Electromagnetic induction

Select LEARNING OBJECTIVES:

- Calculate magnetic flux.
- Apply Faraday's law to find induced emf.
- Combine Faraday's law with Ohm's law to find the induced current. (Not direction).

TEXTBOOK CHAPTERS:

Boxsand :: magnetic induction

WARM UP: Motional emf questions?

As discussed previously there are three unique ways to create a current with electromagnetism. The first method we covered was referred to as motional emf, because the conductor is actually moving with some velocity though a constant uniform magnetic field. Our physics tools in our tool belt allowed us to analyze motional emf; basically we were able to rely on using a magnetic force on the free electrons in the conductor since they had a velocity though the magnetic field. However, in the other two methods of creating current via electromagnetism, the conductor has no velocity. Thus the free electrons have no macroscopic velocity which means that there is no magnetic force on them. Because there is no magnetic force on the free electrons, we need a new model to describe how current is created for the other two experiments.

The new model is Faraday's law, which essentially says that if the magnetic flux changes with respect to time in a given region of space, then a curly electric field is created which can push charges around. The previous sentence is loaded with information. First, we need to introduce the definition of magnetic flux. Then we can explore how the magnetic flux can change as a function of time. Finally, we can put it all together and study how current can be created by this changing magnetic flux.

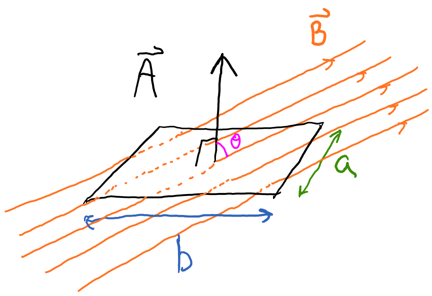
Magnetic flux Φ^B

Faraday's law uses a new quantity called magnetic flux to model how current can be induced. In this section we will explore what magnetic flux is and how to calculate it. In general, flux is the amount of stuff that crosses through a surface area. Some examples of "stuff" are: electric field, magnetic field, water, etc.. As you can imagine, magnetic flux is the amount of magnetic field that crosses through a surface area.

Before we talk about magnetic flux, I like to think about "rain" flux to get a feeling for what flux is. Imagine a hula hoop held out horizontally with respect to the ground. If rain is falling vertically, then a large volume of water is crossing the surface area bounded by the hula hoop; this would be an example of a large value of rain flux. As you begin to rotate the hula hoop so that is becomes more vertical, less and less water passes through the area bounded by the hoop (i.e. less rain flux) until is completely vertical and no water passes through the hoop (no rain flux). CAUTION: This rain example is a metaphor when compared to magnetic flux; rain is actually moving through a hula hoop, but magnetic flux is not moving, flowing, etc..

As seen in the rain flux example, the amount of flux depends on the area of the hoop, the angle of the hoop with respect to the rain, and the intensity of the rain. In terms of magnetic flux, the amount of flux will depend on the area of an arbitrary shape, the angle of the shape with respect to the magnetic field, and the strength of the magnetic field. This functional dependence of flux invites us to use a dot product to mathematically define flux. Below is the mathematical model for magnetic flux with a uniform magnetic field.

Magnetic flux for uniform magnetic field



TYPE of FLUX "MAGNETIC"

DOT PRODUCT

$$\boxed{\Phi^B = \vec{B} \cdot \vec{A}}$$

FLUX

MAGNETIC FIELD VECTOR ON THE SURFACE

AREA VECTOR

*NOTE: Φ^B IS A SCALAR

* SI UNITS $T m^2 \equiv Wb$ "WEBBER"

Note that flux is a scalar. Also, when using SI units (tesla for magnetic field and meters squared for area) the unit is referred to as a webber, "Wb".

Dot product review

- A vector operation that takes in two vectors, and returns a scalar.
- Answers the question, "how parallel are two vectors relative to each other?"

$$\Phi^B = \vec{B} \cdot \vec{A} \qquad \Phi^B = |\vec{B}||\vec{A}|\cos(\theta)$$

DOT PRODUCT (VECTOR OPERATION)

SMALLEST ANGLE BETWEEN $\vec{E}$ AND $\vec{A}$ WHEN PLACED TAIL-TO-TAIL

THIS IS ONLY THE DEFINITION OF FLUX FOR A CONSTANT MAGNETIC FIELD

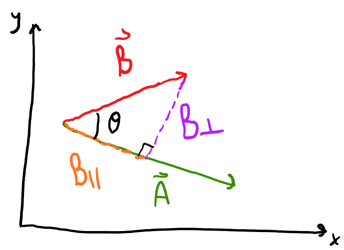
ALSO

$\vec{B} \cdot \vec{A} = B_{\parallel} |\vec{A}|$ WHERE $B_{\parallel} = |\vec{B}| \cos(\theta)$

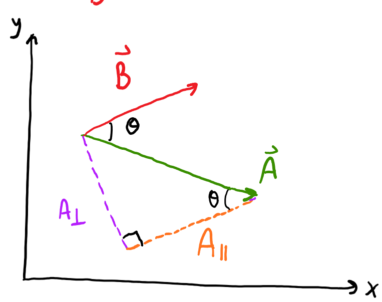
$\vec{B} \cdot \vec{A} = |\vec{B}| A_{\parallel}$ WHERE $A_{\parallel} = |\vec{A}| \cos(\theta)$

PICTORIAL

$\vec{B} \cdot \vec{A} = B_{\parallel} |\vec{A}|$



$\vec{B} \cdot \vec{A} = |\vec{B}| A_{\parallel}$

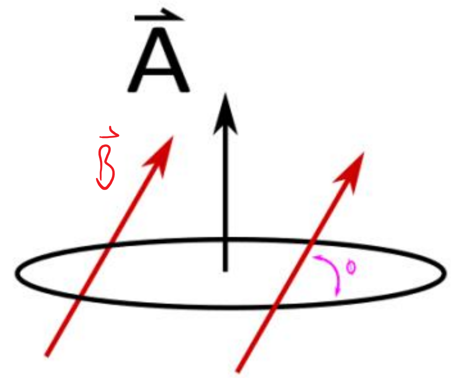


PRACTICE: A circular loop with area 2 m^2, has a uniform magnetic field of 0.02 T going through it at an angle of ϕ=60° with respect to the plane of the loop. What is the magnetic flux through the loop?

1. 0 Wb
2. 0.02 Wb
3. -0.02 Wb
4. 0.035 Wb

$\vec{A}$

2. 0.02 Wb
3. -0.02 Wb
4. 0.035 Wb
5. -0.035 Wb



Faraday's law

Now is a good time to review the three experiments that represent the three different ways to create an induced current via electromagnetism. The experiments are found in the motional emf lecture. As previous stated, the first experiment (i.e. motional emf) prosed no issues; we were able to analyze the experiment using magnetic forces on free charges in uniform magnetic fields. However a magnetic force analysis fails to explain experiments 2 and 3 because there is no magnetic force on the free charges because the conductor is not moving. Michael Faraday introduced a model to account for experiments 2 and 3, and by the beauty of nature Faraday's law also can be used for experiment 1. Faraday's model is known as Faraday's law which states that a changing magnetic flux with respect to time creates an induced curly electric field. It is then this curly electric field that can push free charges around. Remember that electric fields and electric potentials are related, thus we will think in terms of potential (i.e. emf) for now. So let's restate Faraday's law in terms of potential: a changing magnetic flux with respect to time induces an emf which can push free charges around. We can then apply Ohm's law to find the induced current (emf = $I_{induced}$ R). The mathematical representation for Ohm's law is shown below:

MAGNETIC FLUX THROUGH A SINGLE LOOP

$$\boxed{|\mathcal{E}| = \left|-N\frac{\Delta\Phi_B}{\Delta t}\right|}$$

* FARADAY'S LAW

INDUCED EMF

OF LOOPS w/ SAME DIMENSIONS

RATE OF CHANGE OF MAGNETIC FLUX

Hopefully the first thing you notice is that the induced emf can be found by plotting magnetic flux vs time and taking the negative slope. It may seem odd that there is a negative sign included when we are looking only at the magnitude of the emf in the expression above. However, the negative sign plays an important role in determining the direction of the induced current. For now we will not worry about the negative sign or direction of induced current, we will tackle these issues in the next lecture "Lenz's law".

Remember that magnetic flux depends on the magnitude of the area, the magnitude of the magnetic field, and the angle between the area vector and the magnetic field. Thus the magnetic flux can change with respect to time if any one, or combination, of the above three quantities changes with respect to time. The general procedure for finding the induced emf is as follows: at an instant of time (e.g. take a snapshot) find the magnetic flux in variable form, then identify which variables are constant with respect to time and which are changing with respect to time. Once the above two steps are done, it's a matter of algebra to solve for the induced emf. Below is a summary of the algebra for various scenarios where different quantities are constant.

FARADY'S LAW SUMMARY

$$|\bar{\mathcal{E}}| = \left|-N\frac{\Delta\Phi_B}{\Delta t}\right| = \left|-N\frac{\Delta(\vec{B}\cdot\vec{A})}{\Delta t}\right|$$

* IF $|\vec{B}|$ AND θ ARE CONSTANT ; $|\vec{A}|(t)$

$$|\bar{\mathcal{E}}| = N\left|-|\vec{B}|\cos\theta\,\frac{\Delta A(t)}{\Delta t}\right|$$

* IF $|\vec{A}|$ AND θ ARE CONSTANT ; $|\vec{B}|(t)$

$$|\bar{\mathcal{E}}| = N\left|-|\vec{A}|\cos\theta\,\frac{\Delta B(t)}{\Delta t}\right|$$

* IF $|\vec{A}|$ AND $|\vec{B}|$ ARE CONSTANT ; $\theta(t)$

$$|\bar{\mathcal{E}}| = N\left|-|\vec{A}||\vec{B}|\,\frac{\Delta\cos(\theta(t))}{\Delta t}\right|$$

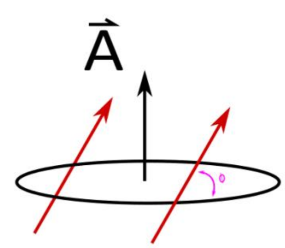
$$|\bar{\mathcal{E}}| = N\left|-|\vec{A}||\vec{B}|\,\frac{\cos\theta_f - \cos\theta_i}{\Delta t}\right|$$

* IF MORE THAN 1 NOT CONSTANT W.R.T. TIME

$$|\bar{\mathcal{E}}| = N\left|-\frac{B_f A_f\cos\theta_f - B_i A_i\cos\theta_i}{\Delta t}\right|$$

PRACTICE: A circular loop with area 2 m^2, has a uniform magnetic field of 0.02 T going through it at an angle of ϕ=60° with respect to the plane of the loop. Which of the following actions will result in an induced current in the loop?

1. Rotate the coil.
2. Decrease the magnetic field.
3. Increase the magnetic field.
4. Nothing, current is already induced in the coil.
5. Move the coil forwards and backwards.
6. Increase the radius of the loop.
7. Decrease the radius of the loop.



PRACTICE: The magnetic flux passing through a coil of wire varies as shown in the plot below. During which time interval(s) will an induced current be present in the coil?

1. t_0 - t_1
2. t_1 - t_2
3. t_2 - t_3
4. t_3 - t_4