

## Wave Optics Foundation Stage (WO.2.L1)

### Lecture 1 Young's Double Slit

#### Textbook Chapters (\* Calculus version)

- **BoxSand** :: KC videos ( [Optics](#) )
- **Knight** (College Physics : A strategic approach 3<sup>rd</sup>) ::
- **\*Knight** (Physics for Scientists and Engineers 4<sup>th</sup>) ::
- **Giancoli** (Physics Principles with Applications 7<sup>th</sup>) ::

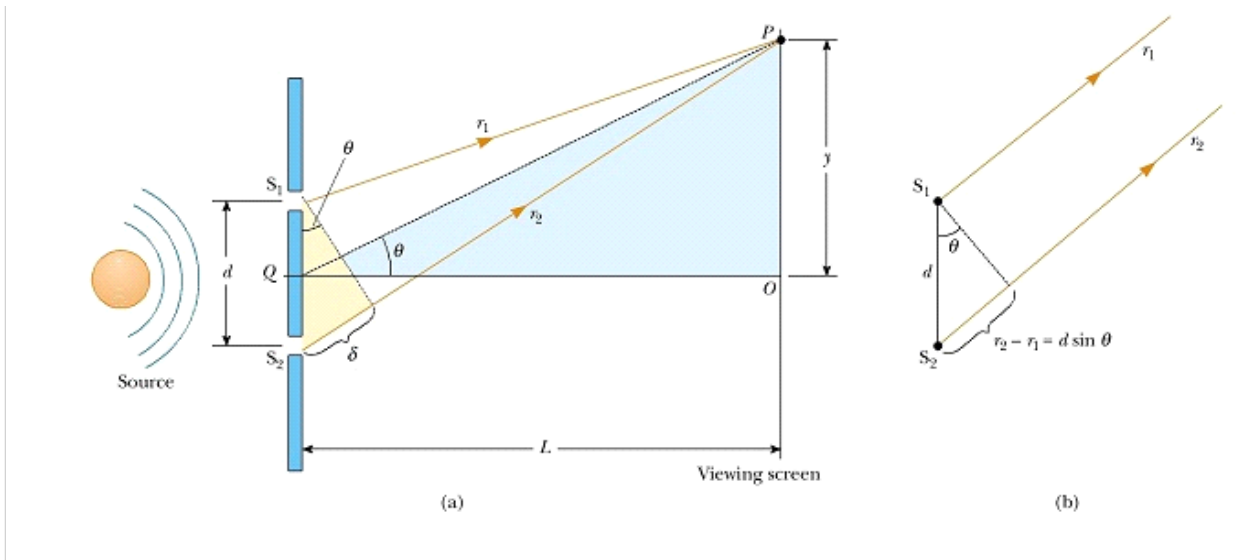
#### Warm up

**WO.2.L1-1:**

**Description:**

**Learning Objectives:** [?] - Can you identify the objectives from the previous lecture, and this lecture, that this question is relevant to?

**Problem Statement:**



**Selected Learning Objectives**

1. Coming soon to a lecture template near you.

**Key Terms**

- Path length difference
- Small Angle Approximation
- Slit Separation
- Viewing Screen
- Coherent
- Constructive
- Destructive

**Key Equations**

$m\lambda = d \sin \theta$	$\tan \theta = \frac{y_m}{L}$
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**Questions**

**Act I: Applications of the Mathematical Model**

**WO.2.L1-2:**

**Problem Statement:** Young's double slit experiment is just one example of two source interference. Which of the following is/are necessary to see a spatial interference pattern on a screen opposite two slits?

1. A single frequency source
2. A coherent source of light
3. Experimental apparatus needs to be in air
4. The source needs to be a light source

Coherent  $\Rightarrow$   $\perp$  Freq.

WO.2.L1-3:

**Problem Statement:** For the double slit experiment, we use a mathematical model  $m\lambda = d \sin \theta$ .

← constructive!

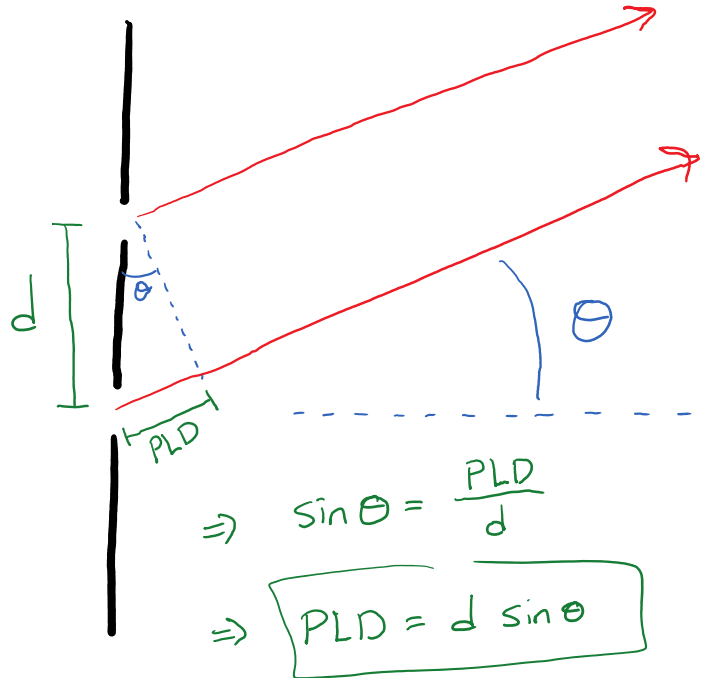
destructive  $\Rightarrow (m + \frac{1}{2})\lambda = d \sin \theta$

Where in this model is the path length difference?

1.  $m$
2.  $\lambda$
3.  $d$
4.  $d \sin \theta$
5.  $\sin \theta$
6.  $\lambda d$

$$\left. \begin{array}{l} \text{PLD} = m\lambda \\ m\lambda = d \sin \theta \end{array} \right\} \text{PLD} = d \sin \theta$$

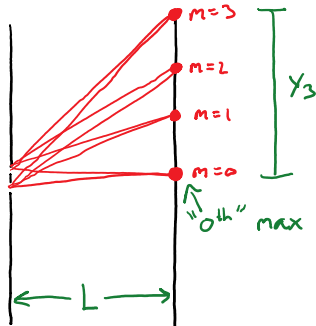
2 diff. methods



WO.2.L1-4:

**Problem Statement:** On observing a double slit experiment, you measure the distance from the slits to the screen to be 1.0 m. You also measure the distance between the central bright spot and the 3rd maximum to be 55 cm. You read on the device that the slits are a distance 4.36  $\mu\text{m}$  apart.

(a) At what angle theta from the central maximum does the 3rd maximum appear?



$$\tan \theta = \frac{y_3}{L}$$

$$\Rightarrow \theta = 28.8^\circ \text{ or } 0.5028 \text{ rad}$$

(b) Find the wavelength of the laser,  $\lambda$ .

$$m\lambda = d \sin \theta$$

$$\lambda = \frac{d}{m} \sin \theta$$

$$\lambda = \frac{d}{m} \sin \theta \quad \leftarrow$$

$$= \frac{4.36 \times 10^{-6}}{3} \sin(0.5628)$$

$$\lambda = 700 \text{ nm}$$

↑ red light!

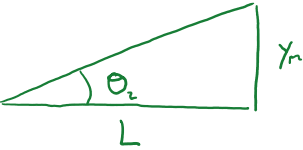
**WO.2.L1-5:**

**Problem Statement:** On observing a double slit experiment, you measure the distance from the slits to the screen to be 1.0 m. You read on the device that the slits are a distance 11.2 μm apart. You also read on the laser that the wavelength it emits is 700 nm. We want to find the distance between the 2nd and 3rd dark fringes on the screen.

(a) What is the angle at which the 2nd dark fringe occurs?

$$\left. \begin{array}{l} (m + \frac{1}{2})\lambda = d \sin \theta \\ \text{1st dark fringe} \Rightarrow m=0 \\ \text{2nd dark fringe} \Rightarrow m=1 \end{array} \right\} \begin{array}{l} \frac{3}{2}\lambda = d \sin \theta_2 \\ \Rightarrow \sin \theta_2 = \frac{3\lambda}{2d} \\ \theta_2 = 5.38^\circ \end{array}$$

(b) What is the distance  $y_2$  for the 2nd dark fringe?



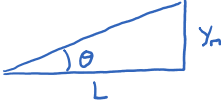
$$\tan \theta_2 = \frac{y_n}{L}$$

$$\Rightarrow y_n = L \tan \theta_2 = 9.4 \text{ cm}$$

(c) What is the distance  $y_3$  for the 3rd dark fringe?

$$(m + \frac{1}{2})\lambda = d \sin \theta \quad m=2$$

$$\sin \theta = \frac{5\lambda}{2d}$$

$$\Rightarrow \theta = 8.99^\circ$$


$$\tan(8.99^\circ) = \frac{y_n}{L}$$

$$\Rightarrow y_n = 15.8 \text{ cm}$$

(c) What is the distance from the 2nd to the 3rd dark fringe on the screen?

$$15.8 \text{ cm} - 9.4 \text{ cm} = 6.4 \text{ cm}$$

Act II: Small Angle Approximation

WO.2.L1-6:

Problem Statement: Work together in teams to complete the following table.

$\theta$ (deg)	$\theta$ (rad)	$\sin \theta$	$\tan \theta$	$\cos \theta$
0.0573	0.001	0.00099...	0.00100...	0.999...
5.73	0.1	0.0998	0.1003	0.995
11.5	0.2	0.1987	0.2027	0.980
17.2	0.3	0.2955	0.3093	0.955
22.9	0.4	0.3894	0.423	0.921
28.6	0.5	0.4794	0.546	0.878
57.3	1.0	0.84	1.56	0.54

(a) Which of the following relationships are true for some portion of the above table?

1.  $\sin \theta = \tan \theta$
2.  $\sin \theta = \cos \theta$
3.  $\cos \theta = \tan \theta$
4.  $\theta = \tan \theta$
5.  $\theta = \sin \theta$
6.  $\theta = \cos \theta$

for "small angles"  $\sin \theta = \tan \theta = \theta!$

(b) For what angles are these equations valid?

$\theta = 0.5 \text{ rad} \Rightarrow$  about 4% error

$\theta = 0.4 \text{ rad} \Rightarrow$  ~ 3% error

$\theta = 0.3 \text{ rad} \Rightarrow$  ~ 1.5% error

(c) Are these equations useful when using degrees? Radians?

$\left. \begin{matrix} \sin \theta = \theta \\ \tan \theta = \theta \end{matrix} \right\}$  only work for  $\theta$  in radians!

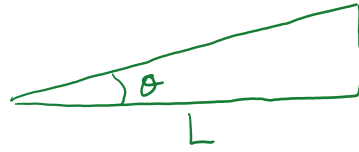
WO.2.L1-7:

Problem Statement: When observing a double slit interference pattern, you note that it is in a regime where the small angle

approximation is a safe approximation to make.

Which of the following relationships can you assume to be true?

1.  $\lambda \gg d$
2.  $\lambda \ll d$
3.  $L \gg y_m$
4.  $L \ll y_m$
5.  $m \gg d$
6.  $m \ll d$



$$y_m \Rightarrow \tan \theta = \frac{y_m}{L}$$

$$\text{if } \theta \ll 1 \Rightarrow \tan \theta \ll 1$$

$$\Rightarrow \frac{y_m}{L} \ll 1 \Rightarrow \boxed{y_m \ll L}$$

$$m\lambda = d \sin \theta$$

$$\theta \ll 1 \Rightarrow \sin \theta \ll 1$$

$$\Rightarrow m \frac{\lambda}{d} \ll 1 \Rightarrow \text{for small } m \quad \boxed{\lambda \ll d}$$

**WO.2.L1-8:**

**Problem Statement:** When observing a double slit interference pattern for 500 nm wavelength light incident on slits with a separation of around 1 mm, you notice that the distance between the 3rd and 4th order bright spots is 5 mm.

If the screen is 15 meters from the slits, what is the precise slit separation?

$$\left. \begin{array}{l} \lambda = 500 \text{ nm} \\ d \sim 1 \text{ mm} \end{array} \right\} d \gg \lambda \Rightarrow \text{small } \theta!$$

$$\frac{m\lambda}{d} = \sin \theta = \theta = \tan \theta = \frac{y_m}{L}$$

$$\Rightarrow y_m = \frac{mL\lambda}{d} \Rightarrow y_4 - y_3 = 5 \text{ mm} = \frac{4L\lambda}{d} - \frac{3L\lambda}{d} = \frac{L\lambda}{d}$$

$$\Rightarrow d = \frac{4L\lambda}{(5 \text{ mm})} = 0.6 \text{ mm}$$

**Act III: Proportional Reasoning and Limits of the Model**

**WO.2.L1-9:**

**Problem Statement:** Suppose the viewing screen in the figure is moved closer to the double slit. What happens to the interference fringes?

1. A plane wave is incident on the double slit

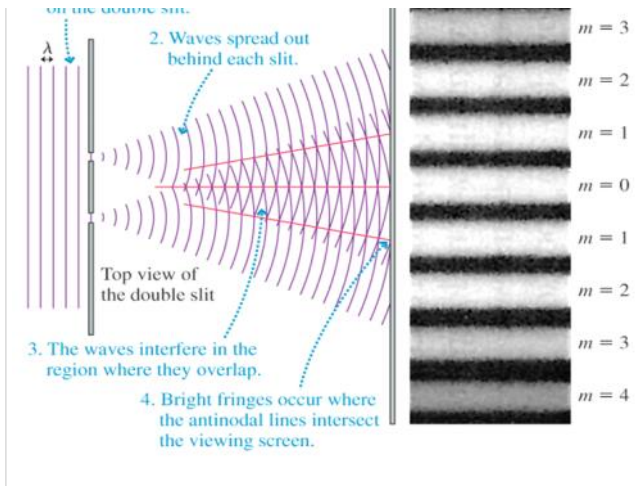


1. They fade out and disappear.
2. They get out of focus.
3. They get brighter and closer together.
4. They get brighter and farther apart.
5. They get brighter but otherwise do not change.

$$\tan \theta = \frac{y_m}{L}$$

$$L \downarrow \Rightarrow y_m \downarrow$$

$\Rightarrow$  closer



Same amount of light in smaller space  $\Rightarrow$  brighter!

#### WO.2.L1-10:

**Problem Statement:** When observing a double slit experiment, you notice that  $y_m$  for some fringe is getting smaller. Which of the following could be causing this?

1. The slit separation is decreasing
2. The slit separation is increasing
3. The wavelength of the light is increasing
4. The wavelength of the light is decreasing
5. The distance between slits and screen is increasing

$$m\lambda = d \sin \theta$$

$$\sin \theta = m \frac{\lambda}{d}$$

$$\theta \downarrow \Rightarrow \lambda \downarrow \text{ or } d \uparrow$$

$$\tan \theta = \frac{y_m}{L}$$

$$y_m \downarrow \Rightarrow L \downarrow \Rightarrow \text{X}$$

$$y_m \downarrow \Rightarrow \theta \downarrow$$

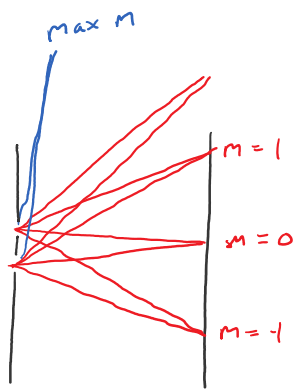
#### WO.2.L1-11:

**Problem Statement:** A particular double slit experiment using 600 nm wavelength light and 50  $\mu\text{m}$  slit separation. How many total fringes would appear on an infinitely large screen?

find max  $m$

max  $m$

$$m\lambda = d \sin \theta \Rightarrow m \quad \text{less when } \theta \rightarrow 90^\circ$$

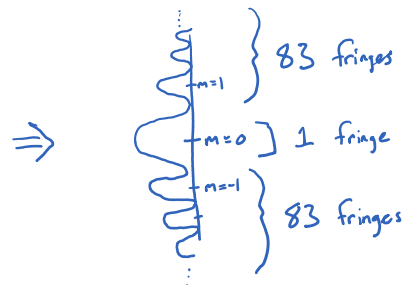


find max m

$$m\lambda = d \sin \theta \Rightarrow m_{\max} \text{ happens when } \theta \rightarrow 90^\circ$$

$$= m_{\max} = \frac{d}{\lambda} \sin(90^\circ) = \frac{50,000 \text{ nm}}{600 \text{ nm}} = 83.33 \dots$$

$\Rightarrow$  83<sup>rd</sup> fringe will appear on screen, 84<sup>th</sup> will not



$$\Rightarrow \boxed{167 \text{ fringes}}$$

**WO.2.L1-12:**

**Problem Statement:** A particular double slit experiment uses a slit separation  $d$ . Is there a limit to the wavelength of the light you can use?

1. Yes,  $\lambda < d$
2. Yes,  $\lambda > d$
3. No, any slit separation will produce an interference pattern

$$\max \theta = 90^\circ$$

$$\Rightarrow m=1 \text{ must have } \theta < 90^\circ$$

$$\Rightarrow m \frac{\lambda}{d} < \sin(90)$$

$$\Rightarrow \lambda < d$$

**WO.2.L1-13:**

**Problem Statement:** In a particular double slit experiment you notice that the small angle approximation is a good approximation. If the wavelength decreases by a factor of 2, what happens to  $y_2$ ?

1. It quadruples
2. It doubles
3. It doesn't change
4. It decreases by a factor of 2
5. It decreases by a factor of 4

$$m\lambda = d \sin \theta$$

$$\Rightarrow m \frac{\lambda}{d} = \sin \theta \approx \theta = \tan \theta = \frac{y_m}{L}$$

$$\Rightarrow 2 \frac{\lambda}{d} = \frac{\gamma_2}{L}$$

$$\lambda d_2 \Rightarrow \gamma_2 d_2$$

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### Conceptual questions for discussion

1. Coming soon to a lecture template near you.
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### Hints

**WO.2.L1-1:** No hints.

**WO.2.L1-2:** No hints.

**WO.2.L1-3:** No hints.

**WO.2.L1-4:** No hints.