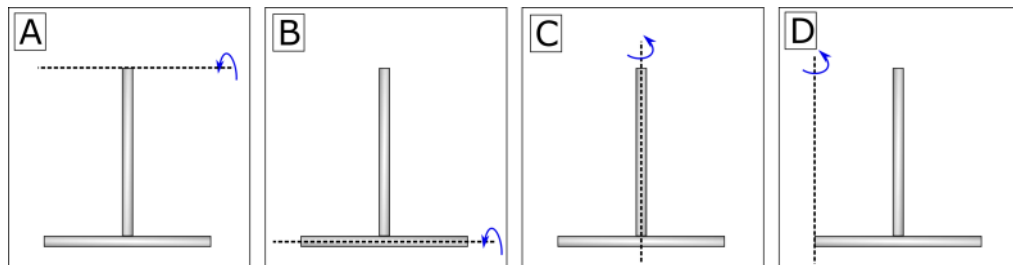


I_{cm}^R

SD.L3.2-06:

Description: Rank moment of inertias. (4 minutes)

Problem Statement: Four T handles are made from two identical rods of equal mass and length. Rank in order, from largest to smallest, the moment of inertia about the reference axis shown by the dotted lines below.



Act III: Dynamics

SD.L3.2-07:

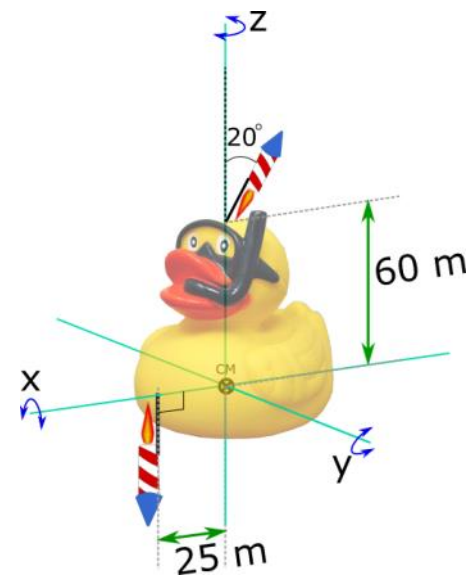
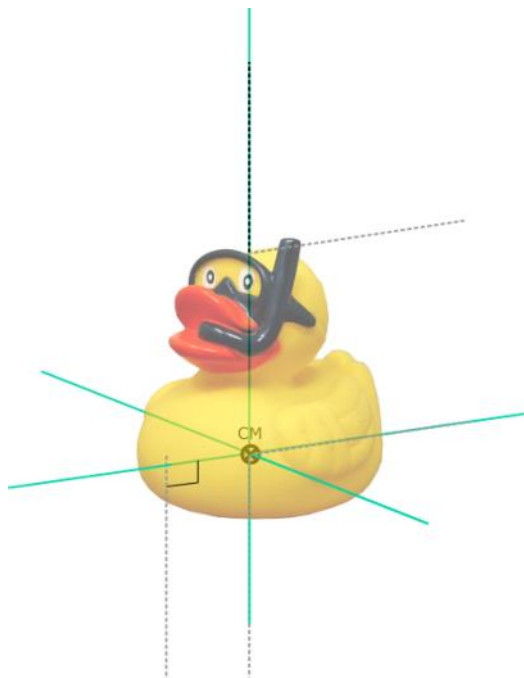
Description: Rotational dynamics problem solving for moment of inertia. (2 minutes + 2 minutes + 6 minutes)

Problem Statement: Mega Puddles, the 417,000-kg giant spaceship designed by U of O architects is falling miserably with two constant 2560 N thrusters stuck on as shown in the image below. While cruising along on his Scooty Puff Jr. Benny determines that the angular acceleration of the spaceship about its center of mass is 0.0477 rad/s^2 .

(a) What axis should be used to determine the moment of inertia?

- (1) x
- (2) y
- (3) z
- (4) some other axis

(b) Draw an eFBD for the spaceship. The ship is far away from other massive objects.



(c) Mega Puddles, the 417,000-kg giant spaceship designed by U of O architects is falling miserably with two constant 2560 N thrusters stuck on as shown in the image below. While cruising along on his Scooty Puff Jr. Benny determines that the angular acceleration of the spaceship about its center of mass is 0.0477 rad/s^2 . **What is the moment of inertia about the reference axis you chose in part (a)?**

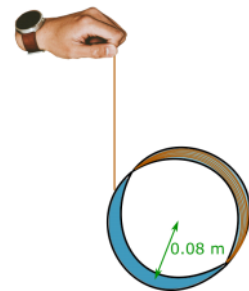
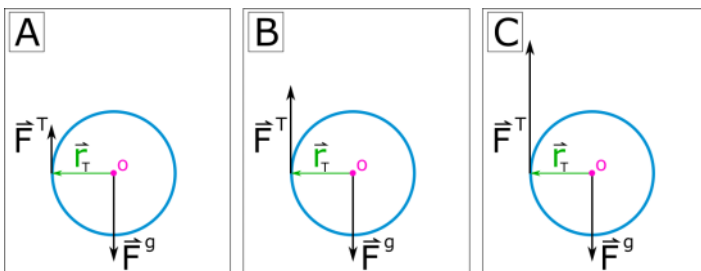
Act IV: Connecting torque analysis to kinematics

SD.L3.2-08:

Description: Rotational dynamics and translational static problem solving for linear and rotational quantities. (4 minutes + 5 minutes + ...)

Problem Statement: A light string is wrapped around a 0.50-kg hollow 0.080-m-radius hoop. The moment of inertia about the center of mass for a hoop is $I_{cm} = m r^2$, where m is the mass of the hoop and r is the radius. The hoop is released from rest above the surface of the earth and the free end of the string is pulled upwards by a hand such that the center of mass of the ring does not move.

(a) Which eFBD correctly represents this scenario?



(b) Which of the following is the correct expression for the angular acceleration of the hoop?

- (1) $-g$
- (2) $r g$
- (3) $-g/r$
- (4) r/g

(c) Find the change in angular position after 0.5 seconds. **(d)** What is the arc length for a point on the rim during this 0.5 seconds?

(e) Which of the following is the correct tangential component of acceleration?

- (1) $-g$
- (2) $r g$
- (3) $-g/r$
- (4) r/g

(f) What is the distance a point on the rim traveled during this 0.5 seconds? **(g)** When a marked spot on the hoop goes through 180 degrees, how much string has been unwound?

(h) How fast (in m/s) is a point on the rim going at the 0.5 second mark?

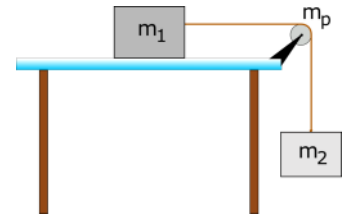
SD.L3.2-09:

Description: Rotational dynamics and translational dynamics problem solving for linear and rotational quantities. (3 minutes + 3 minutes + 4 minutes + 8 minutes + ...)

Problem Statement: A block $m_1 = 5$ kg rests on a horizontal frictionless surface. A rope connecting the block is draped around a uniform solid disk (pulley) of mass $m_p = 2$ kg to a hanging mass of $m_2 = 10$ kg. When released from rest, we wish to determine information about the acceleration of each block, distances they travel after a given amount of time, etc...

(a) You have seen this problem before with two masses and an ideal pulley, only then the pulley had negligible mass. Now we are considering a more real system where the pulley does have mass. We are still assuming negligible friction in the pulley's bearing. Which of the following features of the system have changed?

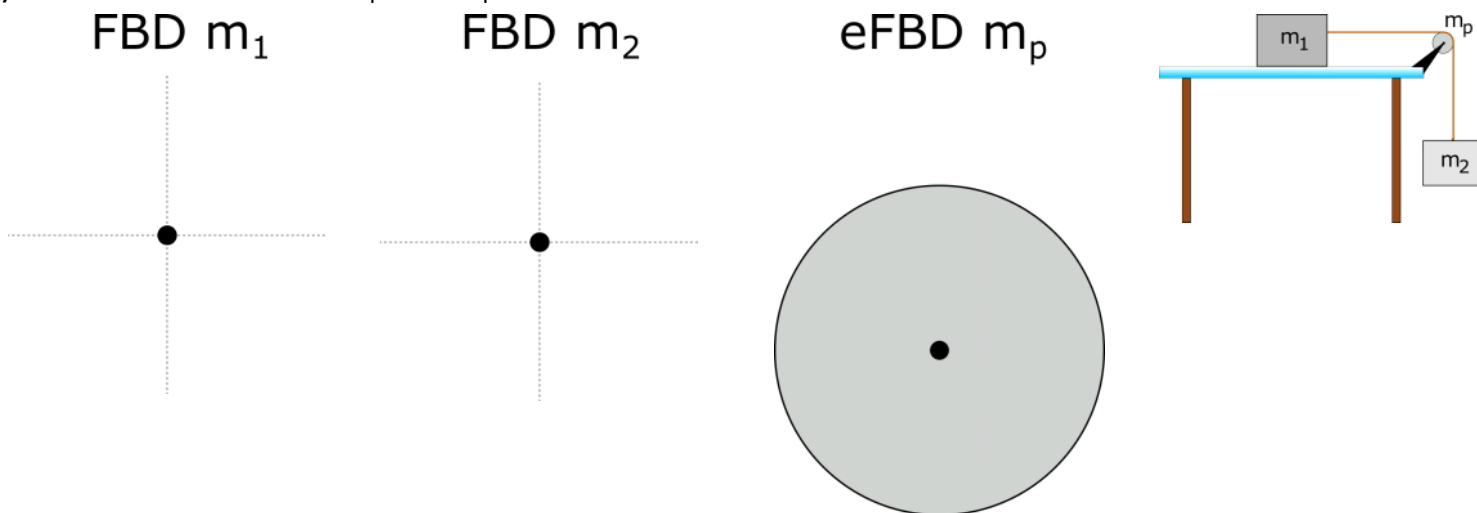
- (1) $|\vec{F}^{T_1}|$ is no longer equal to $|\vec{F}^{T_2}|$
- (2) $|\vec{a}_1|$ is no longer equal to $|\vec{a}_2|$
- (3) The acceleration of block 1 will be less now that the pulley has mass.
- (4) The final speed of mass 2 will be larger after a given unit of time now that the pulley has mass.



(b) Which of the following are constraints on the system?

- (1) $|\vec{a}_1| = |\vec{a}_2|$
- (2) $|\vec{a}_1| = |\mathbf{a}_t|$
- (3) $|\mathbf{a}_t| = |\boldsymbol{\alpha}|r$
- (4) $|\vec{v}_1| = |\vec{v}_2|$
- (5) $|\vec{v}_2| = |\mathbf{v}_t|$
- (6) $|\mathbf{v}_t| = |\boldsymbol{\omega}|r$

(c) Draw the FBD's and eFBD in the provided space below.



(d) A block $m_1 = 5 \text{ kg}$ rests on a horizontal frictionless surface. A rope connecting the block is draped around a uniform solid disk (pulley) of mass $m_p = 2 \text{ kg}$ to a hanging mass of $m_2 = 10 \text{ kg}$. When released from rest, **what is the acceleration of the center of mass of block 1?**

(e) What is the angular acceleration of the 10 cm radius pulley?

(f) How far does the block 1 travel after the first 2 seconds after being released from rest?

(g) How many revolutions does the pulley make in this 2 seconds?

(h) How far does a point on the edge of the pulley travel in this 2 seconds?

(i) What are the tensions in each section of rope?

Conceptual questions for discussion

1. Do you agree with the following statement: Every object has only one moment of inertia. Support your answer with examples.
2. What happens to the moment of inertia about Earth's rotational axis, if anything, when tall buildings are built near the equator?
3. Is there a location on Earth that you could build a tall building without affecting the moment of inertia of Earth about its rotational axis?
4. Use your knowledge of Newton's laws of motion for rotation to explain why sports cars use wheels of lighter mass than normal wheels.
5. Tight rope walkers often use long poles as seen in the cartoon below. Use your knowledge of Newton's laws of motion for rotation to explain why it is advantageous to use such poles when walking along a tight rope.



Hints

SD.L3.2-01: No hints.

SD.L3.2-02: No hints.

SD.L3.2-03: Draw FBDs for each spool to determine which one lands first and where A lands relative to the X.

SD.L3.2-04: The "r" in the moment of inertia equation is the perpendicular distance from the reference axis.

SD.L3.2-05: No hints.

SD.L3.2-06: Break the T-handle up into multiple pieces of equal mass and look at their "r" distance away from the reference axis.

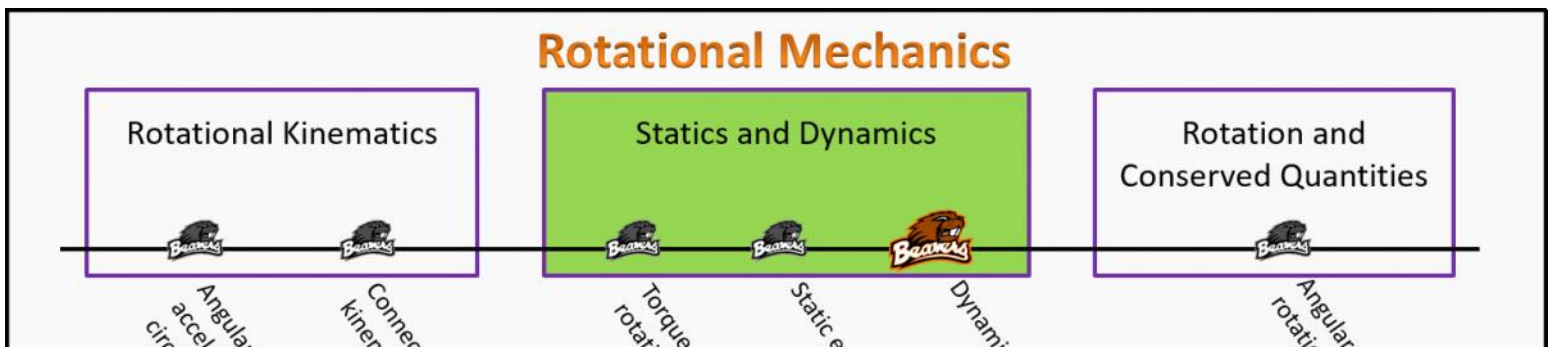
SD.L3.2-07: No hints.

SD.L3.2-08: No hints.

SD.L3.2-09: No hints.

Statics and Dynamics Practice Stage (SD.L3.3)

Lecture 3 Dynamics, Momentum of Inertia



Rotational Mechanics

Rotational Kinematics



Angular position, velocity, and acceleration for non-uniform circular motion

Connecting rotational and linear kinematics

Statics and Dynamics



Torque and Newton's 2nd law for rotation

Static equilibrium, stability

Dynamics, moment of inertia

Rotation and Conserved Quantities



Angular momentum and rotational energy

SD.L3.3-01

Description: Possible static and dynamic, translational and rotational, equilibrium states

Learning Objectives: [x]

Problem Statement: Which of the following are possible physical situations for a system?

- (1) Static translational equilibrium, static rotational equilibrium
- (2) Static translational equilibrium, dynamic rotational equilibrium
- (3) Dynamic translational equilibrium, dynamic rotational equilibrium
- (4) Dynamic translational equilibrium, static rotational equilibrium

SD.L3.3-02

Description: Analyze a system with an object in balance at its center of mass.

Learning Objectives: [x]

Problem Statement: A hammer balances at its center of mass as shown. Which side is heavier?



- | |
|------------------------------|
| (1) the right side |
| (2) the left side |
| (3) the sides weigh the same |

SD.L3.3-03

Description: Features of moment of inertia

Learning Objectives: [x]

Problem Statement: Which of the following statements concerning the moment of inertia I are false?

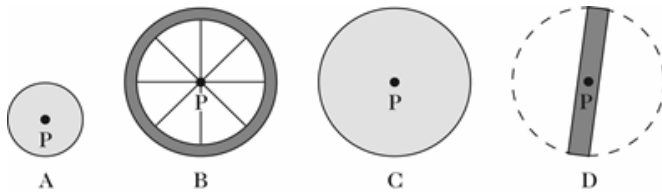
- | |
|--|
| (1) I may be expressed in units of $(\text{kg})(\text{m}^2)$ |
| (2) I depends on the angular acceleration of the object as it rotates. |
| (3) I depends on the location of the rotation axis relative to the particles that make up the object. |
| (4) I depends on the orientation of the rotation axis relative to the particles that make up the object. |

SD.L3.3-04

Description: Ranking final angular speed for different moments of inertia

Learning Objectives: [x]

Problem Statement: In the figure are scale drawings of four objects, each of the same mass and uniform thickness. An equivalent torque is applied to each about point p . Rank each object, from smallest to largest, based on their angular speed after the torque has been applied for an equivalent amount of time on each.



SD.L3.3-05

Description: Finding angular acceleration on solid disk with multiple torques

Learning Objectives: [x]

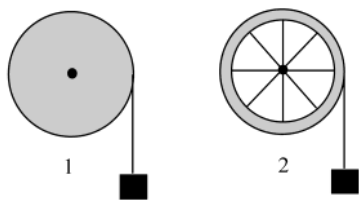
Problem Statement: A uniform solid disk with a mass of 24.3 kg and a radius of 0.314 m is free to rotate about a frictionless axle. Forces of 90 and 125 N are applied to the disk in the same horizontal direction but one is applied to the top and the other is applied to the bottom. What is the angular acceleration of the disk?

SD.L3.3-06

Description: Comparing angular acceleration and forces on wheels with different moments of inertia

Learning Objectives: [x]

Problem Statement: A solid cylinder and a cylindrical shell have the same mass, same radius, and turn on frictionless, horizontal axes. (The cylindrical shell has lightweight spokes connecting the shell to the axle.) A rope is wrapped around each cylinder and tied to a block of mass m . The blocks have the same mass and are held the same height above the ground as shown in the figure. Both blocks are released simultaneously, consider the time from right after the block is released to right before it hits the ground. The ropes do not slip. Which of the following statements are true regarding this situation?



- (1) The block connected to wheel 1 will reach the ground first
- (2) The block connected to wheel 2 will reach the ground first
- (3) Both blocks reach the ground at the same time

(4) The tension in both ropes is equal to mg
(5) The tension in both ropes is less than mg
(6) The tension in both ropes is greater than mg
(7) The tension in both ropes is equal
(8) The tension in the rope from 1 is greater than in the rope from 2
(9) The tension in the rope from 1 is less than in the rope from 2