Magnetic fields

Select LEARNING OBJECTIVES:

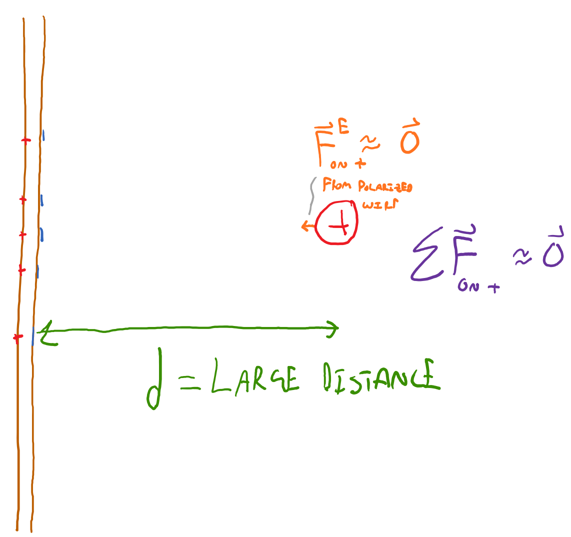
- Calculate the magnitude and direction of the magnetic field due to point charge moving with a velocity.
- Calculate the magnitude and direction of the magnetic field due to a current carrying wire.
- Use the most convenient right hand rule to find directions of magnetic fields.

TEXTBOOK CHAPTERS:

Boxsand :: [Magnetic fields]

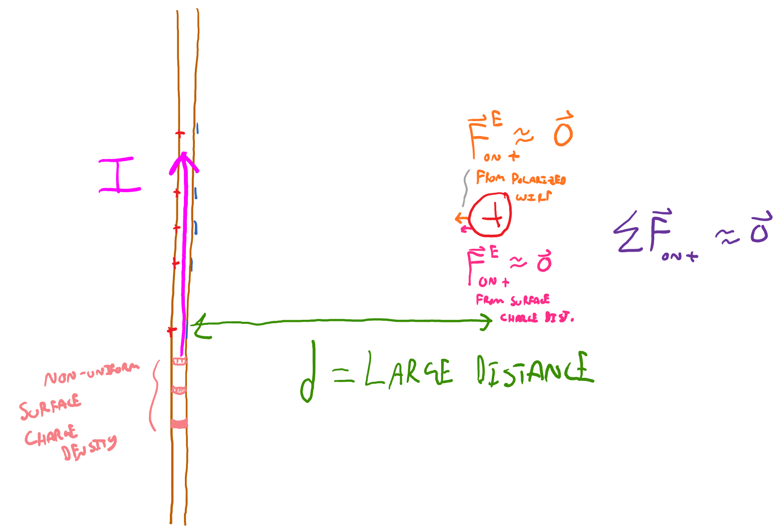
WARM UP: Micro model of current questions?

Consider a very long segment of straight copper wire which is initially net neutral with no current flowing through it. Now imagine placing a positive point charge next to the long copper wire, but not touching it. When done in the depths of space, where all other interactions can be ignored, very little happens to point charge. There is a slight attraction due to the point charge polarizing the wire, then the point charge interacting with the polarized wire's electric field, but this effect is very small, and we can place the point charge far enough away from the wire so that we can safely ignore this interaction. Basically, there are negligible forces acting on the positive point charge, thus the acceleration is also negligible. Since the acceleration due to the electric force is so small, we can assume that over long periods of time the change in position of the positive point particle is very small, thus it is basically still the same distance d away from the wire. Below is a figure of the set up so far.



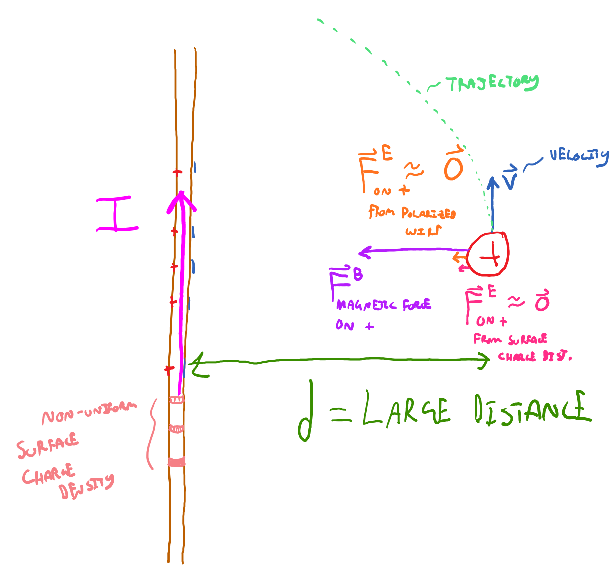
The image above is not to scale, imagine the charged point particle as a very small dot and the wire much longer in length. Newton's third law tells us that there is the same negligible force but in the opposite direction on the wire. Since we are assuming the mass of the copper wire is much larger than the positive point charge, the acceleration of the wire would even be less than that of the charge. We are already assuming the charge acceleration is so small it's negligible, then the wire's acceleration is even more unnoticeable.

Now what if we send current through the wire? To send a current through the wire, we must establish a non-uniform surface charge distribution on the wire. This non-uniform surface distribution may result in another very small electric force on the positive charge. However, since the amount of surface charge required to establish a current is very small, we will also ignore the effects of this addition source of interaction between the wire and the positive point charge. Below is another updated set up of the experiment so far. Again, nothing is to scale.



It should also be noted that there really isn't two separate forces on the positive point charge, there would be only one which is due to two separate phenomena affecting the charge on the wire. Thus the single force would just the electric force due to the charge distribution on the wire. However, since the distribution is determined by two separate phenomena (polarization, and non-uniform surface charge density) the electric force could be thought of as a superposition of the forces from each phenomena. The important part is, you would notice that when you sent current through the wire, there is still a negligible net force on the positive point charge.

So far we have left assumed the positive charge was basically at rest with respect to the wire. If you happened to give the positive particle some velocity, let's say parallel to the wire, you would notice that the positive particle would begin to accelerate towards the wire. This noticeable acceleration suggests that there must be a new force acting on the positive charge when it is moving relative to the wire. We already accounted for the electric forces acting on the positive charge and determined they were negligible, so this new force is truly a "new" force and we call it the magnetic force. Below is another image of this new set up.



Similar experiments can be done in the lab with other types of set ups that suggest we must introduce this new magnetic force into our inventory of known forces.

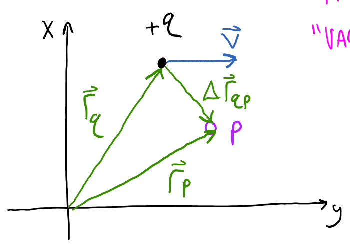
Remember when we first discovered there was a new force, the electric force? After its discovery, we spent a fair amount of time looking at the features of that new force. After we were familiar with the electric force, we introduce the idea of the electric field; charges create electric fields and a different charge placed in that electric field experiences an electric force. Our most advanced and current models used to describe the universe rely on field models. Basically, physicists and scientists think in terms of fields. Because of this, we are going to hold off on talking about the features of the magnetic force, and instead first talk about the magnetic field first.

The philosophy of the magnetic field is the same as the electric field model. That is to say, something will create a magnetic field, and then something else when placed in that magnetic field will experience a magnetic force. The question then becomes, what creates magnetic fields? It turns out we can create magnetic fields a few different ways, and this lecture is all about exploring those different ways.

Magnetic field from charged point particle

One way in which a magnetic field is created is when a charged point particle has a velocity. The functional form for the magnetic field created is given by Biot-Savart's law as shown below.

- Biot-Savart law - Empirically derived mathematical representation for how a point charge creates a magnetic field.



THE MAGNETIC FIELD DEPENDS ON THE PROPERTIES OF SPACE "VACUUM PERMEABILITY"

POINT PARTICLES VELOCITY

CROSS PRODUCT

$$\boxed{\vec{B}_p = \frac{\mu_0}{4\pi}\, q\, \frac{\vec{v} \times \Delta\hat{r}_{qp}}{|\Delta\vec{r}_{qp}|^2}}$$

MAGNETIC FIELD AT POINT P

DIRECTION. POINTS FROM q TO p.

POINT PARTICLE'S CHARGE

DISTANCE BETWEEN POINT PARTICLE AND THE LOCATION OF INTEREST "p"

The first thing you should notice is that the magnetic field is a vector. Perhaps the second thing that might pop out is the cross product in the numerator, but let's ignore that for a moment. The third, and perhaps the most interesting, feature of the magnetic field created by a charged point particle with velocity is that the strength of the magnetic field decreases by $1/\mathbf{r}^2$ as you move away from the point charge. Remember that the strength of the electric field also decreases by $1/\mathbf{r}^2$ as you move away from a point charge. Also, the gravitational field decreases by $1/\mathbf{r}^2$ as you move away from a spherical mass. Now might be a good time to sit back and reflect on just how beautiful the universe is, how can it be that all these seemingly different fields have the exact same $1/\mathbf{r}^2$ dependence? Perhaps there is some underlying connection between all of these fields?

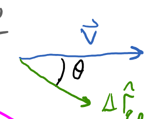
Back on track, your next question might be, how do we deal with the cross product found in this model? It turns out the same way we did before when dealing with torque. The magnitude of the magnetic field is found by taking the magnitude of all the constants and multiplying them with the magnitude of the velocity vector, multiplying again by the magnitude of the unit vector, and finally multiplying by the sine of the smallest angle between the velocity and unit vector when they are tail to tail. Below is the mathematical form of what was stated in the previous sentence.

* UNITS

$$\vec{B} \rightarrow \frac{N}{Am} \equiv \text{TESLA} \equiv T$$

$$\mu_0 = 4\pi \times 10^{-7}\ \frac{Tm}{A}$$

VECTOR OPERATION



SMALLEST ANGLE BETWEEN $\vec{v}$ AND $\Delta\hat{r}_{qp}$

$$\boxed{|\vec{B}_p| = \frac{\mu_0}{4\pi}\,|q|\,\frac{|\vec{v}|\sin\theta}{|\Delta\vec{r}_{qp}|^2}}$$

* W/ MAGNITUDE OF $\vec{B}$, USE RIGHT HAND RULE TO FIND DIRECTION

$$\boxed{|\vec{B}_p| = \frac{\mu_0}{4\pi}|q|\frac{|\vec{v}|\sin\theta}{|\Delta\vec{r}_{qp}|^2}}$$

* W/ MAGNITUDE OF $\vec{B}$, USE RIGHT HAND RULE TO FIND DIRECTION

Convince yourself that you can go from the vector from of Biot-Savart's law to this magnitude form. Unlike torque, we are not going to ignore the vector nature of the magnetic field. Thus we must now introduce how you can find the direction of a vector when it is related to the cross product between two other vectors. The method of finding the direction is known as "the right hand rule for cross products".

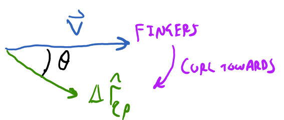
RHR EXAMPLE:

"1ST VECTOR" "2ND VECTOR"

$$\vec{B}_p = \frac{\mu_0}{4\pi} q \frac{\vec{v} \times \Delta\hat{r}_{qp}}{|\Delta\vec{r}_{qp}|^2}$$

RESULTANT VECTOR

VECTOR OPERATION



RHR FOR CROSS PRODUCTS

1. FINGERS IN DIRECTION OF 1ST VECTOR $(\vec{v})$
2. CURL FINGERS TOWARDS 2ND VECTOR $(\Delta\hat{r}_{qp})$
3. THUMB POINTS IN DIRECTION OF RESULTANT $(\vec{B})$

* RESULTANT IS $\perp$ TO BOTH 1ST + 2ND VECTORS

* CONVENTION

⊗ INTO PAGE

⊙ OUT OF PAGE

Be careful, the right hand rule gives us the direction of the resultant vector assuming that all the scalar constants in the expression are positive. In Biot-Savart's law μ_o, 4 π, and $|\Delta\vec{r}_{qp}|$ are all positive, but the charge **q** can be positive or negative. My recommendation is to ignore the sign of the charge when using the right hand rule for Biot-Savart's law; after you find the direction of the magnetic field, then go back and if the charge was positive your direction is correct, if the charge was negative then the magnetic field is just in the opposite direction that you found.

Note that this right hand rule is valid for all cross products. Let's revisit torque as an example. Below is the definition of torque as introduced during our rotational mechanics studies.

"1ST VECTOR"

$$\vec{\tau}_o = \vec{r}_o \times \vec{F}^A$$

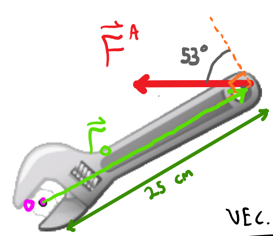
"2ND VECTOR"

RESULTANT VECTOR

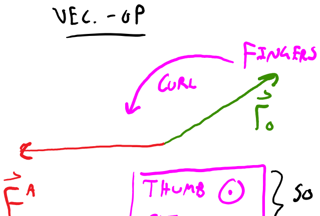
$$|\vec{\tau}_o| = |\vec{r}_o||\vec{F}^A|\sin\theta$$

* USE R.H.R. FOR DIRECTION

EXAMPLE



VEC. - OP



THUMB ⊙ OUT OF PAGE

SO $\vec{\tau}$ IS OUT OF PAGE

PRACTICE: Consider a point particle with positive charge and velocity shown below. What is the direction of