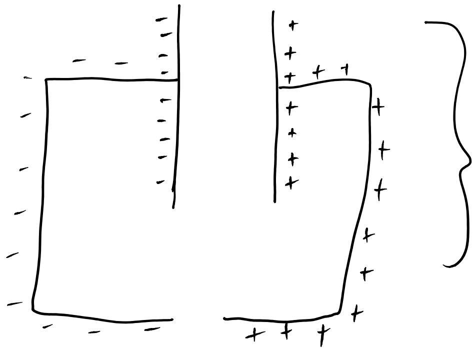
# PH203 Class Template: Circuits

Sunday, March 27, 2016 6:18 PM

## Microscopic Model of charge flow

- if q's are moving + non-zero resistance, $\vec{E} \neq 0$
  b/c $\Sigma\vec{F} \neq 0$

### not connected

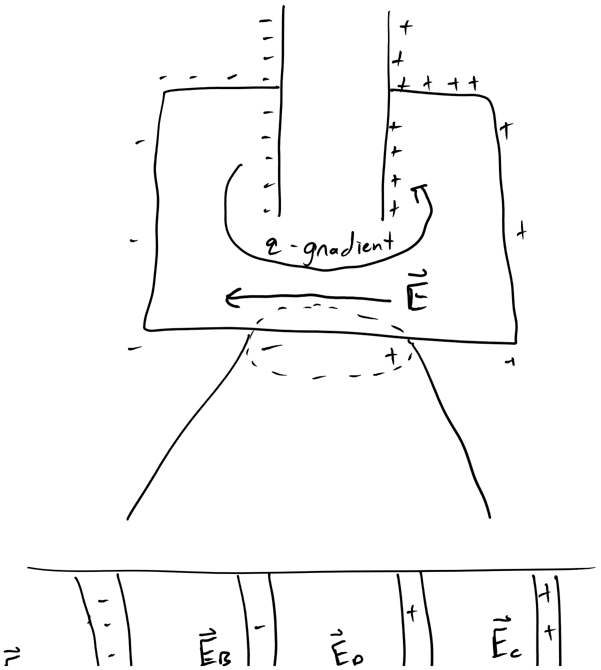


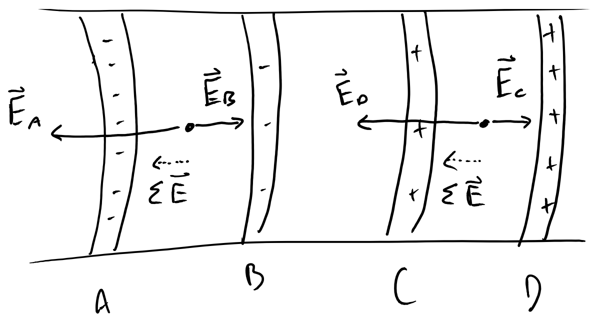
### what happens when connect the wires?

- charges flow + redistribute
  $(- \rightarrow +)$ or $(+ \rightarrow -)$
- create a charge gradient

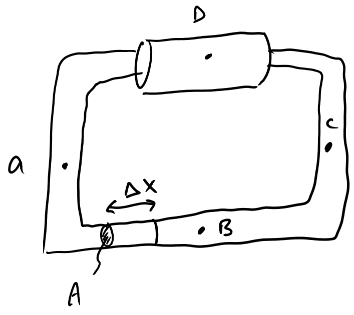
### Connected

- q-gradient creates
  a non-zero E-field





## Microscopic Model of Current (I)



* Conservation of charge:
At all points the $\#e^-/\text{time}/\text{Area}$
is a constant. $(I = \text{const})$

$n$ - density of free $e^-$

$$n = \frac{N}{\text{Volume}}$$ ~ #

$V_d$ - drift speed

$$V_d = \frac{\Delta\bar{x}}{\Delta t}$$

$$\#e^-/\text{time} = n A V_d \leftarrow \left[\frac{\#}{L^3}\right]\left[L^2\right]\left[\frac{L}{T}\right] = \frac{\#}{T} \checkmark$$

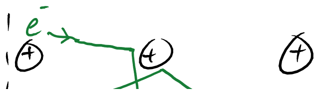
Current $$I = \frac{\Delta q}{\Delta t} = e n A V_d$$

* Convention: current flows in direction +q would flow

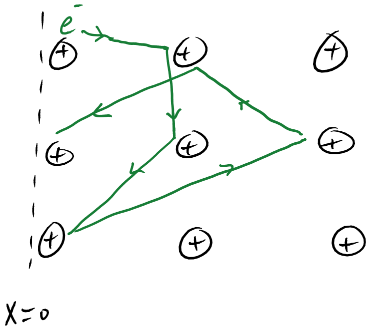
## Charges Navigating the Lattice

$\boxed{I}$ $\vec{E} = 0$, temp $\neq 0$

w/out E-field

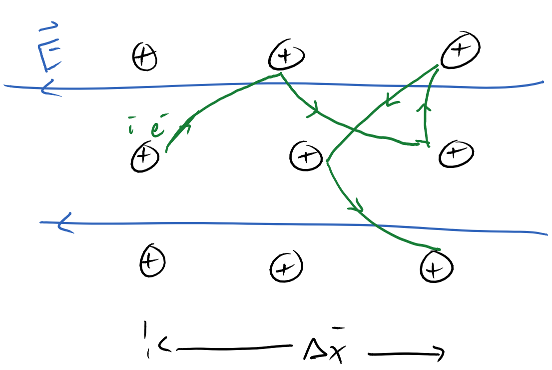


$\Delta\bar{x} = 0$, so $\bar{V}_d = 0$



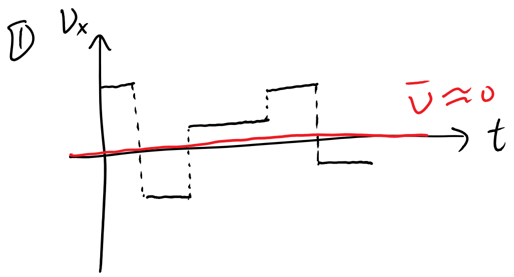
$\Delta\bar{x}=0$, so $\bar{V}_d=0$

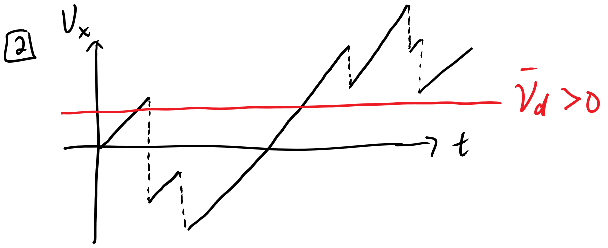
$\boxed{2}$ w/ $\vec{E}\neq 0$, temp $\neq 0$



w/ $\vec{E}\neq 0$, $\Delta\bar{x}\neq 0$

+ $\bar{V}_d\neq 0$





## On the road to Ohm's Law → use macro circuit analysis

$\boxed{1D}$ $\Sigma F = ma \Rightarrow \Sigma F = m\frac{\Delta v}{\Delta t} \Rightarrow \Delta v = \frac{\Sigma F \Delta t}{m} = \frac{eE\Delta t}{m}$ between collisions

Average $\bar{V}_d = \frac{eE}{m}\overline{\Delta t}$ ... combine w/ $I = enAV_d$

average time between collisions

get $$\underbrace{\frac{I}{A}}_{\text{J - current density}} = \underbrace{\frac{ne^2\overline{\Delta t}}{m}}_{\text{material property} \,\equiv\, \sigma \text{ "Conductivity"}} E \Rightarrow \boxed{J = \sigma E}$$

↑ almost Ohm's Law

**Current** $$I = \frac{\Delta q}{\Delta t} = enAV_d$$

**Average** $$\bar{V}_d = \frac{eE}{m}\,\overset{\uparrow}{\overline{\Delta t}}$$

average time Between collisions

$$\underbrace{\frac{I}{A}}_{\text{J - current density}} = \underbrace{\frac{ne^2\overline{\Delta t}}{m}}_{\text{material properties} \,\equiv\, \sigma \text{ conductivity}} E \Rightarrow J = \sigma E$$

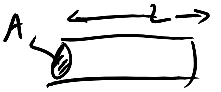
↑ almost Ohm's law.

*Convention: current flow in direction +q would move.

## Definitions

**Resistivity** $\rho \equiv \frac{1}{\sigma}$ material prop (doesn't depend on how much or shape)

**Resistance** $R = \frac{\rho L}{A}$



## Ohm's Law

**Recall** $J = \sigma E$

$$\frac{I}{\cancel{A}} = \frac{L}{R\cancel{A}} E \longrightarrow IR = L\underbrace{E}$$

But.... if E = uniform ... like a wire

$$|\Delta V| = |E| / L$$

Voltage

A    kA

So $\boxed{\Delta \overset{\text{Voltage}}{V} = IR}$

Ohm's Law (macroscopic)

**Charges are not able to move completely freely through a wire, e.g., there is some resistance to their motion. If charges are flowing through a conducting wire, which one of the following statements are true?**

1. **The electric field inside a conductor is zero.**
2. **The electric potential difference between the ends of the wire is zero.**
3. **The electric field is non-zero inside the wire.**
4. **The electric potential difference between the ends of the wire is not zero.**

**You're attempting to make a resistive wire for melting ice off windows that will be connected to a constant voltage source. Which of the following quantities are independent of the length and radius of the wire you use?**

1. **conductivity**
2. **conductance**
3. **free electron density**
4. **resistivity**
5. **resistance**
6. **current through the wire**
7. **voltage difference across the wire**

**If you need to increase the radius of a resistive wire by a factor of 2, but keep the resistance the same, by what factor does the length need to change?**

1. 1/4
2. 1/2
3. 2/3
4. 3/2
5. 2
6. 4
7. 16

**An electric car accelerates for 8.0 s by drawing energy from its 320-V battery pack. During this time the current through the battery is 163 A. How many Coulombs of charge were transferred through the battery?**

1. 40 C
2. 800 C
3. 1300 C
4. 14,400 C
5. 25,600 C

**If the 320 V are multiplied by the Coulombs of charge transferred through the battery during this time, what type of quantity would result?**

1. force
2. electric potential
3. energy
4. electric field
5. current
6. resistance
7. work

## Circuit tools

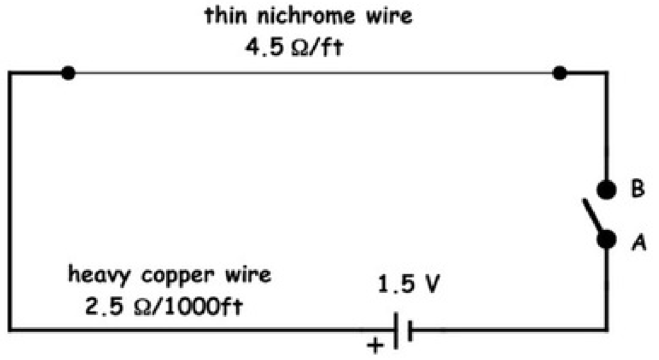
Voltage Loop $\sum \Delta V_{loop} = 0$

Current Junction $\sum I_{in} = \sum I_{out}$

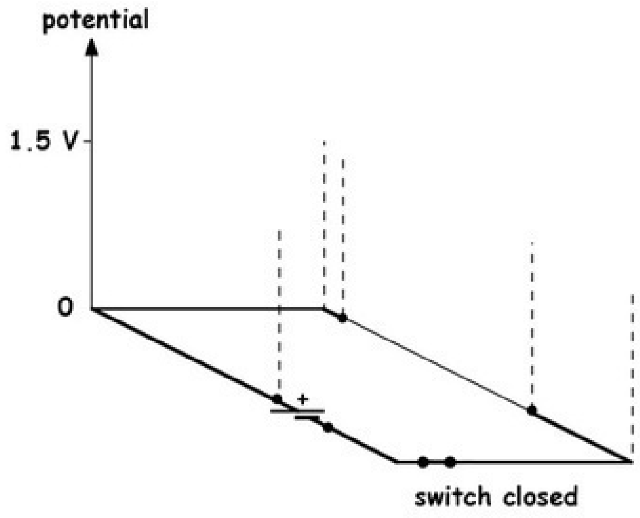
Ohm's Law $\Delta V = I R$

The 3 important eq.

* Assumption $\Delta V_{wire} << \Delta V_{resistor}$

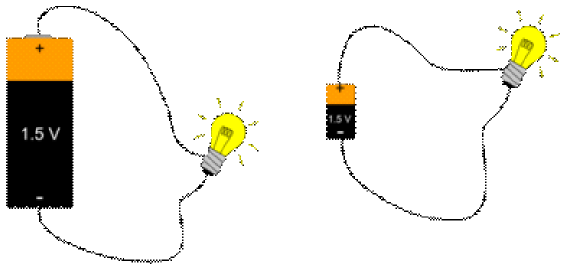


**In the 3D diagram below, sketch the potential around the circuit shown above when the switch is closed.**



What would happen to the light bulb if the battery was replaced by a larger one?

1. The bulb would be brighter.
2. Nothing - it would stay the same brightness.
3. Not enough information.



Power $\left(\frac{\text{Energy}}{\text{time}}\right)$    Recall: $I = \frac{\Delta q}{\Delta t}$ , w/ $\Delta U = \Delta q \Delta V$

$$I = \frac{\Delta U}{\Delta V \Delta t} \Rightarrow \frac{\Delta U}{\Delta t} = I \Delta V$$

$$I = \frac{\Delta U}{\Delta V \Delta t} \Rightarrow \frac{\Delta U}{\Delta t} = I \Delta V$$

$$\boxed{P = I\Delta V = \frac{\Delta V^2}{R} = I^2 R}$$

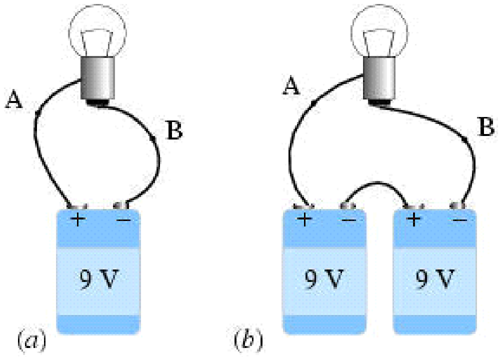
useful when $I = \text{const.}$

useful when $\Delta V = \text{const}$

**A light bulb is connected to a 9-V battery. If a second battery is added in a series as shown in (*b*), how many of the following change?**

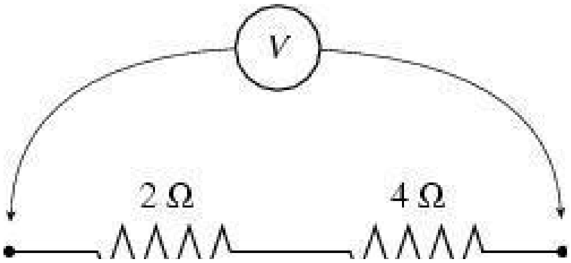
**The current *I* at A, the potential difference $V_{AB}$ between A and B, the resistance *R* of the light bulb.**

1. **All three**
2. **Two**
3. **One**
4. **It depends**

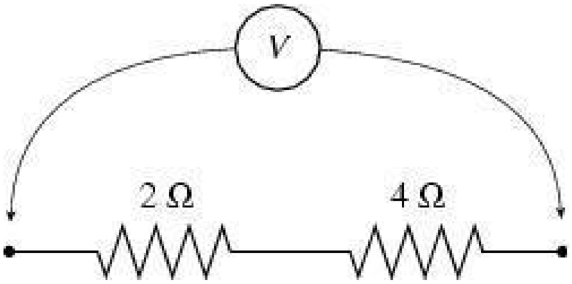


**A constant potential difference *V* is applied across two resistors connected in series as shown. The current through the 2 Ω resistor is 2 A. What is the current through the 4 Ω resistor?**

1. **0 A**
2. **1 A**
3. **2 A**
4. **4 A**
5. **Need to know the potential difference**

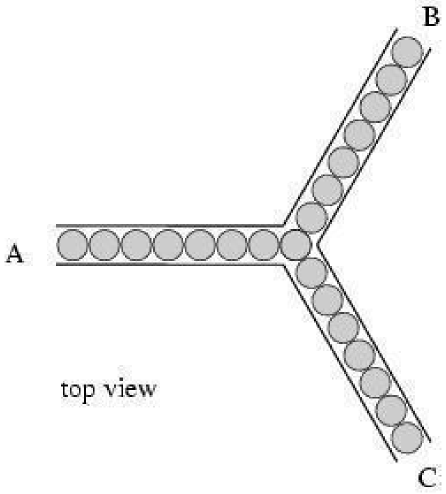


1. **0 A**
2. **1 A**
3. **2 A**
4. **4 A**
5. **Need to know the potential difference**



**Consider the Y-shaped tube shown below. Suppose 2 balls per second are stuffed into the opening at A. The number of balls per second that come out of the tube at B is**

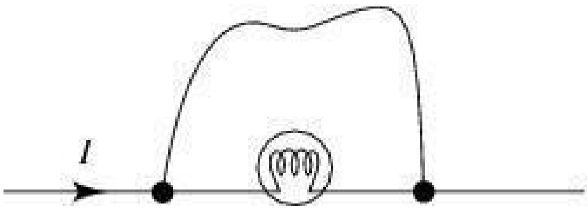
1. **always equal to 2**
2. **always smaller than or equal to 2**
3. **always larger than or equal to 2**
4. **equal to 1**
5. **depends on what happens at C**



**Charge flows through a light bulb. Suppose a wire is connected across the bulb as shown. When the wire is connected,**

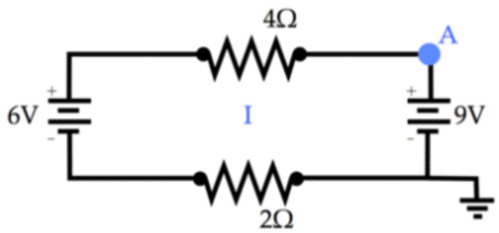
1. **all the charge continues to flow through the bulb.**
2. **half the charge flows through the wire, the other half continues through the bulb.**
3. **all the charge flows through the wire.**
4. **none of the above.**

4. none of the above.

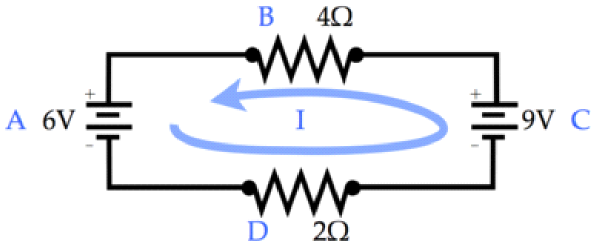


**What is the electrical potential at point A in the circuit?**

1. +9V
2. -9V
3. +6V
4. +3V
5. 0V
6. Can't determine with the information given.

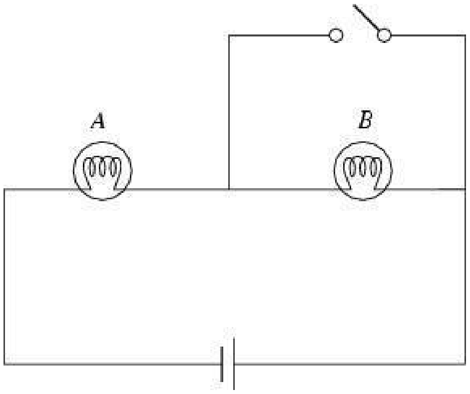


**Consider the following circuit. List the four potential voltage drops (and gains) ΔV encountered going around the loop (don't include units). Express your answer as a sum that should be zero according to Krichhoff's voltage rule: i.e. "1 + 1I + *-12*I + -2I"**

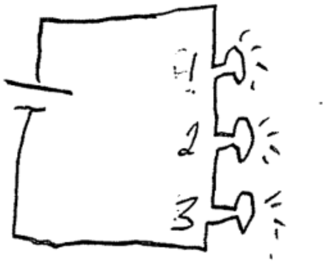


**The circuit below consists of two identical light bulbs burning with equal brightness and a single 12 V battery. When the switch is closed, the brightness of bulb *A***

1. **increases.**
2. **remains unchanged.**
3. **decreases.**



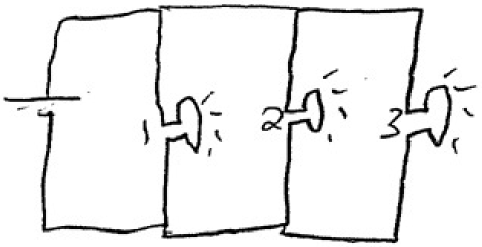
**Consider the following circuit with three light bulbs of equal resistance. Rank the relative brightness of each bulb?**



**If $R_1 < R_2 = R_3$, What will be the relative brightness of each bulb?**

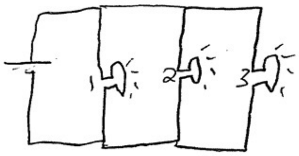
**Consider the following circuit with three light bulbs. Which equation would be most useful for determining the relative brightness of each bulb if the relative resistances are known?**

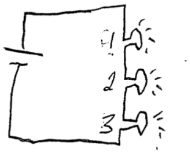
1. **$\Delta V = IR$**
2. **$P = \Delta V^2/R$**
3. **$P = I\Delta V$**
4. **$P = I^2R$**



**Consider the two circuits with three identical light bulbs and an identical voltage source. Which circuit will have brighter bulbs?**

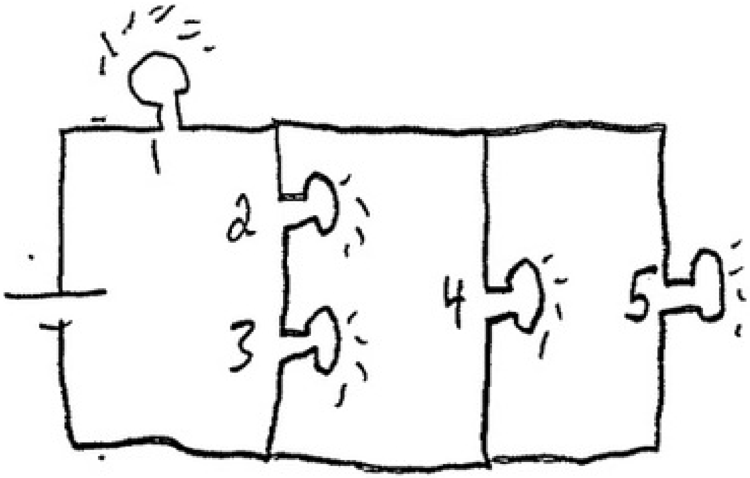
1. **Series**
2. **Parallel**
3. **Equal**





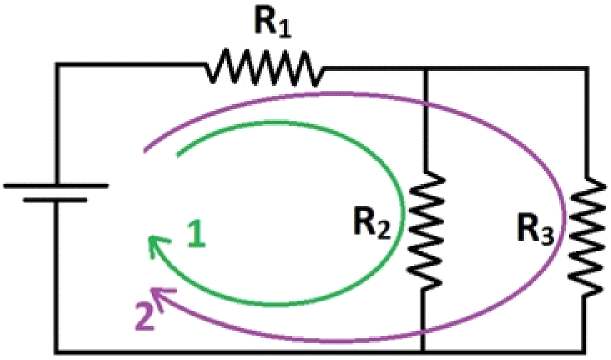
**Consider the following circuit with five equal resistance light bulbs. What is the relative brightness of each bulb?**

1. **1 > 2 = 3 > 4 = 5**
2. **1 < 2 = 3 < 4 = 5**
3. **1 > 2 = 3 = 4 = 5**
4. **1 > 4 = 5 > 2 = 3**
5. **1 < 4 = 5 < 2 = 3**



**Consider the circuit in the figure below where $I_1$ is the current that goes through $R_1$, $I_2$ goes through $R_2$, and $I_3$ goes through $R_3$. What is the voltage drop across resistor $R_1$?**

1. $\Delta V_1 = I_1R_1$
2. $\Delta V_1 = -I_1R_1$
3. $\Delta V_1 = -I_{total}R_1$



**Which equation satisfies Kirchhoff's loop rule when following loop 1?**

1. $+\epsilon + I_1R_1 + I_2R_2 = 0$
2. $+\epsilon - I_1R_1 - I_2R_2 = 0$
3. $+\epsilon - I_1R_1 - I_2R_2 = 0$
4. $+\epsilon + I_1R_1 + I_2R_2 = 0$

**Which equation satisfies Kirchhoff's loop rule when following loop 2?**

1. $+\epsilon + I_1R_1 + I_1R_3 = 0$
2. $+\epsilon - I_1R_1 - I_3R_3 = 0$
3. $+\epsilon - I_1R_1 - I_2R_2 - I_3R_3 = 0$
4. $+\epsilon + I_1R_1 + I_3R_3 = 0$

**Which equation satisfies Kirchhoff's junction rule?**

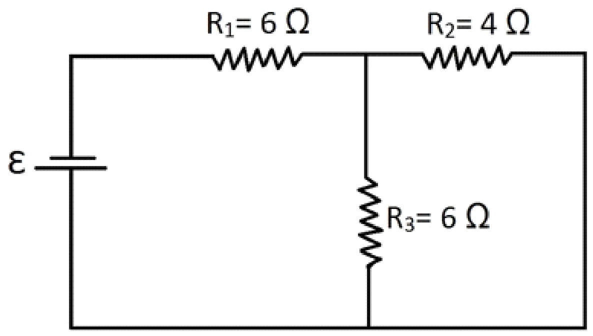
1. $I_1 + I_3 = I_2$
2. $I_1 - I_2 = I_3$
3. $I_1 + I_2 = I_3$
4. $I_2 + I_3 = I_1$
5. $V_1 + V_3 = V_2$
6. $V_1 + V_2 = V_3$

**With the following values, solve for the current in each segment of the circuit:**

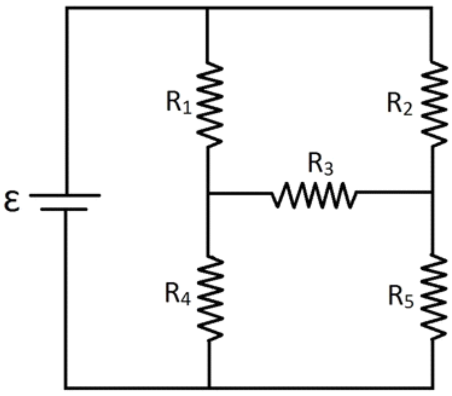
$\varepsilon = 11$ V
$R_1 = 2\ \Omega$
$R_2 = 4\ \Omega$
$R_3 = 6\ \Omega$

1. $I_1 = 2.5$ A, $I_2 = 1.5$ A, $I_3 = 1$ A
2. $I_1 = 1$ A, $I_2 = 1.5$ A, $I_3 = 2.5$ A
3. $I_1 = 2.5$ A, $I_2 = -1.5$ A, $I_3 = 1$ A
4. $I_1 = 2.5$ A, $I_2 = 1.5$ A, $I_3 = -1$ A

**Let $I_1$ be the current through $R_1$, $I_2$ be the current through $R_2$, and $I_3$ be the current through $R_3$. Rank the current that goes through each resistor from least to greatest.**



**Consider the circuit below. What minimum number of loops can be used when applying Kirchhoff's loop rule to completely account for each element?**



**Let $I_1$ be the current through $R_1$, $I_2$ be the current through $R_2$… How many values of I are present in this circuit?**

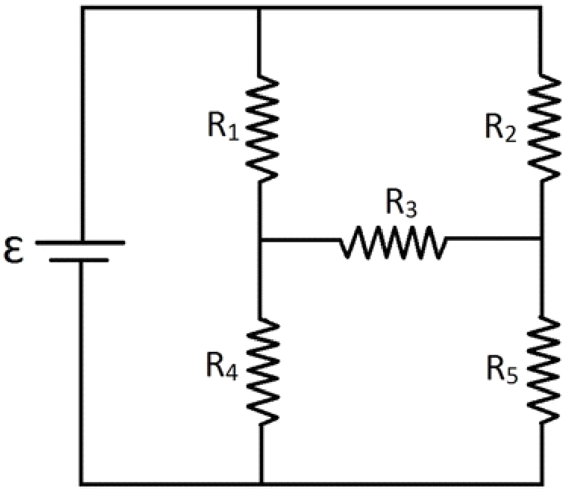
**How many *unique* current junctions are there?**

**Which set of equations is valid for this circuit?**

**a)**

$+\in - \mathbf{I_1 R_1} - \mathbf{I_4 R_4} = 0$

$+\in - \mathbf{I R} - \mathbf{I R} = 0$



**a)**

$+\in -\mathbf{I_1R_1} - \mathbf{I_4R_4} = \mathbf{0}$
$+\in -\mathbf{I_2R_2} - \mathbf{I_5R_5} = \mathbf{0}$
$-\mathbf{I_2R_2} + \mathbf{I_3R_3} + \mathbf{I_1R_1} = \mathbf{0}$
$-\mathbf{I_5R_5} + \mathbf{I_4R_4} - \mathbf{I_3R_3} = \mathbf{0}$

**b)**

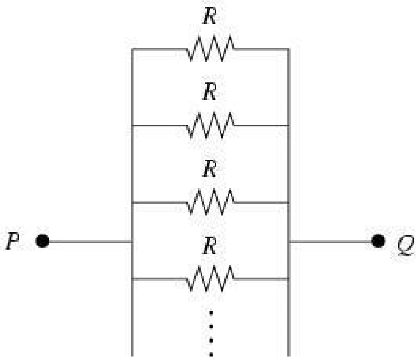
$+\in -\mathbf{I_1R_1} - \mathbf{I_4R_4} = \mathbf{0}$
$+\in -\mathbf{I_2R_2} - \mathbf{I_5R_5} = \mathbf{0}$
$+\in -\mathbf{I_2R_2} + \mathbf{I_3R_3} + \mathbf{I_1R_1} = \mathbf{0}$
$+\in -\mathbf{I_5R_5} + \mathbf{I_4R_4} - \mathbf{I_3R_3} = \mathbf{0}$

**c)**

$-\mathbf{I_1R_1} - \mathbf{I_4R_4} = \mathbf{0}$
$-\mathbf{I_2R_2} - \mathbf{I_5R_5} = \mathbf{0}$
$-\mathbf{I_2R_2} + \mathbf{I_3R_3} + \mathbf{I_1R_1} = \mathbf{0}$
$-\mathbf{I_5R_5} + \mathbf{I_4R_4} - \mathbf{I_3R_3} = \mathbf{0}$

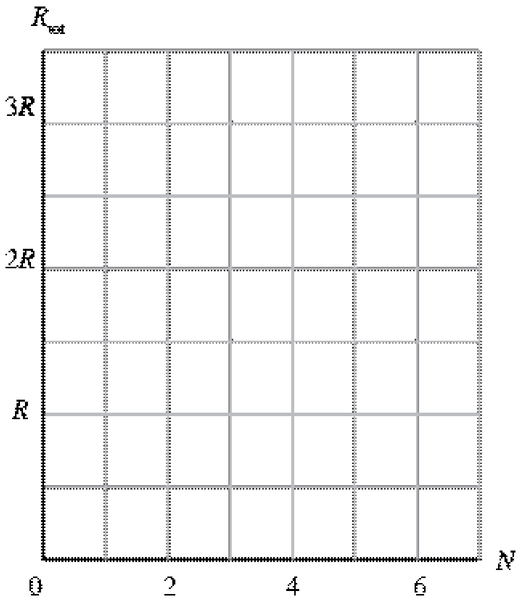
**As more identical resistors, *R*, are added to the parallel circuit shown here, the total resistance between points *P* and *Q***

1. **increases.**
2. **remains the same.**
3. **decreases.**

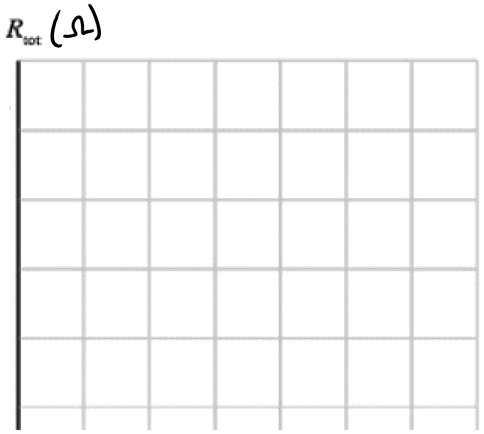


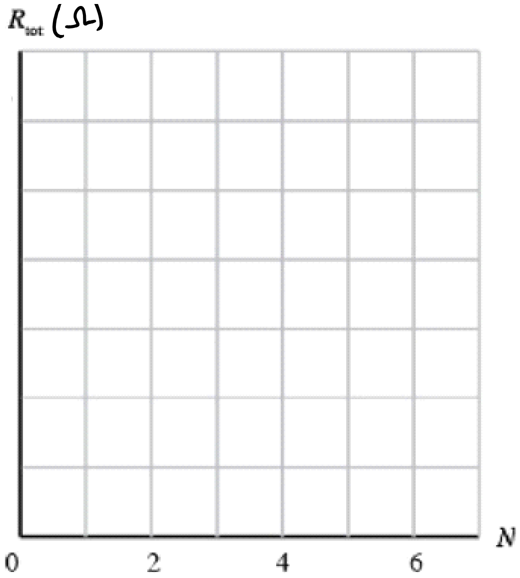
**Draw a graph relating the total resistance $R_{total}$ between points *P* and *Q* versus the total number N of**

**resistors placed between them.**



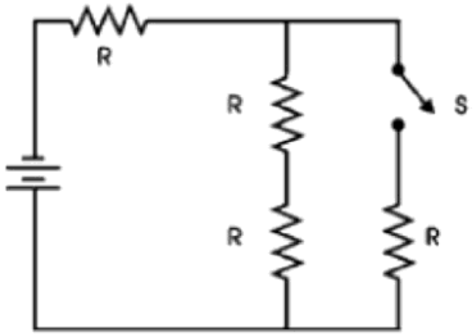
**Draw a graph relating the total resistance as a function of N, where N is the number of equivalent 0.5 Ω resistors added in series.**





**Consider the following circuit. Will more power be dissipated by the circuit when the switch S is open or closed?**

1. **open**
2. **closed**
3. **no difference**



**With the switch S closed, what is the equivalent resistance of the circuit?**

1. **4R**
2. **2R/3**
3. **2R**
4. **5R/3**
5. **5R/6**
6. **R**
7. **R/3**

8. R/2

Use the voltage vs. current curve for the resistors to determine what power is delivered to the circuit by a 12 V battery when the switch is closed?