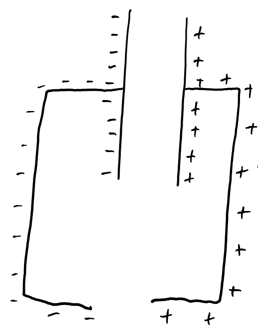
Circuits (completed)

Monday, May 16, 2016 6:27 PM

Microscopic Mode of Charge flow

* if q's are moving + non-zero resistance, $\vec{E} \neq 0$, b/c $\Sigma \vec{F} \neq 0$

not Connected



uniform distribution of q's

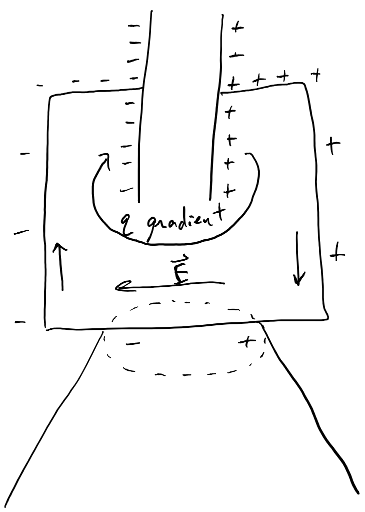
what happens when Connect wires?

* charges flow + redistribute

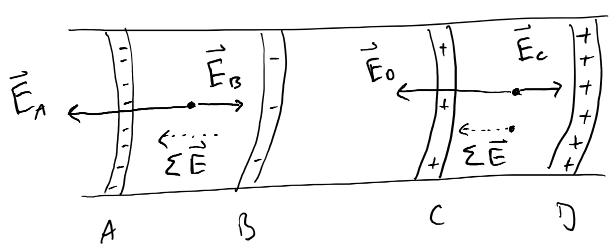
  $(- \to +)$ or $(+ \to -)$

* create a charge gradient

Connected



* q-gradient creates a non-zero E-field



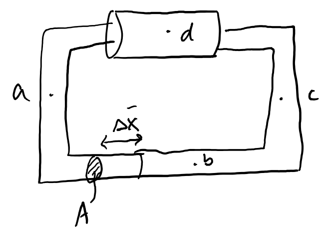
Microscopic Model of Current (I)

* Conservation of Charge:

$I_a = I_b = I_c = I_d$ $\#e^-/$

Microscopic Model of Current

$I_a = I_b = I_c = I_d$



* Conservation of Charge:
At all points the $\#e^-/\text{time}/\text{Area}$ is a constant ($I = \text{const}$)

* n - density of free e^-

$$n = \frac{N}{\text{Volume}}$$

(N ~ #)

* V_d - drift speed

$$V_d = \frac{\Delta \bar{x}}{\Delta t}$$

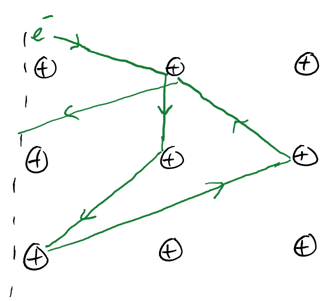
$$\#e^-/\text{time} = n A V_d \quad \left[\frac{\#}{L^3}\right]\left[L^2\right]\left[\frac{L}{T}\right] = \left[\frac{\#}{T}\right] \checkmark$$

Current $\quad I = \dfrac{\Delta q}{\Delta t} = e n A V_d$

* Convention: Current flows in direction $+q$ would flow

Charges Navigating the Lattice

1. $\vec{E} = 0 \quad \text{temp} \neq 0$

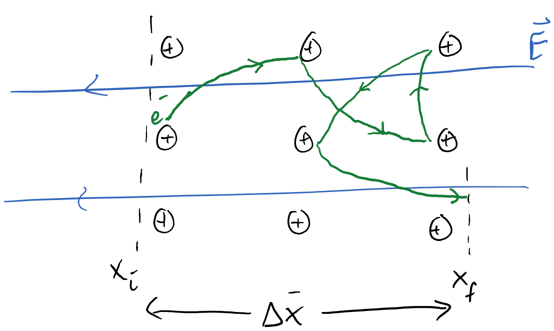


$x_i = x_f$

w/out E-field

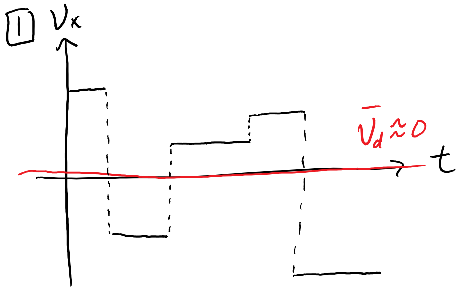
$\Delta \bar{x} = 0$, so $\bar{V}_d = 0$

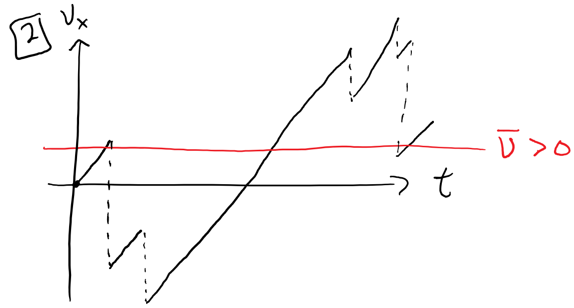
2. $\vec{E} \neq 0$, $\text{temp} \neq 0$



w/ $\vec{E} \neq 0$, $\Delta \bar{x} \neq 0$

& $\bar{V}_d \neq 0$





On the Road to OHm's Law

→ use macro circuit analysis

(1D) $\sum F = ma \Rightarrow \sum F = m\frac{\Delta v}{\Delta t} \Rightarrow \Delta v = \frac{\sum F \Delta t}{m} = \frac{eE\Delta t}{m}$ — between collisions

Average $\bar{v}_d = \frac{eE}{m}\Delta\bar{t}$ - - - - Combine w/ $I = enAv_d$

$\Delta\bar{t}$: average time between collision

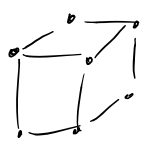
get $\underbrace{\frac{I}{A}}_{J\text{ - current density}} = \underbrace{\frac{ne^2\Delta\bar{t}}{m}}_{\text{material properties } \equiv \sigma \text{ "conductivity"}} E$ (electric field) $\Rightarrow J = \sigma E$ — almost OHm's Law

What will effect conductivity?

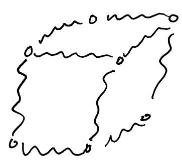
$\sigma = \frac{ne^2\Delta\bar{t}}{m}$, if same material $n = \text{const}$

- - - How can effect $\Delta\bar{t}$?

Temp as Temp $\uparrow$ $\sigma\downarrow$ b/c $\Delta\bar{t}\downarrow$ (e^- collide more often)



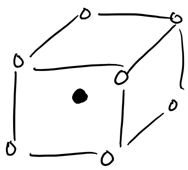
low temp
$\Delta\bar{t}$ high



High temp
$\Delta\bar{t}$ low

←
$\sigma\uparrow$

Impurities



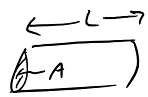
Imperfection in Lattice

Can $\Delta \bar{\tau} \downarrow$ + $\sigma \downarrow$

Defitions

Resistivity (material property) $\rho \equiv \frac{1}{\sigma}$ (doesn't depend on how much or shape)

Resistance $R = \frac{\rho L}{A}$

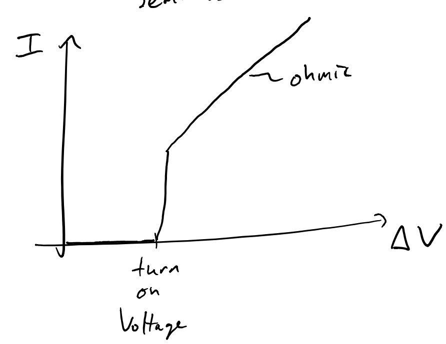


Ohm's Law

Recall $J = \sigma E$ ... $|E| = \frac{|\Delta V|}{L}$

$$\frac{I}{\cancel{A}} = \frac{\cancel{L}}{R\cancel{A}} \frac{\Delta V}{\cancel{L}} \Rightarrow \boxed{\Delta V = IR}$$

Ohm's Law



Charges are not able to move completely freely through a wire, e.g., there is some resistance to their motion. If charges are flowing through a conducting wire, which one of the following statements are true?

1. **The electric field inside a conductor is zero.**
2. **The electric potential difference between the ends of the wire is zero.**
3. **The electric field is non-zero inside the wire.**
4. **The electric potential difference between the ends of the wire is not zero.**

w/ resistance $\Sigma \vec{F} \neq 0$

so $\vec{E} \neq 0$

w/ $E_x = -\frac{\Delta V}{\Delta x}$, $\Delta V \neq 0$

You're attempting to make a resistive wire for melting ice off windows that will be connected to a constant voltage source. Which of the following quantities are independent of the length and radius of the wire you use?

1. **conductivity**
2. **conductance**
3. **free electron density**
4. **resistivity**
5. **resistance**
6. **current through the wire**
7. **voltage difference across the wire**

material properties

$R = \rho \frac{L}{A}$

$\Delta V = IR$, w/ $\Delta V = \text{const}$ but R changing, I must change

If you need to increase the radius of a resistive wire by a factor of 2, but keep the resistance the same, by what factor does the length need to change?

1. **1/4**
2. **1/2**
3. **2/3**
4. **3/2**
5. **2**
6. **4**
7. **16**

$R = \rho \frac{L}{A}$

$= \rho \frac{L}{\pi r^2}$

if $r \to 2r$, $A \to 4A$

so $L \to 4L$ for $R = \text{const.}$

An electric car accelerates for 8.0 s by drawing energy from its 320-V battery pack. During this time the current through the battery is 163 A. How many Coulombs of charge were transferred through the battery?

1. **40 C**
2. **800 C**
3. **1300 C**
4. **14,400 C**
5. **25,600 C**

$I = \frac{\Delta q}{\Delta t} \Rightarrow \Delta q = I \Delta t$

$= 1304\ C$

If the 320 V are multiplied by the Coulombs of charge transferred through the battery during this time, what type of quantity would result?

1. **force**
2. **electric potential**
3. **energy**
4. **electric field**
5. **current**
6. **resistance**
7. **work**

(Volts) (Coulombs) .... $\Delta U = q \Delta V$

Energy!

$(320\ V)(1304\ C) = 417,280\ J$

(2200 lb)

if 100% effecient $\frac{1}{2} m v_f^2 = 417,280$ .... w/ $m = 1000\ kg$

$v_f = 28.9\ m/s$ or 64.6 mph

Circuit tools

Circuit tools

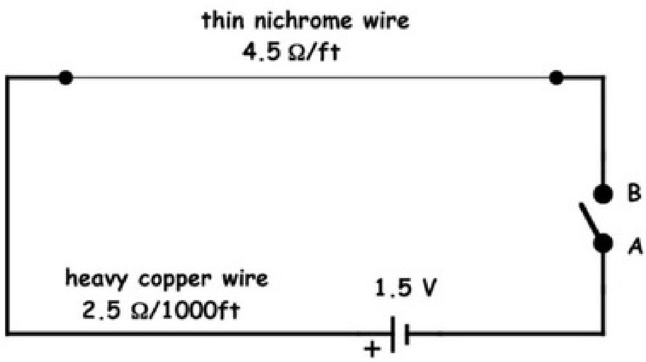
Voltage loop $\quad \Sigma \Delta V_{loop} = 0$

Current Junction $\quad \Sigma I_{in} = \Sigma I_{out}$

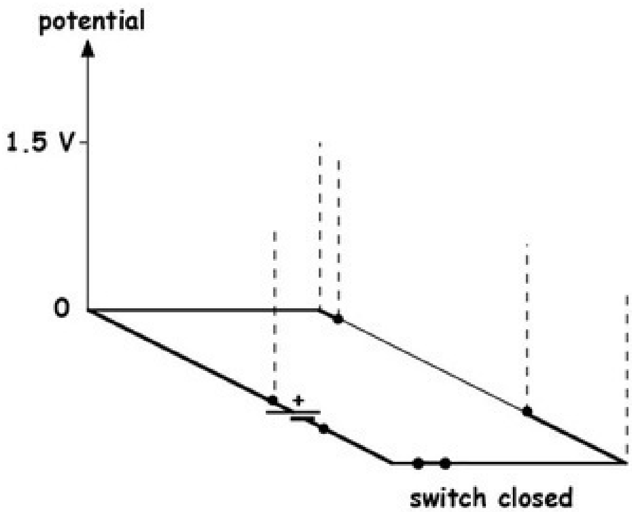
Ohm's Law $\quad \Delta V = I R$

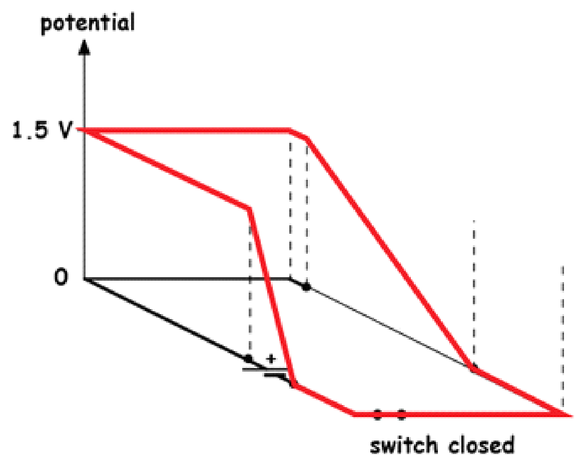
The 3 important eq.

* Assumption $\Delta V_{wire} << \Delta V_{resistor}$



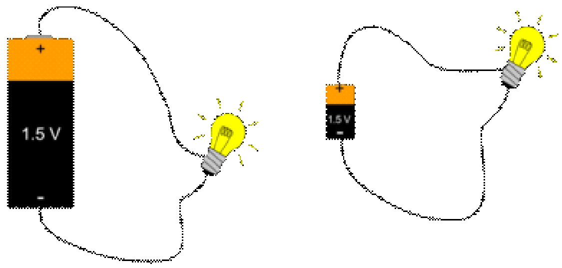
In the 3D diagram below, sketch the potential around the circuit shown above when the switch is closed.





What would happen to the light bulb if the battery was replaced by a larger one?

1. **The bulb would be brighter.**
2. **Nothing - it would stay the same brightness.**
3. **Not enough information.**



what is difference

--- Presumably stored P.E.

---- Hopefully last longer

Power (energy/time)   Recall: $I = \frac{\Delta q}{\Delta t}$, w/ $\Delta U = \Delta q \Delta V$

$$I = \frac{\Delta U}{\Delta V \Delta t} \Rightarrow \frac{\Delta U}{\Delta t} = I \Delta V$$

$$\boxed{P = I\Delta V = \frac{\Delta V^2}{R} = I^2 R}$$

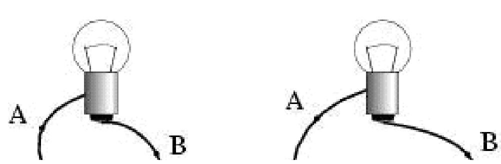
I^2R: useful when I = const.

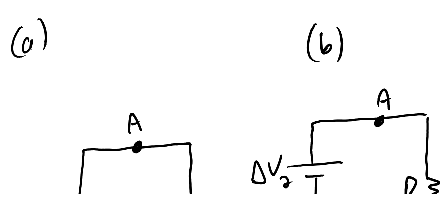
$\frac{\Delta V^2}{R}$: useful when ΔV = const

A light bulb is connected to a 9-V battery. If a second battery is added in a series as shown in (*b*), how many of the following change?

The current *I* at A, the potential difference V_{AB} between A and B, the resistance *R* of the light bulb.

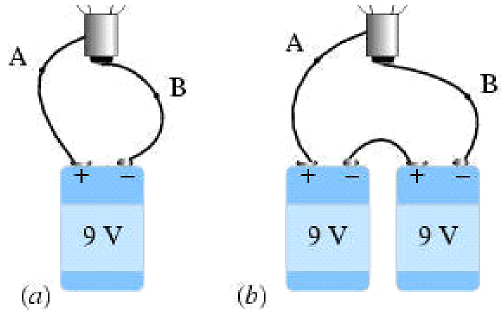
1. **All three**
2. **Two**
3. **One**
4. **It depends**

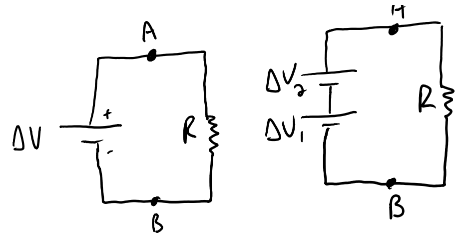




3. One
4. It depends

R-bulb
is a const.





$2\Delta V^{(a)}_{AB} = \Delta V^{(b)}_{AB}$, w/ $\Delta V_{AB} = IR$

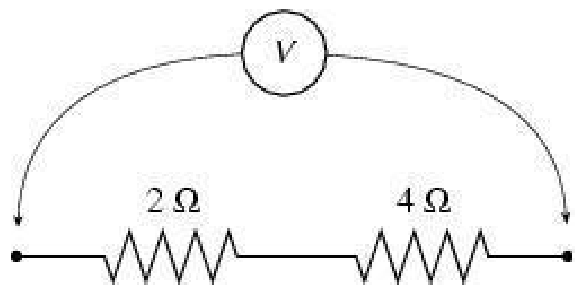
if ΔV changes + R = const.
I must change

A constant potential difference *V* is applied across two resistors connected in series as shown. The current through the 2 Ω resistor is 2 A. What is the current through the 4 Ω resistor?

1. 0 A
2. 1 A
3. 2 A
4. 4 A
5. Need to know the potential difference

$\Sigma I_{in} = \Sigma I_{out}$

No Junctions
means current
is const.

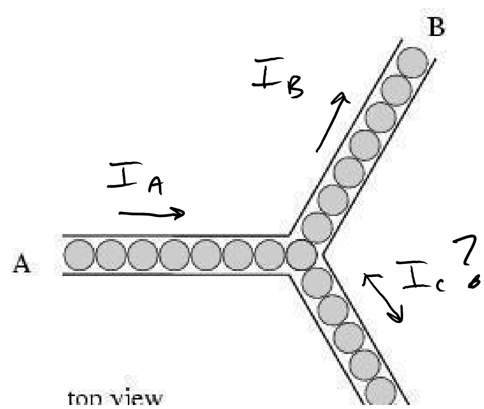


Consider the Y-shaped tube shown below. Suppose 2 balls per second are stuffed into the opening at A. The number of balls per second that come out of the tube at B is

1. always equal to 2
2. always smaller than or equal to 2
3. always larger than or equal to 2
4. equal to 1
5. depends on what happens at C

$\Sigma I_{in} = \Sigma I_{out}$

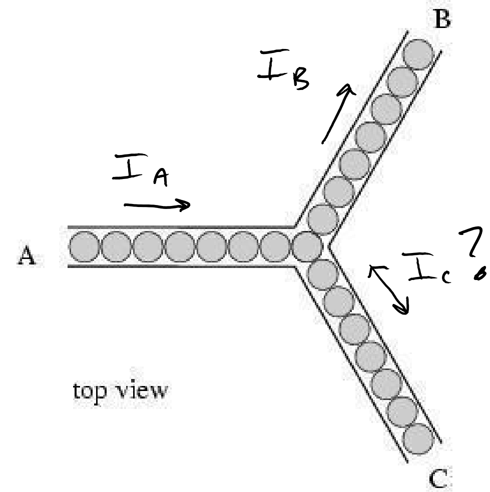
$I_A = I_B + I_C$



1. always equal to 2
2. always smaller than or equal to 2
3. always larger than or equal to 2
4. equal to 1
5. depends on what happens at C

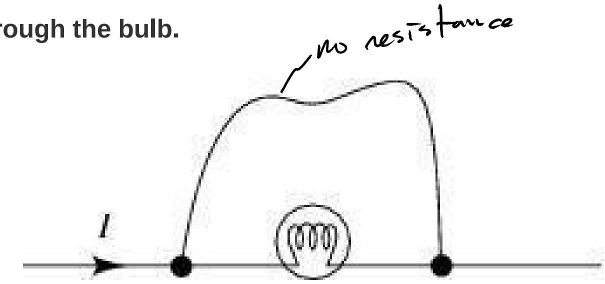
$\sum I_{in} = \sum I_{out}$

$I_A = I_B + I_C$



Charge flows through a light bulb. Suppose a wire is connected across the bulb as shown. When the wire is connected,

1. all the charge continues to flow through the bulb.
2. half the charge flows through the wire, the other half continues through the bulb.
3. all the charge flows through the wire.
4. none of the above.



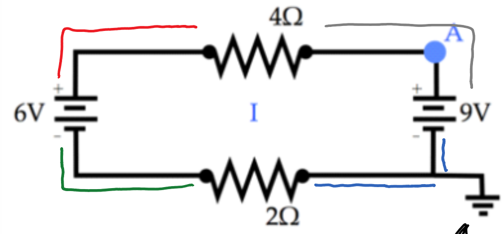
If wire had appreciable resistance
then some current through both
---- How much depends on ratio
of resistance of the two paths

What is the electrical potential at point A in the circuit?

What is the electrical potential at point A in the circuit?

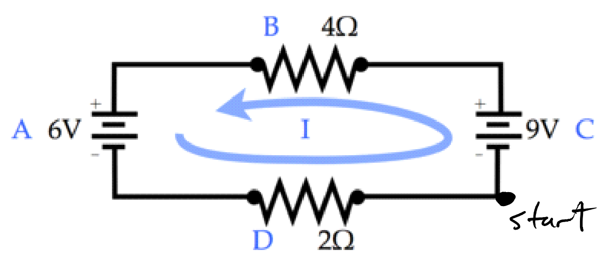
1. +9V
2. -9V
3. +6V
4. +3V
5. 0V
6. Can't determine with the information given.

going from (-) to (+) terminal on a battery steps up in Voltage



Ground
$V \equiv 0$

Consider the following circuit. List the four potential voltage drops (and gains) ΔV encountered going around the loop (don't include units). Express your answer as a sum that should be zero according to Krichhoff's voltage rule: i.e. "1 + 1I + -12I + -2I"



$\Sigma \Delta V_{loop} = 0$

$\Delta V_C + \Delta V_B + \Delta V_A + \Delta V_D = 0$

$+9 - 4I - 6 - 2I = 0$

$3 = 6I$, $\underline{I = \frac{1}{2} A}$,

Power @ B?

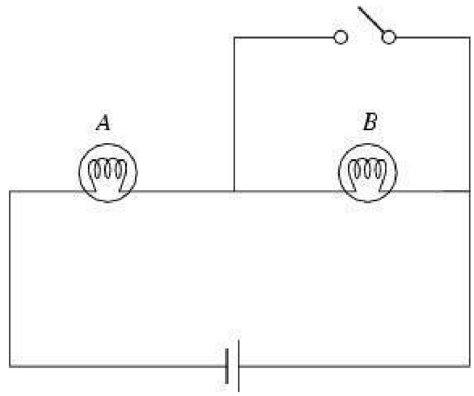
$P = I\Delta V = I^2 R$

$P_B = \left(\frac{1}{2}\right)^2 4 = \underline{1 W}$

The circuit below consists of two identical light bulbs burning with equal brightness and a single 12 V battery. When the switch is closed, the brightness of bulb *A*

1. **increases.**
2. **remains unchanged.**
3. **decreases.**

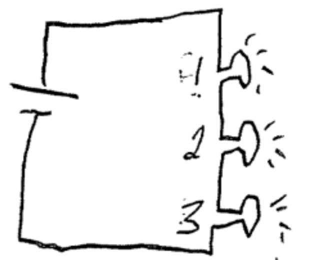
$P = I \Delta V = \frac{\Delta V^2}{R}$



entire Voltage dropped across A when switch closed.

So $\Delta V_A \uparrow$ & so does Power

Consider the following circuit with three light bulbs of equal resistance. Rank the relative brightness of each bulb?



$P = I V$

$= I^2 R \leftarrow$ w/ series $I = const$

$= \frac{\Delta V^2}{R}$ & if $R_1 = R_2 = R_3$

$P_1 = P_2 = P_3$

If $R_1 < R_2 = R_3$, What will be the relative brightness of each bulb?

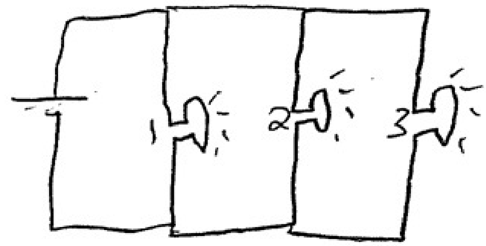
w/ $P = I^2 R$, $P_1 < P_2 = P_3$

Consider the following circuit with three light bulbs. Which equation would be most useful for determining the relative brightness of each bulb if the relative resistances are known?

1. **$\Delta V = IR$**
2. **$P = \Delta V^2/R$**
3. **$P = I\Delta V$**
4. **$P = I^2R$**

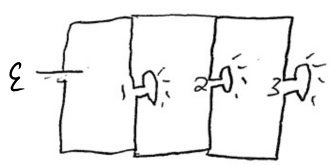
$\Delta V_1 = \Delta V_2 = \Delta V_3$

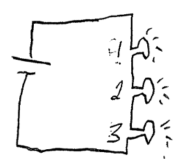
so use $P = \frac{\Delta V^2}{R}$



Consider the two circuits with three identical light bulbs and an identical voltage source. Which circuit will have brighter bulbs?

1. **Series**
2. **Parallel**
3. **Equal**





$P_1 = P_2 = P_3 = \frac{\varepsilon^2}{R}$

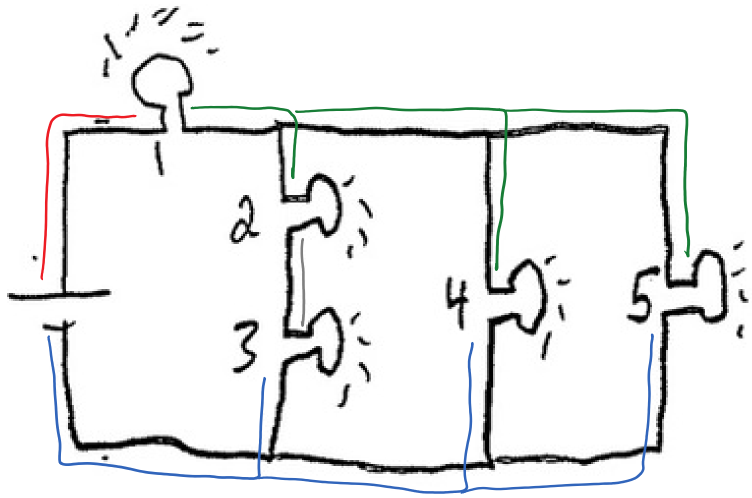
$P_1 = P_2 = P_3 = \frac{\left(\frac{\varepsilon}{3}\right)^2}{R}$

Consider the following circuit with five equal resistance light bulbs. What is the relative brightness of each bulb?

1. 1 > 2 = 3 > 4 = 5

Consider the following circuit with five equal resistance light bulbs. What is the relative brightness of each bulb?

1. 1 > 2 = 3 > 4 = 5
2. 1 < 2 = 3 < 4 = 5
3. 1 > 2 = 3 = 4 = 5
4. 1 > 4 = 5 > 2 = 3
5. 1 < 4 = 5 < 2 = 3



* P_1 greatest ... all current through it

* $P_4 = P_5$ b/c $\Delta V_4 = \Delta V_5$

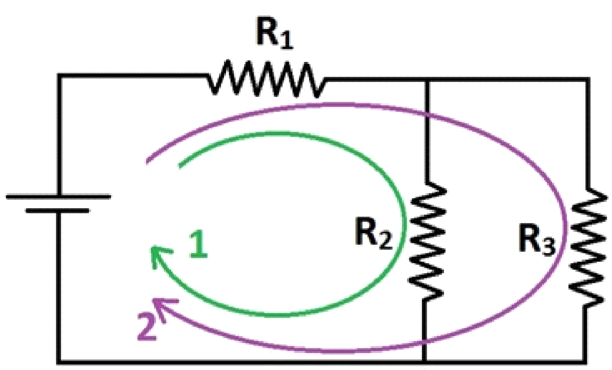
* $P_2 = P_3$ b/c $I_2 = I_3$

* $\Delta V_2 = \Delta V_3 < \Delta V_4 = \Delta V_5$

- - - - So, $P_1 > P_4 = P_5 > P_2 = P_3$

Consider the circuit in the figure below where I_1 is the current that goes through R_1, I_2 goes through R_2, and I_3 goes through R_3. What is the voltage drop across resistor R_1?

1. $\Delta V_1 = I_1R_1$
2. $\Delta V_1 = -I_1R_1$
3. $\Delta V_1 = -I_{total}R_1$



Which equation satisfies Kirchhoff's loop rule when following loop 1?

1. $+\epsilon + I_1R_1 + I_2R_2 = 0$
2. $+\epsilon - I_1R_1 - I_2R_2 = 0$
3. $+\epsilon - I_1R_1 - I_2R_2 = 0$
4. $+\epsilon + I_1R_1 + I_2R_2 = 0$

Which equation satisfies Kirchhoff's loop rule when following loop 2?

1. $+\epsilon + I_1R_1 + I_1R_3 = 0$
2. $+\epsilon - I_1R_1 - I_3R_3 = 0$
3. $+\epsilon - I_1R_1 - I_2R_2 - I_3R_3 = 0$
4. $+\epsilon + I_1R_1 + I_3R_3 = 0$

Which equation satisfies Kirchhoff's junction rule?

1. $I_1 + I_3 = I_2$
2. $I_1 - I_2 = I_3$
3. $I_1 + I_2 = I_3$
4. $I_2 + I_3 = I_1$
5. $V_1 + V_3 = V_2$
6. $V_1 + V_2 = V_3$

With the following values, solve for the current in each segment of the circuit:

ε = 11 V
R_1 = 2 Ω
R_2 = 4 Ω
R_3 = 6 Ω

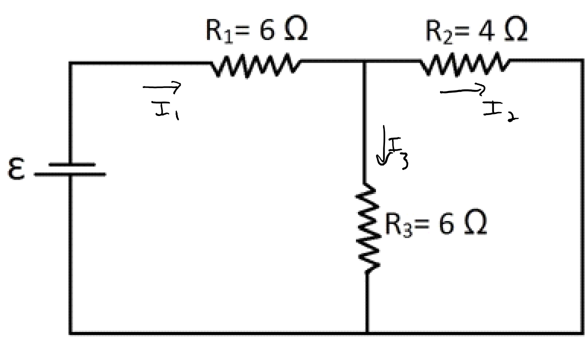
① $11 - 2I_1 - 4I_2 = 0$

② $11 - 2I_1 - 6I_3 = 0$

Junction $I_1 = I_2 + I_3$

1. **I_1 = 2.5 A, I_2 = 1.5 A, I_3 = 1 A**
2. **I_1 = 1 A, I_2 = 1.5 A, I_3 = 2.5 A**
3. **I_1 = 2.5 A, I_2 = -1.5 A, I_3 = 1 A**
4. **I_1 = 2.5 A, I_2 = 1.5 A, I_3 = -1 A**

Let I_1 be the current through R_1, I_2 be the current through R_2, and I_3 be the current through R_3. Rank the current that goes through each resistor from least to greatest.



$I_1 = I_2 + I_3$, $\Delta V = IR$

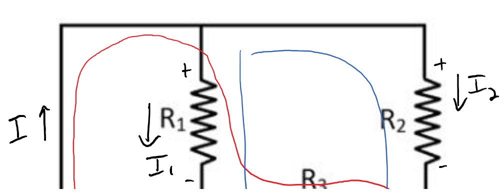
w/ $\Delta V_2 = \Delta V_3$ + $R_3 > R_2$

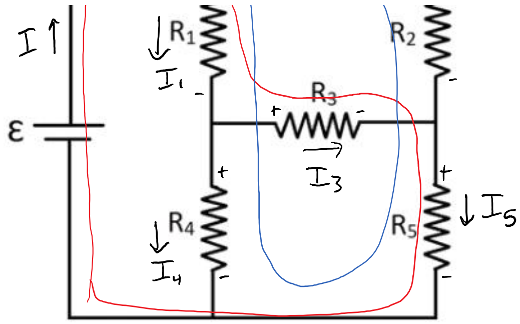
$I_3 < I_2$

So $I_3 < I_2 < I_1$

Consider the circuit below. What minimum number of loops can be used when applying Kirchhoff's loop rule to completely account for each element?

$\boxed{2}$





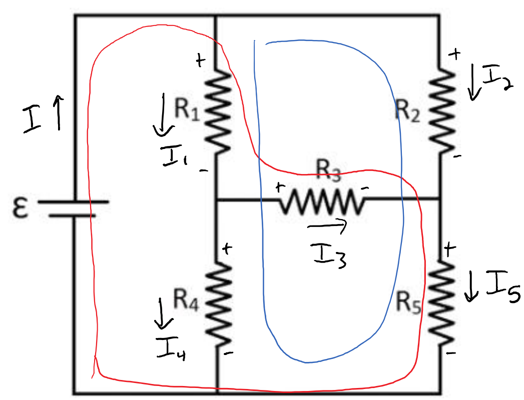
Let I_1 be the current through R_1, I_2 be the current through R_2… How many values of I are present in this circuit?

$\boxed{6}$

How many *unique* current junctions are there?

$\boxed{4}$

Which set of equations is valid for this circuit?



a)
$+\in -\mathbf{I_1R_1} - \mathbf{I_4R_4} = \mathbf{0}$ ✓
$+\in -\mathbf{I_2R_2} - \mathbf{I_5R_5} = \mathbf{0}$ ✓
$-\mathbf{I_2R_2} + \mathbf{I_3R_3} + \mathbf{I_1R_1} = \mathbf{0}$ ✓
$-\mathbf{I_5R_5} + \mathbf{I_4R_4} - \mathbf{I_3R_3} = \mathbf{0}$ ✓

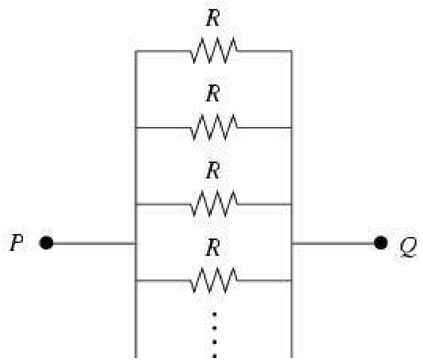
b)
$+\in -\mathbf{I_1R_1} - \mathbf{I_4R_4} = \mathbf{0}$ ✓
$+\in -\mathbf{I_2R_2} - \mathbf{I_5R_5} = \mathbf{0}$ ✓
$+\in -\mathbf{I_2R_2} + \mathbf{I_3R_3} + \mathbf{I_1R_1} = \mathbf{0}$ ×
$+\in -\mathbf{I_5R_5} + \mathbf{I_4R_4} - \mathbf{I_3R_3} = \mathbf{0}$

c)
$-\mathbf{I_1R_1} - \mathbf{I_4R_4} = \mathbf{0}$ ×
$-\mathbf{I_2R_2} - \mathbf{I_5R_5} = \mathbf{0}$
$-\mathbf{I_2R_2} + \mathbf{I_3R_3} + \mathbf{I_1R_1} = \mathbf{0}$
$-\mathbf{I_5R_5} + \mathbf{I_4R_4} - \mathbf{I_3R_3} = \mathbf{0}$

As more identical resistors, *R*, are added to the parallel circuit shown here, the total resistance between points *P* and *Q*

1. **increases.**
2. **remains the same.**
3. **decreases.**

$$\frac{1}{R_{tot}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots \Rightarrow R_{tot} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \cdots\right)^{-1}$$



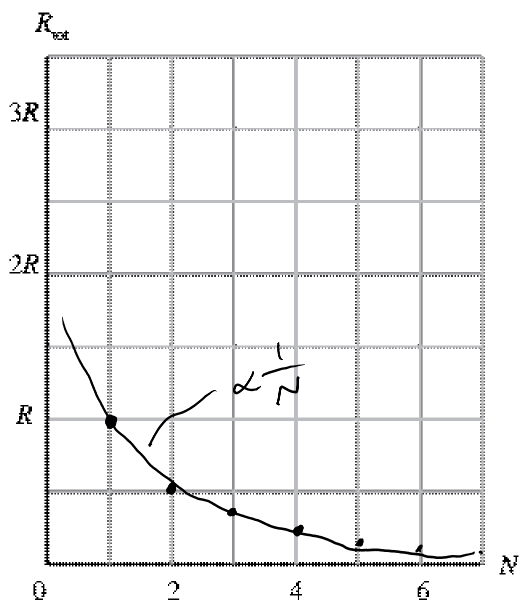
$\Rightarrow$ P —∿∿— Q, R_{tot}

ex. $R = 4\Omega$, $N = 3$

$$R_{tot} = \left(\frac{1}{4} + \frac{1}{4} + \frac{1}{4}\right)^{-1}$$

$$R_{tot} = \left(\frac{3}{4}\right)^{-1} = \frac{4}{3} < R$$

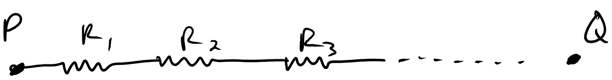
Draw a graph relating the total resistance R_{total} between points *P* and *Q* versus the total number N of resistors placed between them.



if N resistors all resistance R

$$R_{tot} = \left(N \frac{1}{R}\right)^{-1} = \frac{R}{N}$$

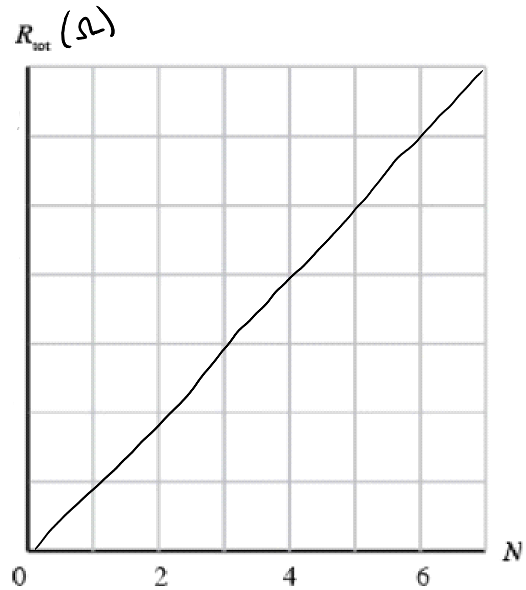
Draw a graph relating the total resistance as a function of N, where N is the number of equivalent 0.5 Ω resistors added in series.



$R_{tot} = R_1 + R_2 + R_3 + \dots$

if all R

$R_{tot} = N R$



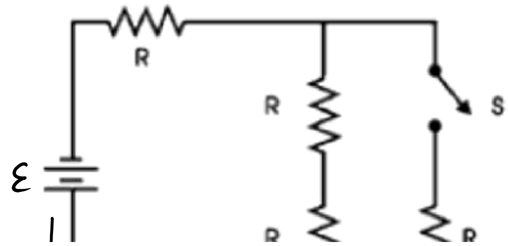
Consider the following circuit. Will more power be dissipated by the circuit when the switch S is open or closed?

1. open
2. closed
3. no difference

As more // branches are added

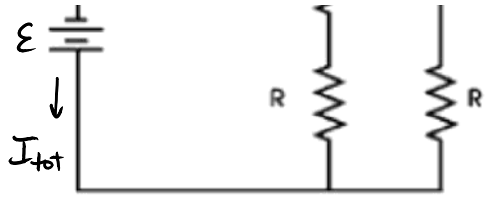
$R_{tot} \downarrow$

w/ $\mathcal{E} = I_{tot} R_{tot}$ $\quad \mathcal{E} = \text{const}$



w/ $\mathcal{E} = I_{tot} R_{tot}$, $\mathcal{E} = \text{const}$,

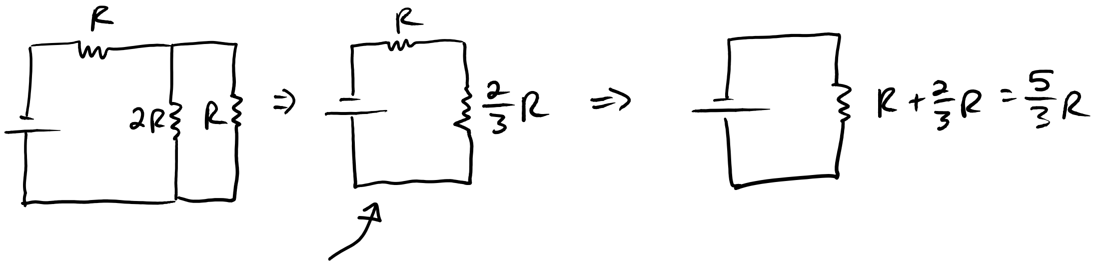
so $I_{tot} \uparrow$



$P_{tot} = I_{tot} \mathcal{E}$ ...so $P_{tot} \uparrow$

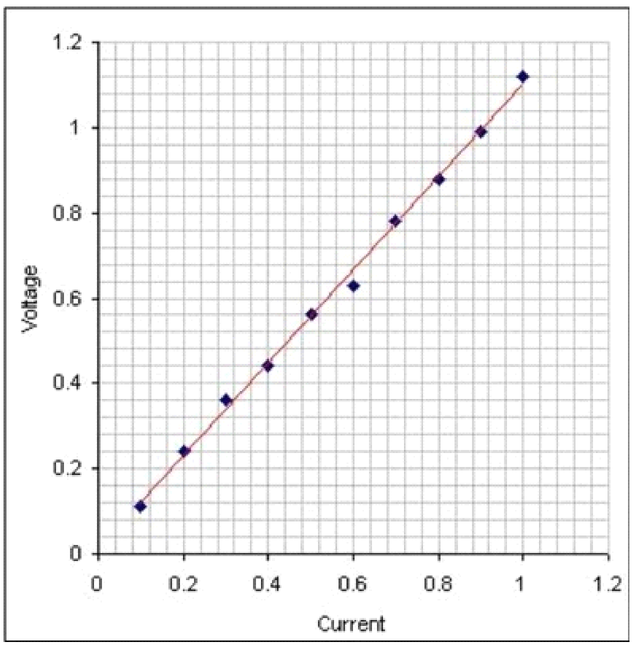
With the switch S closed, what is the equivalent resistance of the circuit?

1. **4R**
2. **2R/3**
3. **2R**
4. **5R/3**
5. **5R/6**
6. **R**
7. **R/3**
8. **R/2**



$\left(\frac{1}{2R} + \frac{1}{R}\right)^{-1} = \frac{2}{3}R$

Use the voltage vs. current curve for the resistors to determine what power is delivered to the circuit by a 12 V battery when the switch is closed?



$R = \frac{\Delta V}{I} \approx 1.125\,\Omega$

$P = \frac{\Delta V^2}{R} \Rightarrow P_{tot} = \frac{\mathcal{E}^2}{R} = \underline{128\,W}$